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Form Factor and Mean-Square Radius for the Vector Current of Composite System of Two Fermions in the Relativistic Quasipotential Approach

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The new expressions for the form factor components and mean-square radius of the vector current of composite system of two relativistic fermions with arbitrary masses are obtained. The pseudoscalar, vector and pseudovector composite systems were considered. For them is received identity, which installs the dependency between masses and quarks spin, forming composite systems. Values of the mean-square radius of the ground-state s-wave level of pseudoscalar $\pi^{\pm}$-, $K^{\pm}$- and K_0-mesons with the Coulomb interaction are calculated. The analysis is performed and the dependency of form factor behavior and mean-square radius concerning the differences of quarks masses are installed. Consideration is conducted within the framework of relativistic quasipotential approach on the basis of covariant Hamiltonian formulation of quantum field theory by transition to the three-dimensional relativistic configurational representation in the case of two relativistic spinor particles with arbitrary masses.

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1. Introduction

The different approaches can be used for description of the form factor behavior of the composite system [1–5]. The using of three-dimensional relativistic covariant two-particle quasipotential (RQP) equation of Logunov-Tavkhelidze [6] for description of the form factors of composite systems were executed in [7, 8]. However, use of the equation Logunov-Tavkhelidze for wave function in the momentum representation has not allowed to research the behavior of the form factor in broad interval of importances of the momentum transfer of the relativistic two-particle bound system. The other model was considered in [9, 10]. This model has used the RQP approach [11, 12] on the basis of covariant Hamiltonian formulation of quantum field theory [13] in which the contribution of small distances in the proton form factor takes into account by means of transition to the three-dimensional relativistic configurational representation (**r**-representation) in the case of two relativistic spinless particles with equal masses m [14]. In the RQP approach [11] for a bound system of two relativistic spinless particles of arbitrary masses developed in [15, 16], new covariant expressions of the components of elastic form factor for the cases of a scalar and vector currents as functions of the invariant variable $\Delta^2_{\mathcal{P},\mathcal{Q}}$, which there is the square of the momentum-transfer vector in the Lobachevsky space, have been found in [17, 18].

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The aim of present study, considered as continuation in [17–20], is to obtain the new expressions for the form factor and mean-square radius of the vector current of composite system of two relativistic fermions with arbitrary masses. Consideration is conducted within the framework of RQP approach [11, 12] on the basis of covariant Hamiltonian formulation of quantum field theory [13] by transition to the **r**-representation in the case of two relativistic spin particles with arbitrary masses m_1 и m_2 [15, 16].

2. Equation for the wave function

Within the framework of RQP approach [11, 12] for spherically symmetric potentials the equation in the **r**-representation [15, 16] for the radial wave function of the composite system with relative orbital angular momentum $\ell \geq 0$, $\varphi_\ell(r,\chi')$, consisting of two relativistic fermions with arbitrary masses m_1, m_2 and spin 1/2, has the form [21][1)]

$$\left(\hat{H}^{\rm rad}_{0,\ell} - \cosh\chi'\right)\varphi_\ell(r,\chi') = -V(r;M_Q)\hat{A}\left(\hat{H}^{\rm rad}_{0,\ell}\right)\varphi_\ell(r,\chi'). \tag{1}$$

Here $M_Q^2 = s_q = Q^2 = (q_1+q_2)^2 = Q_0^2 - \mathcal{Q}^2$, where $q_i, i=1,2$ are 4-momentas of composite particle, the operator

$$\hat{H}^{\rm rad}_{0,\ell} = \cosh\left(i\lambda'\frac{d}{dr}\right) + \frac{\lambda'^2\ell(\ell+1)}{2r(r+i\lambda')}\exp\left(i\lambda'\frac{d}{dr}\right) \tag{2}$$

is the radial part of free Hamiltonian operator (we use the system of units where $\hbar = c = 1$)

$$\hat{H}_0 = 2m'\left[\cosh\left(i\lambda'\frac{\partial}{\partial r}\right) + \frac{i\lambda'}{r}\sinh\left(i\lambda'\frac{\partial}{\partial r}\right) - \frac{\lambda'^2}{2r^2}\Delta_{\theta,\varphi}\exp\left(i\lambda'\frac{\partial}{\partial r}\right)\right], \tag{3}$$

where $\Delta_{\theta,\varphi}$ is its the angular part, $\lambda' = 1/m'$ is the Compton wavelength associated with an effective relativistic particle playing the role of the two-particle system [15, 16], having the mass $m' = \sqrt{m_1 m_2}$, the relative momentum $\mathbf{\Delta}_{q',m'\lambda_Q}$ and carrying the total energy of particles M_Q, which is proportional to the energy $\Delta^0_{q',m'\lambda_Q}$ of one effective relativistic particle of mass m', and the rapidity χ' is used to parametrize the momentum and energy as

$$\mathbf{\Delta}_{q',m'\lambda_Q} = m'\sinh\chi'\mathbf{n}_{\Delta_{q',m'\lambda_Q}}, |\mathbf{n}_{\Delta_{q',m'\lambda_Q}}| = 1, M_Q = 2m'g'\Delta^0_{q',m'\lambda_Q}, \tag{4}$$
$$\Delta^0_{q',m'\lambda_Q} = m'\cosh\chi',$$

where factor g' gives by expression

$$g' = \frac{m'}{2\mu} = \frac{m_1+m_2}{2\sqrt{m_1 m_2}}, \tag{5}$$

and $\mu = m_1 m_2/(m_1+m_2)$ is the reduced mass of two particles of arbitrary masses; $\Delta^0_{q',m'\lambda_Q}$ and $\mathbf{\Delta}_{q',m'\lambda_Q}$ are, respectively, the time and spatial components of the 4-vector $\Lambda^{-1}_{\lambda_Q}q' = \Delta_{q',m'\lambda_Q}$ from the Lobachevsky space associated by the pure Lorentz transformations $\Lambda^{-1}_{\lambda_Q}$

[1)] Similar equation for the case of two spinor particles with the equal masses was received in [22], and earlier in [23], but at a different definition of the wave function and quasipotential.

with the velocity 4-vector of the composite particle $\lambda_{\mathcal{Q}} = (\lambda^0_{\mathcal{Q}}; \boldsymbol{\lambda}_{\mathcal{Q}}) = \mathcal{Q}/\sqrt{\mathcal{Q}^2}$ by the formulas

$$\Lambda^{-1}_{\lambda_{\mathcal{Q}}}\mathbf{q}' = \boldsymbol{\Delta}_{q',m'\lambda_{\mathcal{Q}}} = \mathbf{q}'(-)m'\boldsymbol{\lambda}_{\mathcal{Q}} = \mathbf{q}' - \boldsymbol{\lambda}_{\mathcal{Q}}\left(q'_0 - \frac{\mathbf{q}'\cdot\boldsymbol{\lambda}_{\mathcal{Q}}}{1+\lambda^0_{\mathcal{Q}}}\right), \tag{6}$$
$$(\Lambda^{-1}_{\lambda_{\mathcal{Q}}}q')^0 = \Delta^0_{q',m'\lambda_{\mathcal{Q}}} = q'_0\lambda^0_{\mathcal{Q}} - \mathbf{q}'\cdot\boldsymbol{\lambda}_{\mathcal{Q}} = \sqrt{m'^2 + \boldsymbol{\Delta}^2_{q',m'\lambda_{\mathcal{Q}}}},$$

and all 4-momenta belong to the upper sheet of the mass hyperboloid

$$\Delta^2_{q',m'\lambda_{\mathcal{Q}}} = \Delta^{02}_{q',m'\lambda_{\mathcal{Q}}} - \boldsymbol{\Delta}^2_{q',m'\lambda_{\mathcal{Q}}} = m'^2. \tag{7}$$

Quasipotential $V(r; M_{\mathcal{Q}})$ is local in the sense of Lobachevsky geometry, the group parameter r plays the role of modulus of relativistic relative coordinate $\mathbf{r}$ ($\mathbf{r} = r\mathbf{n}, |\mathbf{n}| = \mathbf{1}$), and the operator $\hat{A}$ is defined as

$$\hat{A}\left(\hat{H}^{\rm rad}_{0,\ell}\right) = \frac{1}{4}\left[a'\left(\hat{H}^{\rm rad}_{0,\ell}\right)^2 + b'\right], \tag{8}$$

where spin parameters a' and b' in (8) are expressed through factor (5) by the formulas

$$a' = \begin{cases} g'^2 & \text{for } \hat{O} = \gamma_5 \text{ (pseudoscalar)};\\ \frac{1}{2}g'^2 & \text{for } \hat{O} = \gamma_\mu \text{ (vector)};\\ -\frac{1}{2}g'^2 & \text{for } \hat{O} = \gamma_5\gamma_\mu \text{ (pseudovector)}; \end{cases} \qquad b' = \begin{cases} 1 - g'^2 & \text{for } \hat{O} = \gamma_5;\\ \frac{3}{4} - \frac{1}{2}g'^2 & \text{for } \hat{O} = \gamma_\mu;\\ \frac{1}{4} + \frac{1}{2}g'^2 & \text{for } \hat{O} = \gamma_5\gamma_\mu. \end{cases} \tag{9}$$

The value of spin parameters a' and b' in (8) for $m_1 = m_2 = m$ coincides with the values of their analogs a and b obtained in [22].

3. Form factor of the relativistic two-particle system

In Refs. [7–10] the form factor of two-particle system was defined as the matrix element of the local current operator between bound states with the 4-momentum $\mathcal{Q}$ and $\mathcal{P}$ through the covariant wave RQP-functions satisfying RQP-equation in the momentum representation. Then, the invariant expression in the momentum representation near the poles of the bound states with the 4-momentum $\mathcal{Q}$ and $\mathcal{P}$ for the matrix element of the local vector-current operator of the two relativistic with arbitrary masses fermions bound state has the form[2)]

$$< \mathcal{P}|J_\nu|\mathcal{Q} >= -\frac{z_1}{\sqrt{M_{\mathcal{P}}M_{\mathcal{Q}}}(4\pi)^3}\int\frac{d\tau_{\mathcal{P}}d\tau_{\mathcal{Q}}d\mathbf{k}_2 d\mathbf{k}'_1 d\mathbf{k}_1}{\sqrt{m_2^2+\mathbf{k}_2^2}\sqrt{m_1^2+\mathbf{k}'^2_1}\sqrt{m_1^2+\mathbf{k}_1^2}}\frac{\Gamma^*_{M_{\mathcal{P}}}(\mathcal{P}k_2)}{\tau_{\mathcal{P}}+i\varepsilon}\times \tag{10}$$
$$\times{\rm tr}[\hat{O}^+(\hat{k}_1+m_1)(\hat{k}'_1+m_1)\hat{O}(\hat{k}_2-m_2)](k_1+k'_1)_\nu\frac{\Gamma_{M_{\mathcal{Q}}}(\mathcal{Q}k_2)}{\tau_{\mathcal{Q}}-i\varepsilon}\times$$
$$\times\delta^{(4)}\left(-\mathcal{Q}+k'_1+k_2-\lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}\right)\delta^{(4)}\left(\mathcal{P}-k_1-k_2+\lambda_{\mathcal{P}}\tau_{\mathcal{P}}\right)+(1\leftrightarrow 2).$$

Here, either as in Refs. [17–20], the functions $\Gamma_{M_{\mathcal{Q}}}(\mathcal{Q}k_2)$ and $\Gamma_{M_{\mathcal{P}}}(\mathcal{P}k_2)$ are the scalar parts of the vertex functions which depends each only on one the Lorentz-invariant scalar

2) Similar approach was used to get the expressions of form factors for scalar and vector currents as for the system of two spinless particles with arbitrary masses [17, 18], so and for the system of spinor particles with equal masses [19, 20].

parameter $\mathcal{Q}k_2$ and $\mathcal{P}k_2$, respectively; for $\hat{O}$, we take the Dirac matrices γ_5, γ_μ, and $\gamma_5\gamma_\mu$ ($\mu = 0,1,2,3$), $\hat{k}_i = k_i^\mu\gamma_\mu$, m_i is the mass of the ith component ($i = 1,2$) carrying the 4-momentum k_i and k_i', and all 4-momenta belong to the upper sheets of the masses hyperboloids

$$k_i^2 = k_{i0}^2 - \mathbf{k}_i^2 = m_i^2, i = 1,2. \tag{11}$$

The expression in (10) corresponds diagram on Fig. 1. On Fig. 1 the solid lines correspond to the constituents that carry the 4-momenta $k_1', k_i, i = 1,2$, while the dashed lines represent spurion quasiparticles. The composite-particle 4-velocities were chosen to be $\lambda_\mathcal{Q} = (\lambda_\mathcal{Q}^0; \boldsymbol{\lambda}_\mathcal{Q}) = \mathcal{Q}/\sqrt{\mathcal{Q}^2} = \mathcal{Q}/M_\mathcal{Q}$, where $M_\mathcal{Q}^2 = s_q = \mathcal{Q}^2 = (q_1 + q_2)^2$, and $\lambda_\mathcal{P} = (\lambda_\mathcal{P}^0; \boldsymbol{\lambda}_\mathcal{P}) = \mathcal{P}/\sqrt{\mathcal{P}^2} = \mathcal{P}/M_\mathcal{P}$, where $M_\mathcal{P}^2 = s_p = \mathcal{P}^2 = (p_1 + p_2)^2$.

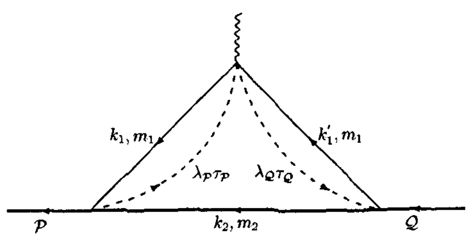

Figure 1: The diagram of matrix element of the local current operator in the case of two relativistic spinor particles with arbitrary masses

Values of the trace in (10) with matrixs $\hat{O} = \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu$ ($\mu = 0,1,2,3$) gives by expression

$$\begin{gathered} \mathrm{tr}[\hat{O}^+(\hat{k}_1 + m_1)(\hat{k}_1' + m_1)\hat{O}(\hat{k}_2 - m_2)] = \\ = -4(\tilde{a}m_2k_1k_1' + \tilde{b}m_1k_1k_2 + \tilde{b}m_1k_1'k_2 + \tilde{a}m_2m_1^2), \end{gathered} \tag{12}$$

which for $m_1 = m_2 = m$ coincides with the values of their analogs obtained in [19, 20], where

$$\tilde{a} = \begin{cases} 1 \text{ for } \hat{O} = \gamma_5, \text{(pseudoscalar)}; \\ 4 \text{ for } \hat{O} = \gamma_\mu, \text{(vector)}; \\ 4 \text{ for } \hat{O} = \gamma_5\gamma_\mu, \text{(pseudovector)}; \end{cases} \quad \tilde{b} = \begin{cases} 1 \text{ for } \hat{O} = \gamma_5; \\ 2 \text{ for } \hat{O} = \gamma_\mu; \\ -2 \text{ for } \hat{O} = \gamma_5\gamma_\mu. \end{cases} \tag{13}$$

Expression (10) for the matrix element of the vector-current operator features additionally its transverse component, which breaks the transverseness condition [18, 20]

$$(\mathcal{P} - \mathcal{Q})^\nu < \mathcal{P}|J_\nu|\mathcal{Q} >= 0.$$

Therefore, the 4-vector in expression (10) can be represented in the form

$$G(\chi_\Delta) < \mathcal{P}|J_\nu|\mathcal{Q} >= F^{(+)}(t)(\mathcal{P} + \mathcal{Q})_\nu + iF^{(-)}(t)(\mathcal{P} - \mathcal{Q})_\nu, \tag{14}$$

where the square of the 4-momentum transfer t and rapidity χ_Δ under $\mathcal{Q}^2 = M_\mathcal{Q}^2 = \mathcal{P}^2 = M_\mathcal{P}^2 = M^2$ are connected by correlation

$$\begin{gathered} t = (\mathcal{P} - \mathcal{Q})^2 = -Q^2 = 2M^2 - 2\mathcal{P}\mathcal{Q} = 2M^2(1 - \cosh\chi_\Delta), \\ (\mathcal{P} + \mathcal{Q})^2 = 2M^2 + 2\mathcal{P}\mathcal{Q} = 4M^2 - t, \end{gathered} \tag{15}$$

and the factor

$$G(\chi_\Delta) = \frac{\chi_\Delta}{\sinh\chi_\Delta} \tag{16}$$

is the relativistic geometric factor in the RQP approach [11], which first appeared substantiated in [24] for the invariant description of the spatial structure of particles in the three dimensional relativistic **r**-representation [14]. The factor (16) has a clear physical meaning, because, as was shown in [24], the factor (16) describes the value of the contribution to the form factor of nucleon of its central sphere, inside which the quarks move, having the radius equal to its Compton wavelength ($r_0 = 1/M$). In the nonrelativistic limit ($\chi_\Delta \to 0$) $G(\chi_\Delta) \to 1$, which corresponds to the point particle. Thus, the factor (16) is the measure of the contribution of relativistic effects caused by the dynamics of quarks.

Taking into consideration Eqs. (10), (14) and (15), we obtain the following expressions for the components of the elastic form factor in the form ($M_{\mathcal{Q}} = M_{\mathcal{P}} = M$)

$$F^{(+)}(t) = -\frac{z_1 G(\chi_\Delta)}{M(4M^2-t)(4\pi)^3}\int \frac{d\tau_{\mathcal{P}} d\tau_{\mathcal{Q}} d\mathbf{k}_2 d\mathbf{k}_1' d\mathbf{k}_1}{\sqrt{m_2^2+\mathbf{k}_2^2}\sqrt{m_1^2+\mathbf{k}_1'^2}\sqrt{m_1^2+\mathbf{k}_1^2}}\frac{\Gamma^*_{M_{\mathcal{P}}}(\mathcal{P}k_2)}{\tau_{\mathcal{P}}+i\varepsilon}\times \tag{17}$$
$$\times \mathrm{tr}[\hat{O}^+(\hat{k}_1+m_1)(\hat{k}_1'+m_1)\hat{O}(\hat{k}_2-m_2)](\mathcal{P}+\mathcal{Q})(k_1+k_1')\frac{\Gamma_{M_{\mathcal{Q}}}(\mathcal{Q}k_2)}{\tau_{\mathcal{Q}}-i\varepsilon}\times$$
$$\times\delta^{(4)}\left(-\mathcal{Q}+k_1'+k_2-\lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}\right)\delta^{(4)}\left(\mathcal{P}-k_1-k_2+\lambda_{\mathcal{P}}\tau_{\mathcal{P}}\right)+(1\leftrightarrow 2),$$

$$F^{(-)}(t) = -\frac{z_1 G(\chi_\Delta)}{itM(4\pi)^3}\int \frac{d\tau_{\mathcal{P}} d\tau_{\mathcal{Q}} d\mathbf{k}_2 d\mathbf{k}_1' d\mathbf{k}_1}{\sqrt{m_2^2+\mathbf{k}_2^2}\sqrt{m_1^2+\mathbf{k}_1'^2}\sqrt{m_1^2+\mathbf{k}_1^2}}\frac{\Gamma^*_{M_{\mathcal{P}}}(\mathcal{P}k_2)}{\tau_{\mathcal{P}}+i\varepsilon}\times \tag{18}$$
$$\times \mathrm{tr}[\hat{O}^+(\hat{k}_1+m_1)(\hat{k}_1'+m_1)\hat{O}(\hat{k}_2-m_2)](\mathcal{P}-\mathcal{Q})(k_1+k_1')\frac{\Gamma_{M_{\mathcal{Q}}}(\mathcal{Q}k_2)}{\tau_{\mathcal{Q}}-i\varepsilon}\times$$
$$\times\delta^{(4)}\left(-\mathcal{Q}+k_1'+k_2-\lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}\right)\delta^{(4)}\left(\mathcal{P}-k_1-k_2+\lambda_{\mathcal{P}}\tau_{\mathcal{P}}\right)+(1\leftrightarrow 2).$$

In order to carry out integration in expressions (17) and (18) with respect to $\mathbf{k}_1', \mathbf{k}_1, \tau_{\mathcal{Q}}$ and $\tau_{\mathcal{P}}$, we perform the pure Lorentz transformations $\Lambda^{-1}_{\lambda_{\mathcal{Q}}}$ and $\Lambda^{-1}_{\lambda_{\mathcal{P}}}$ by formulas (6) in the integrals with respect to $\mathbf{k}_1', \mathbf{k}_2$ and $\mathbf{k}_1$, respectively, and we take into account correlations ($M_{\mathcal{Q}} = M_{\mathcal{P}} = M$) (details see in [17–20])

$$\Lambda^{-1}_{\lambda_{\mathcal{Q}}}\mathcal{Q} = (M_{\mathcal{Q}};\mathbf{0}), \Lambda^{-1}_{\lambda_{\mathcal{P}}}\mathcal{P} = (M_{\mathcal{P}};\mathbf{0}), \mathcal{Q}k_2 = M_{\mathcal{Q}}\Delta^0_{k_2,m_2\lambda_{\mathcal{Q}}}, \mathcal{P}k_2 = M_{\mathcal{P}}\Delta^0_{k_2,m_2\lambda_{\mathcal{P}}}, \tag{19}$$
$$\tau_{\mathcal{Q}(\mathcal{P})} + M_{\mathcal{Q}(\mathcal{P})} = \sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}(\mathcal{P})}}}},\ \boldsymbol{\Delta}_{k_1',m_1\lambda_{\mathcal{Q}}} = -\boldsymbol{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}},\ \boldsymbol{\Delta}_{k_1,m_1\lambda_{\mathcal{P}}} = -\boldsymbol{\Delta}_{k_2,m_2\lambda_{\mathcal{P}}},$$
$$(\mathcal{P}\pm\mathcal{Q})(k_1+k_1') = M\left[1-\frac{t}{2M^2}+\frac{m_1^2-m_2^2}{\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}}s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}}}\right]\left(\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}}\pm\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}}}\right),$$
$$2k_1k_2 = s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}} - m_1^2 - m_2^2,\ 2k_1'k_2 = s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}} - m_1^2 - m_2^2,$$
$$2k_1k_1' = 2m_1^2 - s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}} - s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}} + 2\left(1-\frac{t}{2M^2}\right)\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}}s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}},$$
$$\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}(\mathcal{P})}}}} = \sqrt{m_1^2+\boldsymbol{\Delta}^2_{k_2,m_2\lambda_{\mathcal{Q}(\mathcal{P})}}} + \sqrt{m_2^2+\boldsymbol{\Delta}^2_{k_2,m_2\lambda_{\mathcal{Q}(\mathcal{P})}}},$$
$$\Delta^0_{k_2,m_2\lambda_{\mathcal{Q}(\mathcal{P})}} = \sqrt{m_2^2+\boldsymbol{\Delta}^2_{k_2,m_2\lambda_{\mathcal{Q}(\mathcal{P})}}}.$$

Formulas in (19) also allows to calculate trace (12) with matrixs $\hat{O} = \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu$:

$$\mathrm{tr}[\hat{O}^+(\hat{k}_1+m_1)(\hat{k}_1'+m_1)\hat{O}(\hat{k}_2-m_2)] = -2\Big[4\tilde{a}m_1^2m_2 - 2\tilde{b}m_1\left(m_1^2+m_2^2\right)+$$
$$+2\tilde{a}m_2\left(1-\frac{t}{2M^2}\right)\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}}s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}}+\left(\tilde{b}m_1-\tilde{a}m_2\right)\left(s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}}+s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}\right)\Big].$$

Here parameters $\tilde{a}$ and $\tilde{b}$ are given in (13), and expressions (17) and (18) takes the form

$$F^{(+)}(t) = \frac{2z_1 G(\chi_\Delta)}{(4M^2-t)(4\pi)^3}\int \frac{d\mathbf{\Delta}_{k_2,m_2\lambda_\mathcal{Q}}}{\sqrt{m_2^2+\mathbf{\Delta}^2_{k_2,m_2\lambda_\mathcal{Q}}}\sqrt{m_1^2+\mathbf{\Delta}^2_{k_2,m_2\lambda_\mathcal{Q}}}\sqrt{m_1^2+\mathbf{\Delta}^2_{k_2,m_2\lambda_\mathcal{P}}}}\times \tag{20}$$
$$\times\frac{\Gamma^*_{M_\mathcal{P}}(\mathbf{\Delta}_{k_2,m_2\lambda_\mathcal{P}})}{M_\mathcal{P}-\sqrt{s_{\Delta_{k_2,m_2\lambda_\mathcal{P}}}}-i\varepsilon}\Big[4\tilde{a}m_1^2m_2-2\tilde{b}m_1\left(m_1^2+m_2^2\right)+$$
$$+2\tilde{a}m_2\left(1-\frac{t}{2M^2}\right)\sqrt{s_{\Delta_{k_2,m_2\lambda_\mathcal{P}}}s_{\Delta_{k_2,m_2\lambda_\mathcal{Q}}}}+\left(\tilde{b}m_1-\tilde{a}m_2\right)\left(s_{\Delta_{k_2,m_2\lambda_\mathcal{P}}}+s_{\Delta_{k_2,m_2\lambda_\mathcal{Q}}}\right)\Big]\times$$
$$\times\left[1-\frac{t}{2M^2}+\frac{m_1^2-m_2^2}{\sqrt{s_{\Delta_{k_2,m_2\lambda_\mathcal{P}}}s_{\Delta_{k_2,m_2\lambda_\mathcal{Q}}}}}\right]\left(\sqrt{s_{\Delta_{k_2,m_2\lambda_\mathcal{Q}}}}+\sqrt{s_{\Delta_{k_2,m_2\lambda_\mathcal{P}}}}\right)\times$$
$$\times\frac{\Gamma_{M_\mathcal{Q}}(\mathbf{\Delta}_{k_2,m_2\lambda_\mathcal{Q}})}{M_\mathcal{Q}-\sqrt{s_{\Delta_{k_2,m_2\lambda_\mathcal{Q}}}}+i\varepsilon}+(1\leftrightarrow 2),$$

$$F^{(-)}(t) = \frac{2z_1 G(\chi_\Delta)}{it(4\pi)^3}\int \frac{d\mathbf{\Delta}_{k_2,m_2\lambda_\mathcal{Q}}}{\sqrt{m_2^2+\mathbf{\Delta}^2_{k_2,m_2\lambda_\mathcal{Q}}}\sqrt{m_1^2+\mathbf{\Delta}^2_{k_2,m_2\lambda_\mathcal{Q}}}\sqrt{m_1^2+\mathbf{\Delta}^2_{k_2,m_2\lambda_\mathcal{P}}}}\times \tag{21}$$
$$\times\frac{\Gamma^*_{M_\mathcal{P}}(\mathbf{\Delta}_{k_2,m_2\lambda_\mathcal{P}})}{M_\mathcal{P}-\sqrt{s_{\Delta_{k_2,m_2\lambda_\mathcal{P}}}}-i\varepsilon}\Big[4\tilde{a}m_1^2m_2-2\tilde{b}m_1\left(m_1^2+m_2^2\right)+$$
$$+2\tilde{a}m_2\left(1-\frac{t}{2M^2}\right)\sqrt{s_{\Delta_{k_2,m_2\lambda_\mathcal{P}}}s_{\Delta_{k_2,m_2\lambda_\mathcal{Q}}}}+\left(\tilde{b}m_1-\tilde{a}m_2\right)\left(s_{\Delta_{k_2,m_2\lambda_\mathcal{P}}}+s_{\Delta_{k_2,m_2\lambda_\mathcal{Q}}}\right)\Big]\times$$
$$\times\left[1-\frac{t}{2M^2}+\frac{m_1^2-m_2^2}{\sqrt{s_{\Delta_{k_2,m_2\lambda_\mathcal{P}}}s_{\Delta_{k_2,m_2\lambda_\mathcal{Q}}}}}\right]\left(\sqrt{s_{\Delta_{k_2,m_2\lambda_\mathcal{Q}}}}-\sqrt{s_{\Delta_{k_2,m_2\lambda_\mathcal{P}}}}\right)\times$$
$$\times\frac{\Gamma_{M_\mathcal{Q}}(\mathbf{\Delta}_{k_2,m_2\lambda_\mathcal{Q}})}{M_\mathcal{Q}-\sqrt{s_{\Delta_{k_2,m_2\lambda_\mathcal{Q}}}}+i\varepsilon}+(1\leftrightarrow 2),$$

where we introduced the notations

$$\Gamma_{M_\mathcal{P}}(\mathcal{P}k_2)=\Gamma_{M_\mathcal{P}}(\mathbf{\Delta}_{k_2,m_2\lambda_\mathcal{P}}),\ \Gamma_{M_\mathcal{Q}}(\mathcal{Q}k_2)=\Gamma_{M_\mathcal{Q}}(\mathbf{\Delta}_{k_2,m_2\lambda_\mathcal{Q}}).$$

Farther, in expressions (20) and (21) we shall perform the change of variables [17, 18]

$$\mathbf{\Delta}_{k_2,m_2\lambda_\mathcal{Q}} = g'\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}}\sqrt{\frac{4\mu^2+\mathbf{\Delta}^2_{k',m'\lambda_\mathcal{Q}}}{m'^2+\mathbf{\Delta}^2_{k',m'\lambda_\mathcal{Q}}}}, \tag{22}$$

and shall take into account correlations

$$\sqrt{m_1^2+\mathbf{\Delta}^2_{k_2,m_2\lambda_{\mathcal{Q}(\mathcal{P})}}}=g'f_+^{-1}(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}(\mathcal{P})}}),\ \sqrt{m_2^2+\mathbf{\Delta}^2_{k_2,m_2\lambda_{\mathcal{Q}(\mathcal{P})}}}=g'f_-^{-1}(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}(\mathcal{P})}}), \tag{23}$$
$$m_1=m'\left(g'+\sqrt{g'^2-1}\right)>m_2=m'\left(g'-\sqrt{g'^2-1}\right);$$
$$\sqrt{m_1^2+\mathbf{\Delta}^2_{k_2,m_2\lambda_{\mathcal{Q}(\mathcal{P})}}}=g'f_-^{-1}(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}(\mathcal{P})}}),\ \sqrt{m_2^2+\mathbf{\Delta}^2_{k_2,m_2\lambda_{\mathcal{Q}(\mathcal{P})}}}=g'f_+^{-1}(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}(\mathcal{P})}}),$$
$$m_1=m'\left(g'-\sqrt{g'^2-1}\right)<m_2=m'\left(g'+\sqrt{g'^2-1}\right);$$
$$M_\mathcal{Q}=2g'\Delta^0_{q',m'\lambda_\mathcal{Q}},\ M_\mathcal{P}=2g'\Delta^0_{p',m'\lambda_\mathcal{P}},\ \sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}(\mathcal{P})}}}}=2g'\Delta^0_{k',m'\lambda_{\mathcal{Q}(\mathcal{P})}},$$
$$\frac{d\mathbf{\Delta}_{k_2,m_2\lambda_\mathcal{Q}}}{\sqrt{m_2^2+\mathbf{\Delta}^2_{k_2,m_2\lambda_\mathcal{Q}}}\sqrt{m_1^2+\mathbf{\Delta}^2_{k_2,m_2\lambda_\mathcal{Q}}}}=g'\frac{d\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}}}{\Delta^0_{k',m'\lambda_\mathcal{Q}}}f(\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}}),$$

where

$$f_{\pm}(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}(\mathcal{P})}}) = \frac{\sqrt{m'^2 + \mathbf{\Delta}^2_{k',m'\lambda_{\mathcal{Q}(\mathcal{P})}}}}{m'^2 + \mathbf{\Delta}^2_{k',m'\lambda_{\mathcal{Q}(\mathcal{P})}} \pm m'\sqrt{m'^2 - 4\mu^2}}, \tag{24}$$

$$f(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}}}) = \frac{\sqrt{4\mu^2 + \mathbf{\Delta}^2_{k',m'\lambda_{\mathcal{Q}}}}}{m'^2 + \mathbf{\Delta}^2_{k',m'\lambda_{\mathcal{Q}}}}.$$

Then, considering correlations (22)-(24), the expressions in (20) and (21) takes the form

$$F^{(+)}(t) = \frac{(2m')^4(z_1+z_2)G(\chi_\Delta)}{(4M^2-t)(2\pi)^3}\left(1-\frac{t}{2M^2}\right)^2\int d\Omega_{\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}}}}\Psi^*_M(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})\times \tag{25}$$
$$\times\Bigg\{\Bigg[2\tilde{a}g'^2\frac{\Delta^0_{k',m'\lambda_{\mathcal{P}}}\Delta^0_{k',m'\lambda_{\mathcal{Q}}}}{m'^2} + \frac{1}{1-\dfrac{t}{2M^2}}\left(\tilde{a}+\tilde{b}-2\tilde{b}g'^2+g'^2\left(\tilde{b}-\tilde{a}\right)\frac{\Delta^{02}_{k',m'\lambda_{\mathcal{P}}}+\Delta^{02}_{k',m'\lambda_{\mathcal{Q}}}}{m'^2}\right)\Bigg]\times$$
$$\times\left[\frac{f_+(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})+f_-(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})}{2f(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})} + \frac{m'^2\sqrt{g'^2-1}\big(f_+(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})-f_-(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})\big)}{2g'f(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})\Delta^0_{k',m'\lambda_{\mathcal{P}}}\Delta^0_{k',m'\lambda_{\mathcal{Q}}}\left(1-\dfrac{t}{2M^2}\right)}\right]+$$
$$+\Bigg[-2\tilde{a}g'^2\frac{\Delta^0_{k',m'\lambda_{\mathcal{P}}}\Delta^0_{k',m'\lambda_{\mathcal{Q}}}}{m'^2} + \frac{1}{1-\dfrac{t}{2M^2}}\left(\tilde{a}+\tilde{b}-2\tilde{b}g'^2+g'^2\left(\tilde{b}+\tilde{a}\right)\frac{\Delta^{02}_{k',m'\lambda_{\mathcal{P}}}+\Delta^{02}_{k',m'\lambda_{\mathcal{Q}}}}{m'^2}\right)\Bigg]\times$$
$$\times\Bigg[\frac{m'^2(g'^2-1)\big(f_+(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})+f_-(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})\big)}{2g'^2f(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})\Delta^0_{k',m'\lambda_{\mathcal{P}}}\Delta^0_{k',m'\lambda_{\mathcal{Q}}}\left(1-\dfrac{t}{2M^2}\right)}+$$
$$+\frac{\sqrt{g'^2-1}\big(f_+(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})-f_-(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})\big)}{2g'f(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})}\Bigg]\Bigg\}\frac{\Delta^0_{k',m'\lambda_{\mathcal{P}}}+\Delta^0_{k',m'\lambda_{\mathcal{Q}}}}{m'}\Psi_M(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}}}),$$

$$F^{(-)}(t) = \frac{(2m')^4(z_1+z_2)G(\chi_\Delta)}{-it(2\pi)^3}\left(1-\frac{t}{2M^2}\right)^2\int d\Omega_{\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}}}}\Psi^*_M(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})\times \tag{26}$$
$$\times\Bigg\{\Bigg[2\tilde{a}g'^2\frac{\Delta^0_{k',m'\lambda_{\mathcal{P}}}\Delta^0_{k',m'\lambda_{\mathcal{Q}}}}{m'^2} + \frac{1}{1-\dfrac{t}{2M^2}}\left(\tilde{a}+\tilde{b}-2\tilde{b}g'^2+g'^2\left(\tilde{b}-\tilde{a}\right)\frac{\Delta^{02}_{k',m'\lambda_{\mathcal{P}}}+\Delta^{02}_{k',m'\lambda_{\mathcal{Q}}}}{m'^2}\right)\Bigg]\times$$
$$\times\left[\frac{f_+(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})+f_-(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})}{2f(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})} + \frac{m'^2\sqrt{g'^2-1}\big(f_+(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})-f_-(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})\big)}{2g'f(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})\Delta^0_{k',m'\lambda_{\mathcal{P}}}\Delta^0_{k',m'\lambda_{\mathcal{Q}}}\left(1-\dfrac{t}{2M^2}\right)}\right]+$$
$$+\Bigg[-2\tilde{a}g'^2\frac{\Delta^0_{k',m'\lambda_{\mathcal{P}}}\Delta^0_{k',m'\lambda_{\mathcal{Q}}}}{m'^2} + \frac{1}{1-\dfrac{t}{2M^2}}\left(\tilde{a}+\tilde{b}-2\tilde{b}g'^2+g'^2\left(\tilde{b}+\tilde{a}\right)\frac{\Delta^{02}_{k',m'\lambda_{\mathcal{P}}}+\Delta^{02}_{k',m'\lambda_{\mathcal{Q}}}}{m'^2}\right)\Bigg]\times$$
$$\times\Bigg[\frac{m'^2(g'^2-1)\big(f_+(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})+f_-(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})\big)}{2g'^2f(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})\Delta^0_{k',m'\lambda_{\mathcal{P}}}\Delta^0_{k',m'\lambda_{\mathcal{Q}}}\left(1-\dfrac{t}{2M^2}\right)}+$$
$$+\frac{\sqrt{g'^2-1}\big(f_+(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})-f_-(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})\big)}{2g'f(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})}\Bigg]\Bigg\}\frac{\Delta^0_{k',m'\lambda_{\mathcal{P}}}-\Delta^0_{k',m'\lambda_{\mathcal{Q}}}}{m'}\Psi_M(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}}}),$$

where we have defined the wave function of system in momentum space as

$$\Psi_M(\mathbf{\Delta}_{k',m'\lambda_Q}) = \frac{f(\mathbf{\Delta}_{k',m'\lambda_Q})\Gamma_{M_Q}(\mathbf{\Delta}_{k',m'\lambda_Q})}{2^{3/2}\sqrt{m'}\left(2\Delta^0_{q',m'\lambda_Q} - 2\Delta^0_{k',m'\lambda_Q} + i\varepsilon\right)},$$

and $d\Omega_{\mathbf{\Delta}_{k',m'\lambda_Q}} = m'd\mathbf{\Delta}_{k',m'\lambda_Q}/\Delta^0_{k',m'\lambda_Q}$ is the relativistic three-dimensional volume element in Lobachevsky space. All 4-momenta now belong to the upper sheet of the mass hyperboloid (7). This sheet is embedded in 4-dimensional momentum space and serves as a model of the Lobachevsky space momentum, and the Lorentz group is its motion group. The Lorentz transformations proper mean translations in Lobachevsky space. The role of plane waves corresponding to these translations is played by relativistic plane waves [25]

$$\xi(\mathbf{\Delta}_{k',m'\lambda_Q}, \mathbf{r}) = \left(\frac{\Delta^0_{k',m'\lambda_Q} - \mathbf{\Delta}_{k',m'\lambda_Q}\cdot\mathbf{n}}{m'}\right)^{-1-ir/\lambda'}. \tag{27}$$

The functions in (27) correspond to the main series of unitary irreducible representations of the Lorentz group and satisfy the conditions of completeness and orthogonality

$$\frac{1}{(2\pi)^3}\int d\Omega_{\mathbf{\Delta}_{k',m'\lambda_Q}}\xi(\mathbf{\Delta}_{k',m'\lambda_Q}, \mathbf{r})\xi^*(\mathbf{\Delta}_{k',m'\lambda_Q}, \mathbf{r}') = \delta(\mathbf{r}'-\mathbf{r}), \tag{28}$$
$$\frac{1}{(2\pi)^3}\int d\mathbf{r}\xi(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r})\xi^*(\mathbf{\Delta}_{k',m'\lambda_Q}, \mathbf{r}) = \frac{\Delta^0_{p',m'\lambda_Q}}{m'}\delta(\mathbf{\Delta}_{k',m'\lambda_Q} - \mathbf{\Delta}_{p',m'\lambda_Q}),$$

and the finite-difference equation [15, 16]

$$\left(\hat{H}_0 - 2\Delta^0_{k',m'\lambda_Q}\right)\xi(\mathbf{\Delta}_{k',m'\lambda_Q}, \mathbf{r}) = 0. \tag{29}$$

The vector $\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}}$ from the Lobachevsky space can be represented in the form

$$\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}} = \mathbf{k}'(-)m'\lambda_\mathcal{P} = \Lambda^{-1}_{\lambda_\mathcal{P}}\mathbf{k}' = V(\Lambda_{\lambda_Q}, \mathcal{P})\mathbf{\Delta}_{k',m'\lambda_Q}(-)\frac{m'}{M}\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}}. \tag{30}$$

Here $V(\Lambda_{\lambda_Q}, \mathcal{P}) = \Lambda^{-1}_{\lambda_\mathcal{P}}\Lambda_{\lambda_Q}\Lambda_{\Delta_{\mathcal{P},\mathcal{Q}}}$ is Wigner's rotation matrix, and $\Delta_{\mathcal{P},\mathcal{Q}} = \Lambda^{-1}_{\lambda_Q}\mathcal{P}$ is the 4-momentum transfer in the Lobachevsky space:

$$\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}} = \Lambda^{-1}_{\mathcal{Q}}\boldsymbol{\mathcal{P}} = \boldsymbol{\mathcal{P}}(-)\boldsymbol{\mathcal{Q}} = \boldsymbol{\mathcal{P}} - \frac{\boldsymbol{\mathcal{Q}}}{M}\left(\mathcal{P}_0 - \frac{\boldsymbol{\mathcal{P}}\cdot\boldsymbol{\mathcal{Q}}}{\mathcal{Q}_0 + M}\right) = M\sinh\chi_\Delta\mathbf{n}_\Delta, \tag{31}$$
$$\Delta^0_{\mathcal{P},\mathcal{Q}} = \left(\Lambda^{-1}_{\mathcal{Q}}\mathcal{P}\right)^0 = \frac{\mathcal{P}_0\mathcal{Q}_0 - \boldsymbol{\mathcal{P}}\cdot\boldsymbol{\mathcal{Q}}}{M} = \frac{\mathcal{P}\mathcal{Q}}{M} = M\cosh\chi_\Delta,$$
$$\boldsymbol{\mathcal{P}} = M\sinh\chi_\mathcal{P}\mathbf{n}_\mathcal{P}, \boldsymbol{\mathcal{Q}} = M\sinh\chi_\mathcal{Q}\mathbf{n}_\mathcal{Q}, \mathcal{P}_0 = M\cosh\chi_\mathcal{P}, \mathcal{Q}_0 = M\cosh\chi_\mathcal{Q},$$
$$|\mathbf{n}_\mathcal{P}| = |\mathbf{n}_\mathcal{Q}| = |\mathbf{n}_\Delta| = 1, \Delta^{02}_{\mathcal{P},\mathcal{Q}} - \mathbf{\Delta}^2_{\mathcal{P},\mathcal{Q}} = M^2,$$

where $\chi_\Delta, \chi_\mathcal{P}$, and $\chi_\mathcal{Q}$ are the respective rapidities, and that the square of the 4-momentum transfer t is connected to the 3-momentum transfer $\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}}$ and rapidity χ_Δ by correlation

$$Q^2 = -t = -2M^2 + 2M\sqrt{M^2 + \mathbf{\Delta}^2_{\mathcal{P},\mathcal{Q}}} = 2M^2\left(\cosh\chi_\Delta - 1\right). \tag{32}$$

From Eqs. (30)–(32) we have

$$\Delta^0_{k',m'\lambda_\mathcal{P}} \approx \frac{m'^2\Delta^0_{\mathcal{P},\mathcal{Q}}}{2M\Delta^0_{k',m'\lambda_Q}}, \tag{33}$$

and the factors $(f_+(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}}) \pm f_-(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}}))/2f(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})$ can be simplified to the form

$$\frac{f_+(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}}) + f_-(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})}{2f(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})} \approx 1 + \frac{8M^4\left(g'^2-1\right)}{g'^2\left(2M^2-t\right)^2}\frac{\Delta^0_{k',m'\lambda_\mathcal{P}}\Delta^0_{k',m'\lambda_\mathcal{Q}}}{m'^2}, \tag{34}$$

$$\frac{f_+(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}}) - f_-(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})}{2f(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})} \approx -\frac{16M^4\sqrt{g'^2-1}}{g'\left(2M^2-t\right)^2}\frac{\Delta^0_{k',m'\lambda_\mathcal{P}}\Delta^0_{k',m'\lambda_\mathcal{Q}}}{m'^2},$$

$$\frac{m'\sqrt{m'^2-4\mu^2}}{m'^2+\mathbf{\Delta}^2_{k',m'\lambda_\mathcal{P}}} < 1, \quad \frac{m'^2-4\mu^2}{4\mu^2+\mathbf{\Delta}^2_{k',m'\lambda_\mathcal{P}}} < 1.$$

Therefore, taking into consideration expressions (30)–(34), the components of the elastic form factor in (25) and (26) we can consider as functions of the invariant variable $\mathbf{\Delta}^2_{\mathcal{P},\mathcal{Q}}$, which is the square of the 3-momentum transfer in the Lobachevsky space, and consequently, are convolutions of the covariant wave RQP-functions in this space. Then, by using the Shapiro transformations [25]

$$\psi_M(\mathbf{r}) = \frac{1}{(2\pi)^3}\int d\Omega_{\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}}}\xi(\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}},\mathbf{r})\Psi_M(\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}}),$$

$$\Psi_M(\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}}) = \int d\mathbf{r}\xi^*(\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}},\mathbf{r})\psi_M(\mathbf{r}),$$

the completeness condition in (28), the equation (29), the addition theorem for relativistic plane waves (27) in the form [16],

$$\int d\omega_n\xi\left(\mathbf{\Delta}_{p',m'\lambda_\mathcal{Q}}(-)\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}},\mathbf{r}\right) = \int d\omega_n\xi(\mathbf{\Delta}_{p',m'\lambda_\mathcal{Q}},\mathbf{r})\xi^*\left(\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}},\mathbf{r}\right),$$

and the fact that the free-Hamiltonian operator (3) is Hermitian, we can represent the components of the elastic form factor in (25) and (26) into the form of relativistic Fourier transforms of covariant RQP wave functions in the configuration representation $\psi_M(\mathbf{r})$ (details see in [17–20]):

$$F^{(+)}(t) = \frac{2(2m')^4(z_1+z_2)G(\chi_\Delta)}{4M^2-t}\left(1-\frac{t}{2M^2}\right)\int d\mathbf{r}\xi^*\left(\frac{m'}{M}\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}},\mathbf{r}\right)\times \tag{35}$$

$$\times\mathrm{Re}\left\{R_1^{(+)}(r,\chi') - \frac{t}{2M^2}R_0(r,\chi') + \frac{2(g'^2-1)}{g'^2\left(1-\frac{t}{2M^2}\right)^2}\left[R_2^{(+)}(r,\chi') - \frac{t}{2M^2}R_3(r,\chi')\right]\right\},$$

$$F^{(-)}(t) = \frac{2(2m')^4(z_1+z_2)G(\chi_\Delta)}{-t}\left(1-\frac{t}{2M^2}\right)\int d\mathbf{r}\xi^*\left(\frac{m'}{M}\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}},\mathbf{r}\right)\times \tag{36}$$

$$\times\mathrm{Im}\left\{R_1^{(-)}(r,\chi') - \frac{t}{2M^2}R_0(r,\chi') + \frac{2(g'^2-1)}{g'^2\left(1-\frac{t}{2M^2}\right)^2}\left[R_2^{(-)}(r,\chi') - \frac{t}{2M^2}R_3(r,\chi')\right]\right\},$$

where we introduced the notations:

$$R_0(r,\chi') = 2\tilde{a}g'^2\left[\left(\frac{\hat{H}_0}{2m'}\right)^2\psi_M(\mathbf{r})\right]^*\frac{\hat{H}_0}{2m'}\psi_M(\mathbf{r}), \tag{37}$$

$$R_1^{(+)}(r,\chi') = \left(\tilde{a}+\tilde{b}-2\tilde{b}g'^2\right)\psi_M(\mathbf{r})\left(\frac{\hat{H}_0}{2m'}\psi_M(\mathbf{r})\right)^* +$$

$$+g'^2\left(\tilde{a}+\tilde{b}\right)\left[\left(\frac{\hat{H}_0}{2m'}\right)^2\psi_M(\mathbf{r})\right]^*\frac{\hat{H}_0}{2m'}\psi_M(\mathbf{r})+g'^2\left(\tilde{b}-\tilde{a}\right)\psi_M(\mathbf{r})\left[\left(\frac{\hat{H}_0}{2m'}\right)^3\psi_M(\mathbf{r})\right]^*,$$

$$R_2^{(+)}(r,\chi')=\left(\tilde{a}+\tilde{b}-2\tilde{b}g'^2\right)\psi_M(\mathbf{r})\left(\frac{\hat{H}_0}{2m'}\psi_M(\mathbf{r})\right)^*+$$
$$+g'^2\left(\tilde{b}-\tilde{a}\right)\left[\left(\frac{\hat{H}_0}{2m'}\right)^2\psi_M(\mathbf{r})\right]^*\frac{\hat{H}_0}{2m'}\psi_M(\mathbf{r})-$$
$$-\left(\tilde{a}+\tilde{b}-2\tilde{b}g'^2\right)\left[\left(\frac{\hat{H}_0}{2m'}\right)^2\psi_M(\mathbf{r})\right]^*\frac{\hat{H}_0}{2m'}\psi_M(\mathbf{r})+g'^2\left(\tilde{a}+\tilde{b}\right)\psi_M(\mathbf{r})\left[\left(\frac{\hat{H}_0}{2m'}\right)^3\psi_M(\mathbf{r})\right]^*+$$
$$+g'^2\left(3\tilde{a}-\tilde{b}\right)\left[\left(\frac{\hat{H}_0}{2m'}\right)^3\psi_M(\mathbf{r})\right]^*\left(\frac{\hat{H}_0}{2m'}\right)^2\psi_M(\mathbf{r})-$$
$$-g'^2\left(3\tilde{a}+\tilde{b}\right)\left[\left(\frac{\hat{H}_0}{2m'}\right)^4\psi_M(\mathbf{r})\right]^*\frac{\hat{H}_0}{2m'}\psi_M(\mathbf{r}),$$

$$R_3(r,\chi')=-2\tilde{a}g'^2\left[\left(\frac{\hat{H}_0}{2m'}\right)^2\psi_M(\mathbf{r})\right]^*\frac{\hat{H}_0}{2m'}\psi_M(\mathbf{r})+$$
$$+6\tilde{a}g'^2\left[\left(\frac{\hat{H}_0}{2m'}\right)^3\psi_M(\mathbf{r})\right]^*\left(\frac{\hat{H}_0}{2m'}\right)^2\psi_M(\mathbf{r}),$$

$$R_1^{(-)}(r,\chi')=\left(\tilde{a}+\tilde{b}-2\tilde{b}g'^2\right)\psi_M(\mathbf{r})\left(\frac{\hat{H}_0}{2m'}\psi_M(\mathbf{r})\right)^*+\tag{38}$$
$$+g'^2\left(3\tilde{a}-\tilde{b}\right)\left[\left(\frac{\hat{H}_0}{2m'}\right)^2\psi_M(\mathbf{r})\right]^*\frac{\hat{H}_0}{2m'}\psi_M(\mathbf{r})+g'^2\left(\tilde{b}-\tilde{a}\right)\psi_M(\mathbf{r})\left[\left(\frac{\hat{H}_0}{2m'}\right)^3\psi_M(\mathbf{r})\right]^*,$$

$$R_2^{(-)}(r,\chi')=\left(\tilde{a}+\tilde{b}-2\tilde{b}g'^2\right)\psi_M(\mathbf{r})\left(\frac{\hat{H}_0}{2m'}\psi_M(\mathbf{r})\right)^*-$$
$$-g'^2\left(3\tilde{a}+\tilde{b}\right)\left[\left(\frac{\hat{H}_0}{2m'}\right)^2\psi_M(\mathbf{r})\right]^*\frac{\hat{H}_0}{2m'}\psi_M(\mathbf{r})-$$
$$-\left(\tilde{a}+\tilde{b}-2\tilde{b}g'^2\right)\left[\left(\frac{\hat{H}_0}{2m'}\right)^2\psi_M(\mathbf{r})\right]^*\frac{\hat{H}_0}{2m'}\psi_M(\mathbf{r})+g'^2\left(\tilde{a}+\tilde{b}\right)\psi_M(\mathbf{r})\left[\left(\frac{\hat{H}_0}{2m'}\right)^3\psi_M(\mathbf{r})\right]^*+$$
$$+g'^2\left(9\tilde{a}+\tilde{b}\right)\left[\left(\frac{\hat{H}_0}{2m'}\right)^3\psi_M(\mathbf{r})\right]^*\left(\frac{\hat{H}_0}{2m'}\right)^2\psi_M(\mathbf{r})-$$
$$-g'^2\left(3\tilde{a}+\tilde{b}\right)\left[\left(\frac{\hat{H}_0}{2m'}\right)^4\psi_M(\mathbf{r})\right]^*\frac{\hat{H}_0}{2m'}\psi_M(\mathbf{r}).$$

For s-state ($\ell=0$) of the composite system the integrations in (35) and (36) respecting of angles gives

$$F^{(+)}_{\ell=0}(t) = \frac{16\pi(2m')^3(z_1+z_2)G(\chi_\Delta)}{4M^2-t}\left(1-\frac{t}{2M^2}\right)\int\limits_0^\infty d\rho\frac{\sin(\rho\chi_\Delta)}{\rho\sinh\chi_\Delta}\times \tag{39}$$

$$\times\mathrm{Re}\left\{\tilde{R}^{(+)}_1(\rho,\chi_n)-\frac{t}{2M^2}\tilde{R}_0(\rho,\chi_n)+\frac{2(g'^2-1)}{g'^2\left(1-\frac{t}{2M^2}\right)^2}\left[\tilde{R}^{(+)}_2(\rho,\chi_n)-\frac{t}{2M^2}\tilde{R}_3(\rho,\chi_n)\right]\right\},$$

$$F^{(-)}_{\ell=0}(t) = \frac{16\pi(2m')^3(z_1+z_2)G(\chi_\Delta)}{-t}\left(1-\frac{t}{2M^2}\right)\int\limits_0^\infty d\rho\frac{\sin(\rho\chi_\Delta)}{\rho\sinh\chi_\Delta}\times \tag{40}$$

$$\times\mathrm{Im}\left\{\tilde{R}^{(-)}_1(\rho,\chi_n)-\frac{t}{2M^2}\tilde{R}_0(\rho,\chi_n)+\frac{2(g'^2-1)}{g'^2\left(1-\frac{t}{2M^2}\right)^2}\left[\tilde{R}^{(-)}_2(\rho,\chi_n)-\frac{t}{2M^2}\tilde{R}_3(\rho,\chi_n)\right]\right\},$$

where $\rho = rm'$, the radial part $\hat{H}^{\mathrm{rad}}_{0,\ell}$ of free Hamiltonian operator (3) is defined in (2), the rapidity χ_Δ is connected to the square of the 4-momentum transfer t and the 3-momentum transfer $\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}}$ by correlation (32), and the rapidity χ_n corresponds the level n for s-state of the composite system with the energy $M = M_n = 2m'g'\cosh\chi_n$, and were introduced the notations

$$\tilde{R}_0(\rho,\chi_n) = 2\tilde{a}g'^2\left[\left(\hat{H}^{\mathrm{rad}}_{0,\ell=0}\right)^2\varphi_0(\rho,\chi_n)\right]^*\hat{H}^{\mathrm{rad}}_{0,\ell=0}\varphi_0(\rho,\chi_n), \tag{41}$$

$$\begin{aligned}
\tilde{R}^{(+)}_1(\rho,\chi_n) &= \left(\tilde{a}+\tilde{b}-2\tilde{b}g'^2\right)\varphi_0(\rho,\chi_n)\left(\hat{H}^{\mathrm{rad}}_{0,\ell=0}\varphi_0(\rho,\chi_n)\right)^*+\\
&+g'^2\left(\tilde{a}+\tilde{b}\right)\left[\left(\hat{H}^{\mathrm{rad}}_{0,\ell=0}\right)^2\varphi_0(\rho,\chi_n)\right]^*\hat{H}^{\mathrm{rad}}_{0,\ell=0}\varphi_0(\rho,\chi_n)+\\
&+g'^2\left(\tilde{b}-\tilde{a}\right)\varphi_0(\rho,\chi_n)\left[\left(\hat{H}^{\mathrm{rad}}_{0,\ell=0}\right)^3\varphi_0(\rho,\chi_n)\right]^*,
\end{aligned}$$

$$\begin{aligned}
\tilde{R}^{(+)}_2(\rho,\chi_n) &= \left(\tilde{a}+\tilde{b}-2\tilde{b}g'^2\right)\varphi_0(\rho,\chi_n)\left(\hat{H}^{\mathrm{rad}}_{0,\ell=0}\varphi_0(\rho,\chi_n)\right)^*+\\
&+g'^2\left(\tilde{b}-\tilde{a}\right)\left[\left(\hat{H}^{\mathrm{rad}}_{0,\ell=0}\right)^2\varphi_0(\rho,\chi_n)\right]^*\hat{H}^{\mathrm{rad}}_{0,\ell=0}\varphi_0(\rho,\chi_n)-\\
&-\left(\tilde{a}+\tilde{b}-2\tilde{b}g'^2\right)\left[\left(\hat{H}^{\mathrm{rad}}_{0,\ell=0}\right)^2\varphi_0(\rho,\chi_n)\right]^*\hat{H}^{\mathrm{rad}}_{0,\ell=0}\varphi_0(\rho,\chi_n)+\\
&+g'^2\left(\tilde{a}+\tilde{b}\right)\varphi_0(\rho,\chi_n)\left[\left(\hat{H}^{\mathrm{rad}}_{0,\ell=0}\right)^3\varphi_0(\rho,\chi_n)\right]^*+\\
&+g'^2\left(3\tilde{a}-\tilde{b}\right)\left[\left(\hat{H}^{\mathrm{rad}}_{0,\ell=0}\right)^3\varphi_0(\rho,\chi_n)\right]^*\left(\hat{H}^{\mathrm{rad}}_{0,\ell=0}\right)^2\varphi_0(\rho,\chi_n)-\\
&-g'^2\left(3\tilde{a}+\tilde{b}\right)\left[\left(\hat{H}^{\mathrm{rad}}_{0,\ell=0}\right)^4\varphi_0(\rho,\chi_n)\right]^*\hat{H}^{\mathrm{rad}}_{0,\ell=0}\varphi_0(\rho,\chi_n),
\end{aligned}$$

$$\begin{aligned}
\tilde{R}_3(\rho,\chi_n) &= -2\tilde{a}g'^2\left[\left(\hat{H}^{\mathrm{rad}}_{0,\ell=0}\right)^2\varphi_0(\rho,\chi_n)\right]^*\hat{H}^{\mathrm{rad}}_{0,\ell=0}\varphi_0(\rho,\chi_n)+\\
&+6\tilde{a}g'^2\left[\left(\hat{H}^{\mathrm{rad}}_{0,\ell=0}\right)^3\varphi_0(\rho,\chi_n)\right]^*\left(\hat{H}^{\mathrm{rad}}_{0,\ell=0}\right)^2\varphi_0(\rho,\chi_n),
\end{aligned}$$

$$\tilde{R}_1^{(-)}(\rho,\chi_n) = \left(\tilde{a}+\tilde{b}-2\tilde{b}g'^2\right)\varphi_0(\rho,\chi_n)\left(\hat{H}^{\text{rad}}_{0,\ell=0}\varphi_0(\rho,\chi_n)\right)^* + \tag{42}$$
$$+g'^2\left(3\tilde{a}-\tilde{b}\right)\left[\left(\hat{H}^{\text{rad}}_{0,\ell=0}\right)^2\varphi_0(\rho,\chi_n)\right]^*\hat{H}^{\text{rad}}_{0,\ell=0}\varphi_0(\rho,\chi_n)+$$
$$+g'^2\left(\tilde{b}-\tilde{a}\right)\varphi_0(\rho,\chi_n)\left[\left(\hat{H}^{\text{rad}}_{0,\ell=0}\right)^3\varphi_0(\rho,\chi_n)\right]^*,$$
$$\tilde{R}_2^{(-)}(\rho,\chi_n) = \left(\tilde{a}+\tilde{b}-2\tilde{b}g'^2\right)\varphi_0(\rho,\chi_n)\left(\hat{H}^{\text{rad}}_{0,\ell=0}\varphi_0(\rho,\chi_n)\right)^* -$$
$$-g'^2\left(3\tilde{a}+\tilde{b}\right)\left[\left(\hat{H}^{\text{rad}}_{0,\ell=0}\right)^2\varphi_0(\rho,\chi_n)\right]^*\hat{H}^{\text{rad}}_{0,\ell=0}\varphi_0(\rho,\chi_n)-$$
$$-\left(\tilde{a}+\tilde{b}-2\tilde{b}g'^2\right)\left[\left(\hat{H}^{\text{rad}}_{0,\ell=0}\right)^2\varphi_0(\rho,\chi_n)\right]^*\hat{H}^{\text{rad}}_{0,\ell=0}\varphi_0(\rho,\chi_n)+$$
$$+g'^2\left(\tilde{a}+\tilde{b}\right)\varphi_0(\rho,\chi_n)\left[\left(\hat{H}^{\text{rad}}_{0,\ell=0}\right)^3\varphi_0(\rho,\chi_n)\right]^*+$$
$$+g'^2\left(9\tilde{a}+\tilde{b}\right)\left[\left(\hat{H}^{\text{rad}}_{0,\ell=0}\right)^3\varphi_0(\rho,\chi_n)\right]^*\left(\hat{H}^{\text{rad}}_{0,\ell=0}\right)^2\varphi_0(\rho,\chi_n)-$$
$$-g'^2\left(3\tilde{a}+\tilde{b}\right)\left[\left(\hat{H}^{\text{rad}}_{0,\ell=0}\right)^4\varphi_0(\rho,\chi_n)\right]^*\hat{H}^{\text{rad}}_{0,\ell=0}\varphi_0(\rho,\chi_n).$$

We note that, the transverse component $F^{(-)}_{\ell=0}(t)$ of the elastic form factor for a real-valued potential $V(r)$ vanishes.

4. Form factor and root-mean-square radius for the Coulomb interaction

The expression for the invariant root-mean-square radius (r.m.s.) of the vector current of composite system of two relativistic fermions with arbitrary masses in terms of the wave function s-state according to (39) and (41) has the form [24, 26]

$$< r^2_{0,v} > |_{\ell=0} = \frac{6\partial F^{(+)}_{\ell=0}(t)/\partial t|_{t=0}}{F^{(+)}_{\ell=0}(0)} = \frac{1}{M_n^2}\left\{\frac{1}{2}+\right. \tag{43}$$
$$\left.+\frac{\int\limits_0^\infty d\rho\left\{\rho^2\tilde{R}_1^{(+)}(\rho,\chi_n)-3\tilde{R}_0(\rho,\chi_n)+\frac{2(g'^2-1)}{g'^2}\left[(\rho^2+6)\tilde{R}_2^{(+)}(\rho,\chi_n)-3\tilde{R}_3(\rho,\chi_n)\right]\right\}}{\int\limits_0^\infty d\rho\left[\tilde{R}_1^{(+)}(\rho,\chi_n)+\frac{2(g'^2-1)}{g'^2}\tilde{R}_2^{(+)}(\rho,\chi_n)\right]}\right\}.$$

As example, we consider the form factor (39) and the r.m.s. (43) of meson in the case of the Coulomb (chromodynamic) interaction

$$V_{\text{Coul}} = -\frac{\alpha_s}{r},\ \alpha_s > 0. \tag{44}$$

The radial wave function of exact solution of the integral analogue of the finite-difference RQP-equation (1) with interaction (44) for the s-state and ground level ($n=1$) with the energy M_1, not containing of the i-periodic constants, has the form [21]

$$\varphi_0^{(1)}(\rho,\kappa_1) = C_0^{(1)}(\kappa_1)(\rho-\rho'_{\kappa_1})e^{-(\rho-\rho'_{\kappa_1})\kappa_1}, \tag{45}$$

where

$$\rho'_{\kappa_1} = \frac{\tilde{\alpha}'_s a'}{2}\cos\kappa_1, M_1 = 2m'g'\cos\kappa_1, 0 < \kappa_1 < \pi/2, \tilde{\alpha}'_s = m'\alpha_s,$$

and κ_1 defines by the following quantization condition of the energy levels for the s-state and ground level of a composite system formed by two relativistic spinor quarks having arbitrary masses and interacting via the Coulomb chromodynamic potential (44) [21]:

$$\tilde{\alpha}'_s(b' + a'\cos^2\kappa_1) = 4\sin\kappa_1. \tag{46}$$

The normalization factor $|C_0^{(1)}(\kappa_1)|^2$ gives by expression

$$|C_0^{(1)}(\kappa_1)|^2 = m'\kappa_1^3 e^{-2\kappa_1\rho'_{\kappa_1}}/\pi(2\kappa_1^2\rho'^2_{\kappa_1} - 2\kappa_1\rho'_{\kappa_1} + 1), \tag{47}$$

and it can be found from the normalization condition

$$4\pi\int_0^\infty dr|\varphi_0^{(1)}(r,\kappa_1)|^2 = 1.$$

Then, in accordance with expressions (41) and (45)–(47), the longitudinal component of form factor (39) and the r.m.s. (43) with interaction (44) for the ground level of bound s-state with the energy M_1 can be represented in the form

$$F^{(+),\mathrm{Coul}}_{\ell=0,n=1}(t) = \frac{8(z_1+z_2)(2m')^4\kappa_1^3 G(\chi_\Delta)}{(4M_1^2 - t)(2\kappa_1^2\rho'^2_{\kappa_1} - 2\kappa_1\rho'_{\kappa_1} + 1)\sinh\chi_\Delta}\left(1 - \frac{t}{2M_1^2}\right)\left\{F_1(\kappa_1,\chi_\Delta) - \right. \tag{48}$$

$$\left. - \frac{t}{2M_1^2}F_0(\kappa_1,\chi_\Delta) + \frac{2\,(g'^2-1)}{g'^2\left(1-\dfrac{t}{2M_1^2}\right)^2}\left[F_2(\kappa_1,\chi_\Delta) - \frac{t}{2M_1^2}F_3(\kappa_1,\chi_\Delta)\right]\right\},$$

$$< r^2_{0,v} >^{\mathrm{Coul}}_{\ell=0,n=1} = \frac{1}{M_1^2}\left\{\frac{1}{2} + \frac{1}{2\kappa_1^2}\left[1 + \frac{E_1 + \dfrac{2(g'^2-1)}{g'^2}E_2}{D_1 + \dfrac{2(g'^2-1)}{g'^2}D_2}\right] - \right. \tag{49}$$

$$\left. - \frac{3\left\{D_0 + \dfrac{2(g'^2-1)}{g'^2}\left[D_3 - 2D_2\right]\right\}}{D_1 + \dfrac{2(g'^2-1)}{g'^2}D_2}\right\},$$

where were introduced the notations

$$F_i(\kappa_1,\chi_\Delta) = \frac{4\kappa_1 A_i\chi_\Delta}{(\chi_\Delta^2 + 4\kappa_1^2)^2} + \frac{B_i\chi_\Delta}{\chi_\Delta^2 + 4\kappa_1^2} + C_i\arctan\frac{\chi_\Delta}{2\kappa_1},\ i = 0,1,2,3, \tag{50}$$

$$A_0 = 2\tilde{a}g'^2\cos^3\kappa_1,\ B_0 = 2\tilde{a}g'^2\cos^2\kappa_1\sin\kappa_1\frac{3b' - a'\cos^2\kappa_1}{b' + a'\cos^2\kappa_1}, \tag{51}$$

$$C_0 = 4\tilde{a}b'g'^2\cos\kappa_1\sin^2\kappa_1\frac{b' - a'\cos^2\kappa_1}{(b' + a'\cos^2\kappa_1)^2},$$

$$A_1 = \cos\kappa_1\left(\tilde{a} + \tilde{b} - 2\tilde{b}g'^2\sin^2\kappa_1\right),\ B_1 = \sin\kappa_1\left(\tilde{a} + \tilde{b} - 2\tilde{b}g'^2 - 2\tilde{b}g'^2\cos^2\kappa_1\right),$$

$$C_1 = 2(\tilde{a} - \tilde{b})g'^2\cos\kappa_1\sin^2\kappa_1,$$

$$A_2 = \cos\kappa_1 \sin^2\kappa_1 \left(\tilde{a} + \tilde{b} - 2\tilde{b}g'^2 \sin^2\kappa_1\right),$$
$$B_2 = \sin\kappa_1 \left[\sin^2\kappa_1 \left(\tilde{a} + \tilde{b} - 2\tilde{b}g'^2 \sin^2\kappa_1\right) - 2(\tilde{a} + \tilde{b})\cos^2\kappa_1\right],$$
$$C_2 = -2\cos\kappa_1 \sin^2\kappa_1 \left[\tilde{a} + \tilde{b} - 2\tilde{b}g'^2 \sin^2\kappa_1 - (\tilde{a} + \tilde{b})g'^2\left(3\cos^2\kappa_1 - 1\right)\right],$$
$$A_3 = 2\tilde{a}g'^2 \cos^3\kappa_1 (3\cos^2\kappa_1 - 1),\; B_3 = \frac{2\tilde{a}g'^2\cos^2\kappa_1 \sin\kappa_1}{b' + a'\cos^2\kappa_1}\left[a'\cos^2\kappa_1(3\cos^2\kappa_1 + 1) + \right.$$
$$\left. +3b'(5\cos^2\kappa_1 - 1)\right],\; C_3 = \frac{4\tilde{a}b'g'^2 \cos\kappa_1 \sin^2\kappa_1}{b' + a'\cos^2\kappa_1}\left[\frac{2b'(3\cos^2\kappa_1 - 1)}{b' + a'\cos^2\kappa_1} + 3\cos^2\kappa_1 + 1\right],$$
$$E_i = 5A_i + 2\kappa_1 B_i,\; i = 1, 2,\; D_i = A_i + \kappa_1 B_i + 2\kappa_1^2 C_i, i = 0, 1, 2, 3. \tag{52}$$

We emphasize that, in expressions (51), we excluded the coupling constant $\tilde{\alpha}'_s$ by means of not only the quantization condition of the energy levels in (46), but also of the identity that we installed in the process of the calculations for the spin parameters $a', b', \tilde{a}$ and $\tilde{b}$ (are given in (9) and (13)) and which has the form[3)]

$$a'(\tilde{a} + \tilde{b}) - 2\tilde{b}g'^2(a' + b') \equiv 0. \tag{53}$$

Table 1: Values of the r.m.s. and parameters for $\pi^\pm$-, $K^\pm$- and K_0-mesons

Mesons	M_1, GeV	m_u, GeV	m_d, GeV	m_s, GeV	m', GeV	g'	κ_1	$\tilde{\alpha}'_s$
$\pi^\pm$	0.13957	0.12802	0.15632		0.14147	1.00499	1.05773	14.93515
$K^\pm$	0.49368	0.12802		0.46733	0.24460	1.21700	0.59308	4.16075
K_0	0.49760		0.15632	0.46733	0.27028	1.15370	0.64702	4.66997

Mesons	$< r^2_{0,v} >_{\text{theor}}$, fm^2	$< r^2_{0,v} >_{\text{exp}}$, fm^2
$\pi^\pm$	0.43642	0.439 ± 0.007
$K^\pm$	0.33946	0.34 ± 0.05
K_0	-0.10969	-0.090 ± 0.021

By using obtained in (49), (51) and (52) the results, we calculate the r.m.s. for the ground level of bound s-state of pseudoscalar $\pi^\pm$-, $K^\pm$- and K_0-mesons ($n = 1, \ell = 0, a' = g'^2, b' = 1 - a', \tilde{a} = \tilde{b} = 1$) with masses: $M_{\pi^\pm} = 0.13957\,\text{GeV}$, $M_{K^\pm} = 0.49368\,\text{GeV}$ and $M_{K_0} = 0.49760\,\text{GeV}$ [27]. For this the r.m.s. (49) for the ground level of bound s-state with the energy M_1 (in units of GeV) we represent in the form

$$< r^2_{0,v} >^{\text{Coul}}_{\ell=0,n=1} = \frac{0.03894}{M_1^2}\left\{\frac{1}{2} + \frac{1}{2\kappa_1^2}\left[1 + \frac{(1+\tan^2\kappa_1)\hat{E}_1 + \dfrac{2(g'^2-1)}{g'^2}\hat{E}_2}{(1+\tan^2\kappa_1)\hat{D}_1 + \dfrac{2(g'^2-1)}{g'^2}\hat{D}_2}\right] - \right. \tag{54}$$
$$\left. - \frac{3\left[(1+\tan^2\kappa_1)\hat{D}_0 + \dfrac{2(g'^2-1)}{g'^2}\left(\hat{D}_3 - 2\hat{D}_2\right)\right]}{(1+\tan^2\kappa_1)\hat{D}_1 + \dfrac{2(g'^2-1)}{g'^2}\hat{D}_2}\right\} \text{fm}^2.$$

---

3) Analogous identity was installed and for the component system of two spinor particles with equal masses ($g' = 1$) in [20].

Here we introduced the notations

$$\hat{E}_i = 5\hat{A}_i + 2\kappa_1 \tan\kappa_1 \hat{B}_i, i = 1,2, \hat{D}_i = \hat{A}_i + \kappa_1 \tan\kappa_1 \hat{B}_i + 2\kappa_1^2 \tan^2\kappa_1 \hat{C}_i, i = 0,1,2,3, \quad (55)$$

$$\hat{A}_0 = \tilde{\sigma}, \hat{B}_0 = \frac{\tilde{\sigma}(3\hat{A}_1 - 4)}{\hat{A}_1}, \hat{C}_0 = \frac{2\tilde{\sigma}(\hat{A}_1 - 1)(\hat{A}_1 - 2)}{\hat{A}_1^2},$$

$$\hat{A}_1 = \sigma'(1 + \tan^2\kappa_1) + 1, \hat{B}_1 = \hat{A}_1 - 2, \hat{C}_1 = \tilde{\sigma} - 1,$$

$$\hat{A}_2 = \hat{A}_1 \tan^2\kappa_1, \hat{B}_2 = \hat{A}_1(\tan^2\kappa_1 - 2) - 2\tan^2\kappa_1, \hat{C}_2 = -2\hat{A}_1 - (\tilde{\sigma} + 1)(\tan^2\kappa_1 - 2),$$

$$\hat{A}_3 = -\tilde{\sigma}(\tan^2\kappa_1 - 2), \hat{B}_3 = \tilde{\sigma}\left[\tan^2\kappa_1 + 4 - \frac{4(\hat{A}_1 - 1)(\tan^2\kappa_1 - 2)}{\hat{A}_1}\right],$$

$$\hat{C}_3 = \frac{2\tilde{\sigma}(\hat{A}_1 - 1)}{\hat{A}_1}\left[\tan^2\kappa_1 + 4 - \frac{2(\hat{A}_1 - 1)(\tan^2\kappa_1 - 2)}{\hat{A}_1}\right],$$

where the spin parameters $\sigma' = b'/a'$ and $\tilde{\sigma} = \tilde{a}/\tilde{b}$ satisfies the identity

$$\tilde{\sigma} + 1 - 2g'^2(\sigma' + 1) \equiv 0 \quad (56)$$

with the values

$$\sigma' = \begin{cases} \frac{1}{g'^2} - 1 & \text{for } \hat{O} = \gamma_5 \text{ (pseudoscalar)}; \\ \frac{3}{2g'^2} - 1 & \text{for } \hat{O} = \gamma_\mu \text{ (vector)}; \\ -\frac{1}{2g'^2} - 1 & \text{for } \hat{O} = \gamma_5\gamma_\mu \text{ (pseudovector)}; \end{cases} \qquad \tilde{\sigma} = \begin{cases} 1 & \text{for } \hat{O} = \gamma_5; \\ 2 & \text{for } \hat{O} = \gamma_\mu; \\ -2 & \text{for } \hat{O} = \gamma_5\gamma_\mu. \end{cases} \quad (57)$$

The results of calculations for the r.m.s. (54) and the values of all parameters for the ground level of bound s-state of pseudoscalar $\pi^\pm$-, $K^\pm$- and K_0-mesons are provided in Tabl. 1. The values of r.m.s. for the ground level of bound s-state of pseudoscalar $\pi^\pm$-, $K^\pm$- and K_0-mesons belong to the confidence intervals of their experimental values [28–30].

In Fig. 2 we built the graphs of function $< r_{0,v}^2 >_{\ell=0,n=1}^{\rm Coul} = < r_0^2 >$, represented by expression (54), as the function of variable $\kappa_1 = \kappa$ for the ground level of bound s-state of pseudoscalar $\pi^\pm$-, $K^\pm$- and K_0-mesons with the values of parameters from Tabl. 1. From Fig. 2 we see that graphs to functions $< r_0^2 >$ for the r.m.s. (54) of the ground level of bound s-state of pseudoscalar $\pi^\pm$-, $K^\pm$- and K_0-mesons with the values of parameters from Tabl. 1 has singularities at "critical" values of the rapidity κ. The "critical" values of the rapidity $\kappa_{\pi^\pm}^{\rm conf} \approx 1.47110$, $\kappa_{K^\pm}^{\rm conf} \approx 0.96437$, $\kappa_{K_0}^{\rm conf} \approx 1.04870$ are defined the boundary of the quarks confinement region, forming mesons: at $\kappa \geq \kappa_i^{\rm conf}\,(i = \pi^\pm, K^\pm, K_0)$ the Coulomb constant $\tilde{\alpha}'_s < 0$. The "critical" values of the rapidity $\kappa_{\pi^\pm}^{\rm cr} \approx 0.83910$, $\kappa_{K^\pm}^{\rm cr} \approx 0.60387$, $\kappa_{K_0}^{\rm cr} \approx 0.63973$, probably, are defined or the boundary of the phase transition of quarks-gluon matter, or the boundary of the change to spatial configuration of the quarks, or at transition through these "critical" values of the rapidity $\kappa_i^{\rm cr}\,(i = \pi^\pm, K^\pm, K_0)$ takes place change as the phase of state for the quarks-gluon matter, so and its spatial configuration: at $\kappa = \kappa_i^{\rm cr}\,(i = \pi^\pm, K^\pm, K_0)$ their the Coulomb constant $\tilde{\alpha}'_s > 0$, and for small values of the rapidity their the Coulomb constant are small.

In conclusion, we shall note, that for large $-t = Q^2 \gg M_1^2$ the rapidity behaves as $\chi_\Delta \approx \ln(Q/M_1)^2$ and, consequently, the leading behavior of form factor $F_v(t) = F_{\ell=0,n=1}^{(+),\rm Coul}(t)/F_{\ell=0,n=1}^{(+),\rm Coul}(0)$ gives by expression

$$F_v(t)|_{-t\gg M_1^2} \approx \frac{8\pi\kappa_1^3 \tan^2\kappa_1(1 + \tan^2\kappa_1)\hat{C}_0}{\left(1 + \tan^2\kappa_1\right)\hat{D}_1 + \frac{2\left(g'^2 - 1\right)}{g'^2}\hat{D}_2} \frac{\ln(Q/M_1)^2}{(Q/M_1)^2}\left[1 + O\left(\ln^{-1}(Q/M_1)^2\right)\right]. \quad (58)$$

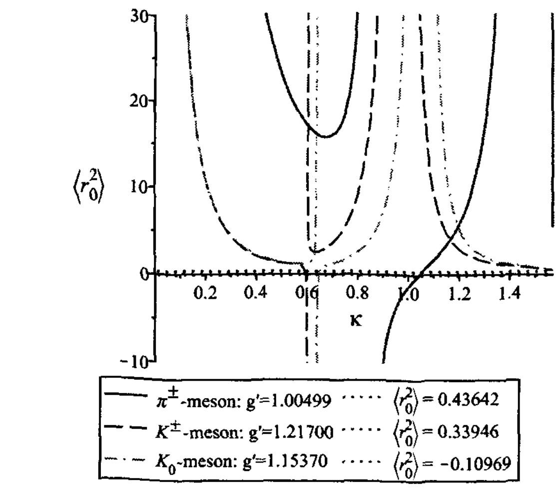

Figure 2: Behavior of function $< r_0^2 >$ as a function of the variable κ for the ground level of bound s-state of pseudoscalar $\pi^{\pm}$-, $K^{\pm}$- and K_0-mesons with the values of parameters from Tabl. 1.

Such behavior of the form factor $F_v(t)$ for large $-t$ complies with the prediction of the dimensional quark counting rules [31, 32], which gives $F(t) \sim |t|^{-1}$. In the case of relativistic spinless particles the decrease of the form factor occurs under the law $F(t) \sim (|t| \ln^3 |t|)^{-1}$ [17].

5. Conclusions

In the present study, we have obtained a new covariant expressions for the elastic form factor components and mean-square radius of a bound system formed by two relativistic spin 1/2 particles with arbitrary masses for the case of vector current. The pseudoscalar, vector and pseudovector composite systems were considered. For them is received identity, which installs the dependency between masses and quarks spin, forming composite systems.

As example, the expressions for the longitudinal component of form factor and mean-square radius of a bound system that formed by two relativistic spin 1/2 particles with arbitrary masses in the case of Coulomb potential were obtained. It is installed that the covariant wave RQP-function of Coulomb potential at $Q^2 = -t \gg 1$ give the decrease for this form factor under the law $F_\pi \sim |t|^{-1}$, which predicts the dimensional quark counting rules. Values of the mean-square radius for the ground level of bound s-state of pseudoscalar $\pi^{\pm}$-, $K^{\pm}$- and K_0-mesons with the Coulomb interaction are calculated and its belong to the confidence intervals of their experimental values. The influence of quarks masses on the behavior of mean-square radius for the ground level of bound s-state of pseudoscalar $\pi^{\pm}$-, $K^{\pm}$- and K_0-mesons are learned.

The consideration is conducted within the framework of relativistic quasipotential approach on the basis of covariant Hamiltonian formulation of quantum field theory, by transition to the three-dimensional relativistic configurational representation in the case of bound system of two relativistic spin particles with arbitrary masses.

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