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New analysis of the nucleon spin sum rules in the analytic approach in QCD

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We study the difference between the higher order analytic perturbation theory (APT) and the standard perturbative description in the analysis of recent Jefferson Lab (JLab) data on the first moments of spin structure functions $g_1^{p,n}$ at $0.05 < Q^2 < 3\,\mathrm{GeV}^2$. It is shown that values of the higher twist parameters extracted from the mentioned data by using the APT approach provides a better convergence of the higher-twist series than with the common perturbative QCD. We obtain a good quantitative description of all the JLab data sets down to $Q \simeq 280\,\mathrm{MeV}$.

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1. Introduction

The analysis of lepton Deep Inelastic Scattering (DIS) by Operator Product Expansion (OPE) has been since long time a powerful tool to test predictions of QCD [1]. In particular, the proton and neutron spin sum rules, $\Gamma_1^{p,n}$, and the Bjorken sum rule (BSR) [2] are a renown target ground for testing different nonperturbative methods in the low energy domain. Higher-twist parameters (known also as the colour polarizabilities) are important ingredients of the nucleon spin structure. Their extraction from experimental studies is relatively complicated as they are most pronounced at low momentum transfer $Q^2 < 1\ \mathrm{GeV}^2$. Although very accurate Jefferson Lab (JLab) data on the lowest moments of the spin-dependent proton and neutron structure functions $\Gamma_1^{p,n}(Q^2)$ in the range $0.05 < Q^2 < 3\,\mathrm{GeV}^2$ are now available [3], higher twist contributions are shadowed by unphysical singularities of QCD coupling. As it was shown in Ref. [4, 5], to cure this problem one can use singularity-free Analytic Perturbation Theory (APT) [6] which allowed quite accurate extraction of higher twist and fairly well description of data down to rather low Q.

The APT based on the causality principle implemented as analyticity imperative in the complex Q^2-plane for the QCD coupling α_S and the higher functions in the form of the Källen-Lehmann spectral representation (for a review on APT concepts and algorithm see Ref. [7]) and give possibility to study low-energy nonperturbative QCD region. But three-loop level expressions for higher APT functions involve the special Lambert function (Refs. [8, 9]), making them difficult to use. Therefore it is necessary to apply the the effective logarithm approach proposed in Ref. [10] for higher APT functions. This leads to a loss of accuracy of the extracted nonperturbative parameters from the precision data.

In the present paper we continue investigation started in Ref. [5]. By using exact higher APT functions up to N^{2}LO order and the recent JLab data on Γ_1^p at $0.05 < Q^2 < 3.2\,\mathrm{GeV}^2$ [3], we extract values of higher twist coefficients. We also compare our results with those obtained by using the effective logarithm approach. We demonstrate that the usage of the exact solution allows us to describe JLab data down to $Q \sim 280$MeV.

2. Theoretical Framework: PT and APT approaches

The lowest Cornwall-Norton moments of spin-dependent proton and neutron structure functions $g_1^{p,n}$ are defined as follow

$$\Gamma_1^{p,n}(Q^2) = \int_0^1 dx\, g_1^{p,n}(x, Q^2)\,, \tag{1}$$

with $x = Q^2/2M\nu$, the energy transfer ν and the nucleon mass M. At large $Q > \Lambda$, the moments $\Gamma_1^{p,n}(Q^2)$ is given by the OPE series in powers of $1/Q^2$ with the expansion coefficients related to nucleon matrix elements of operators of a definite twist (defined as the dimension minus the spin of the operator), and coefficient functions in the form of perturbative QCD series in powers of α_S. The total expression for the $\Gamma_1^{p,n}(Q^2)$ including the HT contributions reads

$$\Gamma_1^{p,n}(Q^2) = \frac{1}{12}\left[\left(\pm a_3 + \frac{1}{3}a_8\right) E_{NS}(Q^2) + \frac{4}{3}a_0^{inv}\, E_S(Q^2)\right] + \sum_{i=2}^{\infty} \frac{\mu_{2i}^{p,n}}{Q^{2i-2}}, \tag{2}$$

where the triplet and octet axial charges are $a_3 \equiv g_A = 1.267 \pm 0.004$ [11] and $a_8 = 0.585 \pm 0.025$ [12], respectively, $\mu_{2i}^{p,n}$ - higher twist coefficients. E_{NS} is the nonsinglet Wilson coefficient calculated as series in powers of α_S up to N^{2}LO order [13]

$$E_{NS}(Q^2) = 1 - \frac{\alpha_S}{\pi} - d_1 \left(\frac{\alpha_S}{\pi}\right)^2 - d_2 \left(\frac{\alpha_S}{\pi}\right)^3 - O(\alpha_S^4), \qquad (3)$$

where for $n_f = 3$ in the MS scheme coefficients $d_1 = 3.583$ [14], $d_2 = 20.215$ [13]. As for the singlet axial charge a_0, it is convenient to work with its RG invariant definition in the MS scheme, $a_0^{inv} = a_0(Q^2 = \infty)$, in which all of the Q^2 dependency is factorized into the definition of the singlet Wilson coefficient $E_S(Q^2)$ [15]

$$E_S(Q^2) = 1 - \frac{\alpha_S}{\pi} - 1.096 \left(\frac{\alpha_S}{\pi}\right)^2 - O(\alpha_S^3). \qquad (4)$$

The perturbative running coupling, $\alpha_S(Q^2)$, is obtained by integration of the renormalization group equation with the three-loop β-function. It is clearly that the low-energy behavior of the Wilson coefficients determined by infrared behavior of the strong running coupling $\alpha_S(Q^2)$. In this region the standard running coupling have unphysical singularities, that substantially complicates the analysis of low-energy data. This problem may be solved by using APT approach (see Ref. [4]).

Let start our analysis from the difference of the proton and neutron spin sum rules. In this case the singlet and octet contributions are canceled out giving rise to the more fundamental Bjorken sum rule, which at $Q^2 \to \infty$ comes to its renowned form $\Gamma_1^{p-n} = g_A/6$. The Bjorken integral is defined as integral of difference between the proton $g_1^p(x, Q^2)$ and the neutron $g_1^n(x, Q^2)$ structure function over Bjorken variable x, with fixed momentum transfer Q^2 [2]

$$\Gamma_1^{p-n}(Q^2) = \int_0^1 \left[g_1^p(x, Q^2) - g_1^n(x, Q^2)\right] dx. \qquad (5)$$

At finite $Q^2 > \Lambda^2$, the Bjorken integral (5) is given by the OPE series.

$$\Gamma_1^{p-n}(Q^2) = \frac{g_A}{6}\left[1 - \Delta^{PT}(Q^2)\right] + \sum_{i=2}^{\infty} \frac{\mu_{2i}^{p-n}(Q^2)}{Q^{2i-2}}, \qquad (6)$$

where $\Delta^{PT}(Q^2) \equiv 1 - E_{NS}(Q^2)$ is the standard perturbative part of the Bjorken sum rule correction.

In the framework of the analytic approach we can write the expression for $\Gamma_1^{p-n}(Q^2)$ in the form which is analogous to one in the standard PT (6). The three-loop APT approximation to $\Delta(Q^2)$ write as follows

$$\Delta^{APT}(Q^2) = \frac{\mathcal{A}_1(Q^2)}{\pi} + d_1 \frac{\mathcal{A}_2(Q^2)}{\pi^2} + d_2 \frac{\mathcal{A}_3(Q^2)}{\pi^3}, \tag{7}$$

where the coefficients d_1 and d_2 are the same as in Eq. (3) and the functions $\mathcal{A}_k$ are derived from the spectral representation and correspond to the discontinuity of the k-th power of the PT running coupling constant

$$\mathcal{A}_k(Q^2) = \frac{1}{\pi} \int_0^{+\infty} \frac{\mathrm{Im}([\alpha_S(-\sigma)]^k)\, d\sigma}{\sigma + Q^2}. \tag{8}$$

The function $\mathcal{A}_1(Q^2)$ defines the APT running coupling constant, $\alpha_S^{APT}(Q^2) = \pi \mathcal{A}_1(Q^2)$, which in the one-loop case is given by [6]

$$\mathcal{A}_1^{(1)}(Q^2) = \frac{1}{\beta_0} \left[\frac{1}{L} + \frac{\Lambda^2}{\Lambda^2 - Q^2} \right], \tag{9}$$

where $L = \ln(Q^2/\Lambda^2)$ and $\beta_0 = 11 - 2n_f/3$ is the one-loop coefficient of the β-function. The higher functions $\mathcal{A}_k$ are related to the lower ones recursively by differentiating [6]

$$\mathcal{A}_{k+1}^{(1)} = -\frac{1}{k\beta_0} \frac{d\mathcal{A}_k^{(1)}}{dL}. \tag{10}$$

Analogous three-loop expressions involve the special Lambert function and are more intricate, and they can be found in Refs. [8, 9]. Meanwhile, even for the three-loop APT case, there exists a possibility to employ the effective logarithm approach proposed in Ref. [10]. In the present context, in the region $Q < 5$ GeV one may use simple model one-loop expressions with some effective logarithm L^* (see Ref. [5])

$$\mathcal{A}_{1,2,3}^{(3)}(L) \to \mathcal{A}_{1,2,3}^{mod} = \mathcal{A}_{1,2,3}^{(1)}(L^*), \quad L^* = \ln(Q/\Lambda_{eff}^{(1)})^2,$$

$$\Lambda_{eff}^{(1)} \simeq \frac{1}{k} \Lambda^{(3)}, \quad (k \simeq 2). \tag{11}$$

Now we exam the effective logarithm APT approach (11) in more detail. In Fig. 1 we show the ratio of the exact three-loop APT running coupling to the running coupling in the effective logarithm APT approach:

$$N(Q) = \frac{\alpha_S^{APT;N^2LO}(L)}{\alpha_S^{APT;LO}(L^*)} = \frac{\mathcal{A}_1^{(3)}(L)}{\mathcal{A}_1^{(1)}(L^*)}. \tag{12}$$

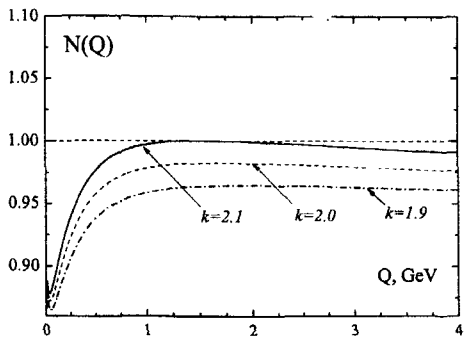

FIG. 1. The ratio of the three-loop exact APT running coupling to effective logarithm APT approach with different k at $\Lambda^{(3)} = 380\,\text{MeV}$.

From Fig. 1 one can see that the maximal error of the model in the low-energy region no more 10%. On the other hand, in Ref. [5] this approach was successfully applied for higher twist analysis of low energy data on Bjorken sum rule. It should be stressed the behavior of α_S has a significant influence on the behavior of the BSR correction. Therefore, error of the effective logarithm approach may reduce the values of HT terms. This motivates study in the next section.

3. Higher twist coefficients

In this section we present an analysis of the JLab data [3] using the exact three-loop PT and APT expressions for the QCD correction $\Delta(Q^2)$ in Eq. (6). From this JLab data we extract the values of the HT coefficients μ_{2i}^{p-n}. The minimal borders of fitting domains in Q^2 are settled from the

ad hoc restriction $\chi^2 < 1/D.o.f.$ and monotonous behavior of the resulting fitted curves.

Note, that in the following when calculating the observables at any particular order of theory we will employ the prescription for the BSR correction in the infrared region, where the order of power α_S-series in QCD correction is matched with the loop order in α_S itself.

Now we test the effective logarithm APT approach (11). We performed best 3-parametric fits for different values of the coefficient k in the range $1.9 \div 2.1$. Our results are shown in Fig. 2.

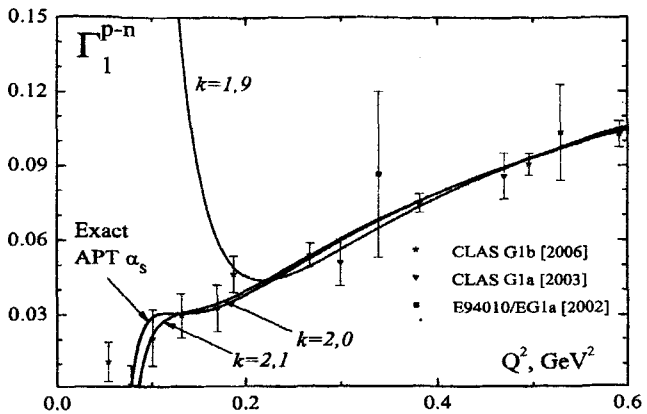

FIG. 2. Best 3-parametric fit results for Γ_1^{p-n} data in the exact APT and in the effective logarithm APT approach with different values of k.

From Fig. 2 one can see that fit results are quite sensitive to value of the coefficient k. Small deviations of the coefficient k can destroy fit, for example with $k = 1.9$. On the other hand, the effective logarithm APT approach with $k = 2.0$ and $k = 2.1$ accurately describes the results and very close to exact. This is due to the fact that the basic models error is in the region $Q << \Lambda$, where experimental points are excluded from the analysis because does not belong to the fitting domains.

Next, we repeat the analysis made in Ref. [5]. We extract HT coefficients from BSR data with using exact APT running coupling and compare our

results with those obtained by using effective logarithm APT approach with $k = 2.0$. In Table 1 we present the combined fit results of the BSR $\Gamma_1^{p-n}(Q^2)$ data in APT and conventional PT approaches. As seen from Table 1, our

Table 1: Combined fit results of BSR data for the HT terms in APT and standard PT approaches.

Approach	Q^2_{min}, GeV^2	μ_4^{p-n}/M^2	μ_6^{p-n}/M^4	μ_8^{p-n}/M^6
N^{2}LO APT	0.47	−0.055(3)	0	0
effective logarithm	0.17	−0.062(4)	0.008(2)	0
with $k = 2.0$	0.10	−0.068(4)	0.010(3)	−0.0007(3)
	0.47	−0.049(3)	0	0
N^{2}LO APT	0.17	−0.069(4)	0.014(1)	0
exact	0.078	−0.065(7)	0.011(3)	−0.0003(7)
	0.66	−0.01(6)	0	0
N^{2}LO PT	0.66	−0.005(15)	0.005(16)	0
exact	0.47	0.03(1)	−0.10(2)	0.06(1)

fits in APT give the HT values indicating a better convergence of the OPE series due to decreasing magnitudes and alternating signs of consecutive terms, in contrast to the usual PT fit results. Also effective logarithm APT approach gives results for values of the HT terms, which is close to results in exact APT. However, the application of effective logarithm APT approach slightly modifies values of HT coefficients leaving corresponding curves close to each other (see Fig. 2).

In Fig. 3, we show best fits of the combined data set for the BSR function $\Gamma_1^{p-n}(Q^2)$ in the frame of N^{2}LO APT and PT approach at fixed $\Lambda = 380\,\text{MeV}$. These curves correspond to the HT values from Table 1.

Thus, the best APT fit allows one to describe well all the BSR data at scales down to $Q \sim 280\,\text{MeV}$ with only the first three terms of the OPE series, unlike the usual PT case, where such fits happened to be impossible (due to the unphysical singularities issue) even for an increasing number of HT terms. This means, that the lower bound of the perturbative QCD applicability (supported by power HT terms) now may be shifted down to $Q \sim 280\,\text{MeV}$.

To show the stability of our fits, in Figs. 4 and 5 we present the characteristic value of the fit, $\chi^2/D.o.f.$, as function of twist-4 term μ_4^{p-n} at

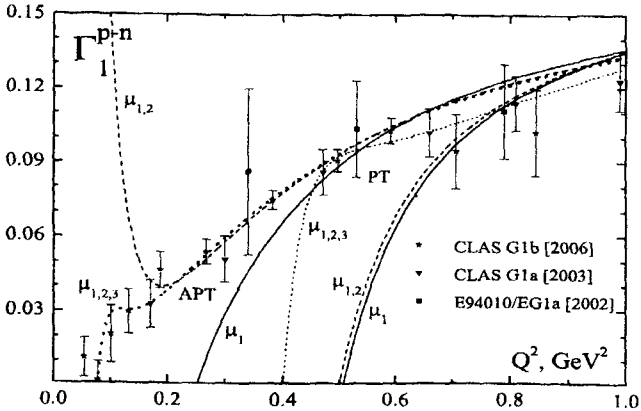

FIG. 3. Best 1,2,3-parametric fits of the JLab data on the Bjorken SR calculated in exact APT and standard PT approaches.

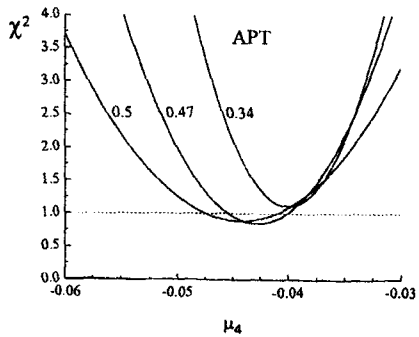

FIG. 4. Behavior of $\chi^2/D.o.f.$ vs. μ_4^{p-n} in the APT at different values of Q^2_{min} (the numbers beside curves, in GeV^2).

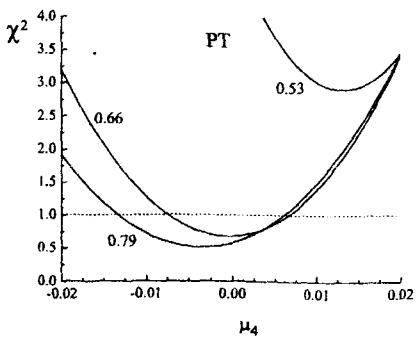

FIG. 5. Behavior of $\chi^2/D.o.f.$ vs. μ_1^{p-n} in the PT at different values of Q^2_{min} (the numbers beside curves, in GeV^2).

different values of the boundary, Q^2_{min} (the numbers beside curves, in GeV^2) in the APT and PT cases, respectively. We see that at lower values $Q^2_{min} \sim 0.5\,\text{GeV}^2$ the APT description turns out to be more precise and stable by comparison the standard PT case. Note that it would be reasonable to take the spread between different minima as an optimistic error

bar.

Let us turn now to the proton structure function moment $\Gamma_1^p(Q^2)$ [see Eq. (2)]. We performed an analysis similar to described above for the Bjorken sum rule. Our results are presented in Fig. 6. We obtain quanti-

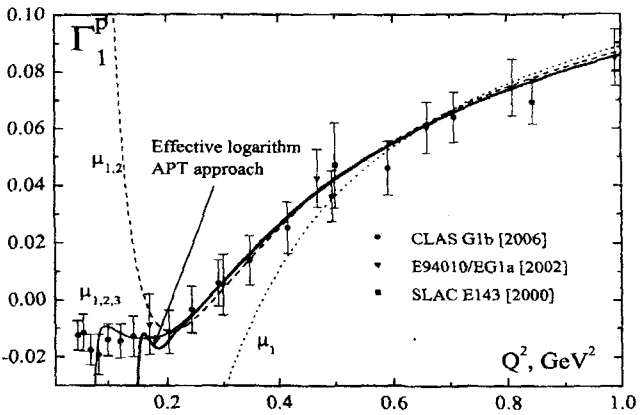

FIG. 6. Best 1,2,3-parametric fits of the JLab and SLAC data on the proton sum rule calculated with the exact APT and best 3-parametric fit with the effective logarithm APT approach.

tative results as well as for the BSR. Thus, we found that application of the effective logarithm APT approach instead of the exact APT leads to an error that is smaller than the error, which retrieves the values of higher twists.

4. Conclusion

We have reconsidered lowest moments of the spin-dependent proton and neutron structure functions $\Gamma_1^{p,n}(Q^2)$ and $\Gamma_1^{p-n}(Q^2)$ by using the APT approach, which, in contrast to PT, does not lead to any unphysical singularities at low Q^2-scales, $Q^2 < 1\,\mathrm{GeV}^2$. High precision JLab data given in the low domain, $0.05 < Q^2 < 3\,\mathrm{GeV}^2$, require reliable methods of their description. Taking into account the analytic properties of the moments of the deep inelastic structure functions, we have used an analytically improved

theoretical description of the first moments of spin structure functions $g_1^{p,n}$. In previous paper [5] to describe above JLab data [3] and extract the HT coefficients it was successfully applied the approximate formulae for the three-loop analytic running coupling. Here we have repeated the analysis performed in Ref. [5] using the exact solution of the RG equation to the three-loop running coupling. We have got that the APT result practically does not change. The reason is that the region where the error of approximation can achieved about 10% disposes outside the fitting domain. So, our analysis has shown that in the description of the spin-dependent sum rules up to the scale of about the QCD parameter Λ can be successfully used the effective logarithm APT approach.

To conclude we note that in addition to the previous result [5, 16] where a good quantitative description of all the Jefferson Lab data sets down to $Q \simeq 350$ MeV has been achieved, in this paper we have achieved a good quantitative description above data down to $Q \simeq 280$ MeV.

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Note added. Just before completing this paper we became aware of the paper [17] where, for the first time, the order α_S^4 contribution to the Bjorken sum rule has been computed. Using this result, $d_3 = 175.7$, we compared the convergence properties of the PT expansion and the APT series for the QCD correction $\Delta(Q^2)$ in Eq. (6). As input, we used the central value of four-loop running coupling, which was extracted from the decay widths of the τ-lepton: $\alpha_S(M_\tau^2 = 1.78^2\,\mathrm{GeV}^2) = 0.322 \pm 0.024$ [18], fixing in such a way the value of the QCD correction in Eq. (6). The successive terms of

the PT series respectively constitute 58%, 20% , 12%, and 10% of the total. At the same time, the corresponding contributions to the APT series make up 74%, 20%, 5%, and 1% of the total. The convergence of the APT series seems to be somewhat better behaved than is that of the PT expansion at such small $Q \simeq 1.78\,\mathrm{GeV}$.

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