Nonlinear Dynamics and Applications. Vol. 26 (2020) 472 - 482

Form Factor for a System of the Two Fermions an Equal Masses in the Relativistic Quasipotential Approach

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The new relativistic form factor of the two relativistic with equal masses fermions bound state for the scalar and the vector-current cases are obtained. Consideration is conducted within the framework of relativistic quasipotential approach on the basis of covariant Hamiltonian formulation of quantum field theory by transition to the three-dimensional relativistic configurational representation in the case of two relativistic spin particles with equal masses.

PACS numbers:
Keywords:

1. Introduction

To describe the behavior of the form factors the different pole vector-dominance models (VDM) are often used [1, 2]. However the models VDM fail in description experimently of the observed for large importances of the momentum transfer of the system $-t = Q^2$ the quick decrease of proton electromagnetic form factor at time-like region according to the law of dipole $\sim t^{-2}$. The using of three-dimensional relativistic covariant two-particle quasipotential (RQP) equation of Logunov-Tavkhelidze [3] for description of the form factors of composite systems was executed in [4, 5]. However, use of the equation Logunov-Tavkhelidze for wave function in the momentum representation has not allowed to research the behavior of the form factor in broad interval of importances of the momentum transfer of the relativistic two-particle bound system. The other model in which the contribution of small distances in the proton form factor take into account was considered in [6, 7]. This model is based on invariant description of the structure of the particles in relativistic configurational space that was carried in [8] in the case of interaction between two relativistic spinless particles with equal masses m. In the RQP approach for a bound system of two relativistic spinless particles of arbitrary masses developed in [9, 10], new covariant expressions of elastic form factor for the cases of a scalar and vector currents as functions of the invariant variable $\Delta^2_{\mathcal{P},Q}$, which there is the square of the momentum-transfer vector in the Lobachevsky space, has been obtained in [11, 12].
The aim of present study, considered as continuation in [11, 12] is to obtain the new expression for the elastic form factor of the two relativistic with equal masses fermions bound state in the case of scalar and vector currents. Consideration is conducted within the framework of RQP approach [13, 14] on the basis of covariant Hamiltonian formulation of quantum field theory [15] by transition to the three-dimensional relativistic configurational representation in the case of two relativistic spin particles with equal masses [8].

2. Equation for the Wave Function

Within the framework of RQP approach [13, 14] for spherically symmetric potentials equation for the wave function in the $\mathbf{r}$-representation [8] in the case of two relativistic

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particles with equal masses and spin 1/2 has the form [16] (we use the system of units where $\hbar = c = 1$)

$$\frac{1}{2m}(M_Q - \widehat{H}_0)\psi_{M_Q}(\mathbf{r}) = V(\mathbf{r})\hat{A}\Big(\frac{\widehat{H}_0}{2m}\Big)\psi_{M_Q}(\mathbf{r}). \tag{1}$$

Here $M_Q^2 = s_q = \mathcal{Q}^2 = (q_1+q_2)^2 = \mathcal{Q}_0^2 - \boldsymbol{\mathcal{Q}}^2$, where $q_i, i = 1,2$ – 4-momentas of composite particle, the operator

$$\widehat{H}_0 = 2m\left[\cosh\left(i\lambda\frac{\partial}{\partial r}\right) + \frac{i\lambda}{r}\sinh\left(i\lambda\frac{\partial}{\partial r}\right) - \frac{\lambda^2}{2r^2}\Delta_{\theta,\varphi}\exp\left(i\lambda\frac{\partial}{\partial r}\right)\right] \tag{2}$$

is the operator of free Hamiltonian while $\Delta_{\theta,\varphi}$ is its the angular part, and $\lambda = 1/m$ is the Compton wavelength; quasipotential $V(\mathbf{r})$ is local in the sense of Lobachevsky geometry, the group parameter r plays the role of modulus of relativistic relative coordinate $\mathbf{r}$ ($\mathbf{r} = r\mathbf{n}, |\mathbf{n}| = 1$), and the operator $\hat{A}$ is defined as

$$\hat{A}\Big(\frac{\widehat{H}_0}{2m}\Big) = \frac{1}{4}\left[a\Big(\frac{\widehat{H}_0}{2m}\Big)^2 + b\right], \tag{3}$$

$$a = \begin{cases} 1 & , \hat{O} = \gamma_5 \,(\text{pseudoscalar}); \\ 1/2 & , \hat{O} = \gamma_\mu \,(\text{vector}); \\ -1/2 & , \hat{O} = \gamma_5\gamma_\mu \,(\text{pseudovector}); \end{cases} \qquad b = \begin{cases} 0 & , \hat{O} = \gamma_5 \,(\text{pseudoscalar}); \\ 1/4 & , \hat{O} = \gamma_\mu \,(\text{vector}); \\ 3/4 & , \hat{O} = \gamma_5\gamma_\mu \,(\text{pseudovector}). \end{cases} \tag{4}$$

For simplicity, we assume, in just the same way as in [16, 17], that the quasipotential has a bispinor structure of the $I \otimes I$ form and that the vertex function also has a specific momentum-variable-independent spinor structure that is proportional to the matrix $\hat{O}$; for $\hat{O}$, we choose the Dirac matrices γ_5, γ_μ, and $\gamma_5\gamma_\mu$ ($\mu = 0, 1, 2, 3$). This choice of the matrix $\hat{O}$ has allowed us find exact solutions of the RQP equation (1) [16].

By using the expansion of the RQP wave function $\psi_{M_Q}(\mathbf{r})$ on a Legendre function $P_\mu(z)$ of the first kind,

$$\psi_{M_Q}(\mathbf{r}) = \sum_{\ell=0}^{\infty}(2\ell+1)i^\ell\frac{\varphi_\ell(r,\chi)}{r}P_\ell\Big(\frac{\boldsymbol{\Delta}_{q,m\lambda_Q}\cdot\mathbf{r}}{|\boldsymbol{\Delta}_{q,m\lambda_Q}|r}\Big),$$

Eq. (1) transformed to the equation for the partial wave function[1)]:

$$\left(\widehat{H}_{0,\ell}^{\text{rad}} - \cosh\chi\right)\varphi_\ell(r,\chi) = -V(r)\hat{A}\left(\widehat{H}_{0,\ell}^{\text{rad}}\right)\varphi_\ell(r,\chi). \tag{5}$$

Here the operator $\hat{A}$ is defined in (3), the operator

$$\widehat{H}_{0,\ell}^{\text{rad}} = \cosh\left(i\lambda\frac{d}{dr}\right) + \frac{\lambda^2\ell(\ell+1)}{2r(r+i\lambda)}\exp\left(i\lambda\frac{d}{dr}\right) \tag{6}$$

is the radial part of free Hamiltonian operator (2), and χ is the rapidity related with the relative 3-momentum and energy by the formulas $\boldsymbol{\Delta}_{q,m\lambda_Q} = m\sinh\chi\mathbf{n}_{\Delta_{q,m\lambda_Q}}, |\mathbf{n}_{\Delta_{q,m\lambda_Q}}| = 1, M_Q = 2\Delta^0_{q,m\lambda_Q}, \Delta^0_{q,m\lambda_Q} = m\cosh\chi$, where $\Delta^0_{q,m\lambda_Q}$ and $\boldsymbol{\Delta}_{q,m\lambda_Q}$ are, respectively, the

1) In [17], this equation in the case of two relativistic particles with equal masses and spin 1/2 it was obtained for other the determination of wave function and quasipotential.

time and spatial components of the 4-vector $\Lambda_{\lambda_Q}^{-1} q = \Delta_{q,m\lambda_Q}$ from the Lobachevsky space with the velocity 4-vector of the composite particle $\lambda_Q = (\lambda_Q^0; \boldsymbol{\lambda}_Q) = \mathcal{Q}/\sqrt{\mathcal{Q}^2}$:

$$\Lambda_{\lambda_Q}^{-1}\mathbf{q} = \boldsymbol{\Delta}_{q,m\lambda_Q} = \mathbf{q}(-)m\boldsymbol{\lambda}_Q = \mathbf{q} - \boldsymbol{\lambda}_Q\left(q_0 - \frac{\mathbf{q}\cdot\boldsymbol{\lambda}_Q}{1+\lambda_Q^0}\right), \tag{7}$$
$$(\Lambda_{\lambda_Q}^{-1}q)^0 = \Delta_{q,m\lambda_Q}^0 = q_0\lambda_Q^0 - \mathbf{q}\cdot\boldsymbol{\lambda}_Q = \sqrt{m^2 + \boldsymbol{\Delta}_{q,m\lambda_Q}^2},$$

and all 4-momenta belong to the upper sheet of the mass hyperboloid

$$\Delta_{q,m\lambda_Q}^2 = \Delta_{q,m\lambda_Q}^{02} - \boldsymbol{\Delta}_{q,m\lambda_Q}^2 = m^2. \tag{8}$$

3. Form Factor for a System of the Two Fermions

In Ref. [6, 7] the form factor of two-particle system was defined as the matrix element of the local current operator between bound states with the 4-momentum $\mathcal{Q}$ and $\mathcal{P}$ through the covariant wave RQP-functions satisfying RQP-equation in the momentum representation. Then, the invariant expression in the momentum representation for the matrix element of the local current operator of the two relativistic with equal masses fermions bound state in the case of scalar current has the form

$$< \mathcal{P}|J|\mathcal{Q} >= -\frac{z_1}{(2\pi)^3}\int d\tau_{\mathcal{P}} d\tau_{\mathcal{Q}} d^{(4)}k_1 d^{(4)}k_1' d^{(4)}k_2 \Gamma_{\mathcal{P}}^{\alpha\beta+}(k_1, k_2; \lambda_{\mathcal{P}}\tau_{\mathcal{P}}) \times \tag{9}$$
$$\times S_\gamma^{(+)\alpha}(k_1, m)\frac{1}{(\tau_{\mathcal{P}} + i\varepsilon)(\tau_{\mathcal{Q}} - i\varepsilon)} S_\delta^{(+)\gamma}(k_1', m)\Gamma_{\mathcal{Q}}^{\delta\kappa}(k_1', k_2; \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}})\, S_\kappa^{(+)\beta}(k_2, -m)\times$$
$$\times\delta^{(4)}(-\mathcal{Q} + k_1' + k_2 - \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}})\,\delta^{(4)}(\mathcal{P} - k_1 - k_2 + \lambda_{\mathcal{P}}\tau_{\mathcal{P}}) + (1 \leftrightarrow 2).$$

Here $\Gamma_{\mathcal{Q}}^{\delta\kappa}(k_1', k_2; \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}})$ and $\Gamma_{\mathcal{P}}^{\alpha\beta}(k_1, k_2; \lambda_{\mathcal{P}}\tau_{\mathcal{P}})$ are the vertex functions, where $\alpha, \beta, \ldots, \kappa (= 0, 1, 2, 3)$ are bispinor indices; $S^{(+)}(k_i, m) = \theta(k_{i0})(\hat{k}_i + m)\delta(k_i^2 - m^2)$ are the positive-frequency parts of the spinor Green's functions, where $\hat{k}_i = k_i^\mu\gamma_\mu$, and $k_i\, (p_i, q_i, i = 1, 2)$ are 4-momenta of the constituents with masses $m_1 = m_2 = m$, and all 4-momenta of the particles belong to the upper sheet of the mass hyperboloid (8). The vertex functions for simplicity, same as in [16] at conclusion of the equation (1), has a specific momentum-variable-independent spinor structure that is proportional to the matrix $\hat{O}$,

$$\Gamma_{\mathcal{Q}}^{\delta\kappa}(k_1', k_2; \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}) = \hat{O}^{\delta\kappa}\Gamma_{\mathcal{Q}}(k_1', k_2; \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}),\ \Gamma_{\mathcal{P}}^{\alpha\beta}(k_1, k_2; \lambda_{\mathcal{P}}\tau_{\mathcal{P}}) = \hat{O}^{\alpha\beta}\Gamma_{\mathcal{P}}(k_1, k_2; \lambda_{\mathcal{P}}\tau_{\mathcal{P}}),$$

and for $\hat{O}$, we choose the Dirac matrices γ_5, γ_μ, and $\gamma_5\gamma_\mu$ $(\mu = 0, 1, 2, 3)$.

The expression in (9) corresponds diagram on figure 1, where because of the transition to different proper times of the system before $(\tau_{\mathcal{Q}})$ and after $(\tau_{\mathcal{P}})$ the interaction event, the 4-velocities of composite particle before $[\lambda_{\mathcal{Q}}$, where $\lambda_{\mathcal{Q}} = \mathcal{Q}/\sqrt{\mathcal{Q}^2}$, $\mathcal{Q}^2 = (q_1 + q_2)^2 = s_q = M_{\mathcal{Q}}^2]$ and after $[\lambda_{\mathcal{P}}$, where $\lambda_{\mathcal{P}} = \mathcal{P}/\sqrt{\mathcal{P}^2}, \mathcal{P}^2 = (p_1 + p_2)^2 = s_p = M_{\mathcal{P}}^2]$ the interaction event are also different.

Within this approach [11, 12], the vertex functions $\Gamma_{\mathcal{Q}}(k_1', k_2; \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}})$ and $\Gamma_{\mathcal{P}}(k_1, k_2; \lambda_{\mathcal{P}}\tau_{\mathcal{P}})$ for $\lambda_{\mathcal{Q}} \upuparrows \mathcal{Q}$ and $\lambda_{\mathcal{P}} \upuparrows \mathcal{P}$ depend each on only one the Lorentz-invariant scalar parameter – $\mathcal{Q}k_2$ and $\mathcal{P}k_2$, respectively, and we introduce the notation

$$\Gamma_{\mathcal{Q}}(k_1', k_2; \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}) = \Gamma_{M_{\mathcal{Q}}}(\mathcal{Q}k_2),\ \Gamma_{\mathcal{P}}(k_1, k_2; \lambda_{\mathcal{P}}\tau_{\mathcal{P}}) = \Gamma_{M_{\mathcal{P}}}(\mathcal{P}k_2).$$

We execute the integration respecting of $k_{10}', k_{i0}, i = 1, 2$ in (9) in just the same way as we performed integration in [11, 12]. As a result, expression (9) for the current takes the

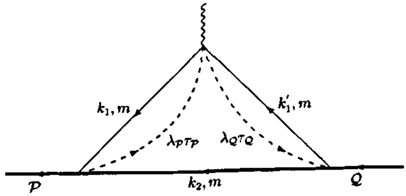

Figure 1: The diagram of matrix element of the local current operator in the case of two relativistic spin particles with equal masses.

form

$$< \mathcal{P}|J|\mathcal{Q} >= -\frac{z_1}{(4\pi)^3}\int \frac{d\tau_{\mathcal{P}}d\tau_{\mathcal{Q}}d\mathbf{k}_1 d\mathbf{k}_1' d\mathbf{k}_2}{\sqrt{m^2+\mathbf{k}_2^2}}\frac{\Gamma^*_{M_{\mathcal{P}}}(\mathcal{P}k_2)}{\sqrt{m^2+\mathbf{k}_1^2}(\tau_{\mathcal{P}}+i\varepsilon)}\times \tag{10}$$
$$\times \mathrm{Tr}[\hat{O}^+(\hat{k}_1+m)(\hat{k}_1'+m)\hat{O}(\hat{k}_2-m)]\frac{\Gamma_{M_{\mathcal{Q}}}(\mathcal{Q}k_2)}{\sqrt{m^2+\mathbf{k}_1'^2}(\tau_{\mathcal{Q}}-i\varepsilon)}\times$$
$$\times\delta^{(4)}(-\mathcal{Q}+k_1'+k_2-\lambda_{\mathcal{Q}}\tau_{\mathcal{Q}})\,\delta^{(4)}(\mathcal{P}-k_1-k_2+\lambda_{\mathcal{P}}\tau_{\mathcal{P}})+(1\leftrightarrow 2).$$

In order to perform integration with respect to $\mathbf{k}_1', \mathbf{k}_1, \tau_{\mathcal{Q}}$ and $\tau_{\mathcal{P}}$ in expression (10), we make, in the integrals with respect to $\mathbf{k}_1'$ and $\mathbf{k}_1$, the pure Lorentz transformations (7) $L = \Lambda^{-1}_{\lambda_{\mathcal{Q}}}$ and $L = \Lambda^{-1}_{\lambda_{\mathcal{P}}}$, respectively - $\Lambda^{-1}_{\lambda_{\mathcal{Q}}}\mathcal{Q} = (M_{\mathcal{Q}};\mathbf{0})$ and $\Lambda^{-1}_{\lambda_{\mathcal{P}}}\mathcal{P} = (M_{\mathcal{P}};\mathbf{0})$ - and consider that $\mathcal{Q}k_2$ and $\mathcal{P}k_2$ are Lorentz scalars - $\mathcal{Q}k_2 = \Lambda^{-1}_{\lambda_{\mathcal{Q}}}(\mathcal{Q}k_2) = (\Lambda^{-1}_{\lambda_{\mathcal{Q}}}\mathcal{Q})(\Lambda^{-1}_{\lambda_{\mathcal{Q}}}k_2) = M_{\mathcal{Q}}\Delta^0_{k_2,m\lambda_{\mathcal{Q}}}$ and $\mathcal{P}k_2 = M_{\mathcal{P}}\Delta^0_{k_2,m\lambda_{\mathcal{P}}}$ - and that the integration measures $d\Omega_{\mathbf{k}_i} = md\mathbf{k}_i/\Delta^0_{k_i,m\lambda_{\mathcal{Q}}}, i = 1,2$, and the delta functions in (10) on the mass hyperboloid (8) are invariant under the Lorentz transformations $\Lambda^{-1}_{\lambda_{\mathcal{Q}(\mathcal{P})}}$ (7). We thus recast expression in (10) into the form $(M_{\mathcal{Q}} = M_{\mathcal{P}} = M)$

$$< \mathcal{P}|J|\mathcal{Q} >= \frac{z_1+z_2}{(2\pi)^3}\int d\Omega_{\mathbf{\Delta}_{k,m\lambda_{\mathcal{Q}}}}\Psi^*_M(\mathbf{\Delta}_{k,m\lambda_{\mathcal{P}}})\hat{B}(\Delta^0_{k,m\lambda_{\mathcal{P}}},\Delta^0_{k,m\lambda_{\mathcal{Q}}})\Psi_M(\mathbf{\Delta}_{k,m\lambda_{\mathcal{Q}}}), \tag{11}$$

where we have defined the wave function for the system in momentum space as

$$\Psi_{M_{\mathcal{Q}}}(\mathbf{\Delta}_{k,m\lambda_{\mathcal{Q}}}) = \frac{\Gamma_{M_{\mathcal{Q}}}(\mathbf{\Delta}_{k,m\lambda_{\mathcal{Q}}})}{2^{3/2}\sqrt{m}\Delta^0_{k,m\lambda_{\mathcal{Q}}}(M_{\mathcal{Q}}-2\Delta^0_{k,m\lambda_{\mathcal{Q}}}+i\varepsilon)},$$
$$\mathbf{\Delta}_{k,m\lambda_{\mathcal{Q}}} = \mathbf{\Delta}_{k_2,m\lambda_{\mathcal{Q}}} = -\mathbf{\Delta}_{k_1',m\lambda_{\mathcal{Q}}};\ \mathbf{\Delta}_{k_2,m\lambda_{\mathcal{P}}} = -\mathbf{\Delta}_{k_1,m\lambda_{\mathcal{P}}},$$

and for the matrixes $\hat{O} = \gamma_5, \gamma_\mu; \gamma_5\gamma_\mu$ $(\mu = 0,1,2,3)$ under the Lorentz transformations $\Lambda^{-1}_{\lambda_{\mathcal{Q}(\mathcal{P})}}$ (7) we find

$$-\mathrm{Tr}\left[\hat{O}^+(\hat{k}_1+m)(\hat{k}_1'+m)\hat{O}(\hat{k}_2-m)\right] = 4m\left(\tilde{a}k_1k_1'+\tilde{b}k_1'k_2+\tilde{b}k_1k_2+\tilde{a}m^2\right) = \tag{12}$$
$$= \hat{B}(\Delta^0_{k,m\lambda_{\mathcal{P}}},\Delta^0_{k,m\lambda_{\mathcal{Q}}}) = 2m\Big\{2\tilde{a}(2\Delta^0_{k,m\lambda_{\mathcal{P}}})(2\Delta^0_{k,m\lambda_{\mathcal{Q}}})-$$
$$-(\tilde{a}-\tilde{b})\left[(2\Delta^0_{k,m\lambda_{\mathcal{P}}})^2+(2\Delta^0_{k,m\lambda_{\mathcal{Q}}})^2\right]+4m^2(\tilde{a}-\tilde{b})\Big\},$$

$$\tilde{a} = \begin{cases} 1\ ;\ \hat{O} = \gamma_5\,(\text{pseudoscalar}); \\ 4\ ;\ \hat{O} = \gamma_\mu\,(\text{vector}); \\ -4\ ;\ \hat{O} = \gamma_5\gamma_\mu\,(\text{pseudovector}); \end{cases} \quad \tilde{b} = \begin{cases} 1\ ,\ \hat{O} = \gamma_5\,(\text{pseudoscalar}); \\ 2\ ,\ \hat{O} = \gamma_\mu\,(\text{vector}); \\ 2\ ,\ \hat{O} = \gamma_5\gamma_\mu\,(\text{pseudovector}). \end{cases} \tag{13}$$

The vector $\mathbf{\Delta}_{k,m\lambda_{\mathcal{P}}}$ from the Lobachevsky space can be represented in the form

$$\mathbf{\Delta}_{k,m\lambda_{\mathcal{P}}} = \mathbf{k}(-)m\lambda_{\mathcal{P}} = \Lambda^{-1}_{\lambda_{\mathcal{P}}}\mathbf{k} = \Lambda^{-1}_{\lambda_{\mathcal{P}}}\Lambda_{\lambda_{\mathcal{Q}}}\mathbf{\Delta}_{k,m\lambda_{\mathcal{Q}}} =$$
$$= \left(\Lambda^{-1}_{\lambda_{\mathcal{P}}}\Lambda_{\lambda_{\mathcal{Q}}}\Lambda_{\Delta_{\mathcal{P},\mathcal{Q}}}\right)\left(\Lambda^{-1}_{\Delta_{\mathcal{P},\mathcal{Q}}}\mathbf{\Delta}_{k,m\lambda_{\mathcal{Q}}}\right) = V(\Lambda_{\lambda_{\mathcal{Q}}},\mathcal{P})\mathbf{\Delta}_{k,m\lambda_{\mathcal{Q}}}(-)\frac{m}{M}\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}}.$$

Here $V(\Lambda_{\lambda_{\mathcal{Q}}},\mathcal{P}) = \Lambda^{-1}_{\lambda_{\mathcal{P}}}\Lambda_{\lambda_{\mathcal{Q}}}\Lambda_{\Delta_{\mathcal{P},\mathcal{Q}}}$ is Wigner's rotation matrix and $\Delta_{\mathcal{P},\mathcal{Q}} = \Lambda^{-1}_{\lambda_{\mathcal{Q}}}\mathcal{P}$ is the 4-momentum transfer in the Lobachevsky space:

$$\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}} = \Lambda^{-1}_{\mathcal{Q}}\boldsymbol{\mathcal{P}} = \boldsymbol{\mathcal{P}}(-)\boldsymbol{\mathcal{Q}} = \boldsymbol{\mathcal{P}} - \frac{\boldsymbol{\mathcal{Q}}}{M}\left(\mathcal{P}_0 - \frac{\boldsymbol{\mathcal{P}}\cdot\boldsymbol{\mathcal{Q}}}{\mathcal{Q}_0+M}\right) = M\sinh\chi_\Delta\mathbf{n}_\Delta, \qquad (14)$$
$$\Delta^0_{\mathcal{P},\mathcal{Q}} = \left(\Lambda^{-1}_{\mathcal{Q}}\mathcal{P}\right)^0 = \frac{\mathcal{P}_0\mathcal{Q}_0 - \boldsymbol{\mathcal{P}}\cdot\boldsymbol{\mathcal{Q}}}{M} = \frac{\mathcal{P}\mathcal{Q}}{M} = M\cosh\chi_\Delta,$$
$$\boldsymbol{\mathcal{P}} = M\sinh\chi_{\mathcal{P}}\mathbf{n}_{\mathcal{P}}, \boldsymbol{\mathcal{Q}} = M\sinh\chi_{\mathcal{Q}}\mathbf{n}_{\mathcal{Q}}, \mathcal{P}_0 = M\cosh\chi_{\mathcal{P}}, \mathcal{Q}_0 = M\cosh\chi_{\mathcal{Q}},$$
$$|\mathbf{n}_{\mathcal{P}}| = |\mathbf{n}_{\mathcal{Q}}| = |\mathbf{n}_\Delta| = 1, {\Delta^{0}}^2_{\mathcal{P},\mathcal{Q}} - \mathbf{\Delta}^2_{\mathcal{P},\mathcal{Q}} = M^2,$$

where $\chi_\Delta, \chi_{\mathcal{P}}$, and $\chi_{\mathcal{Q}}$ are the respective rapidities, and that the square of the 4-momentum transfer t is related to the 3-momentum transfer $\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}}$ as

$$Q^2 = -t = -(\mathcal{P}-\mathcal{Q})^2 = -2M^2 + 2M\sqrt{M^2 + \mathbf{\Delta}^2_{\mathcal{P},\mathcal{Q}}} = 2M^2\left(\cosh\chi_\Delta - 1\right). \qquad (15)$$

The elastic form factor $F(t)$ in the case of scalar current we define as

$$F(t) = <\mathcal{P}|J|\mathcal{Q}>. \qquad (16)$$

It follows that one can consider the elastic form factor $F(t)$ as a function of the invariant variable $\mathbf{\Delta}^2_{\mathcal{P},\mathcal{Q}}$, which is the square of the 3-momentum transfer in the Lobachevsky space. Taking into consideration expressions (11)–(16), the form factor $F(t)$ then represents a convolution of covariant wave RQP-functions in this space. Consequently, by using for relativistic plane waves[2)] the completeness condition

$$\frac{1}{(2\pi)^3}\int d\Omega_{\mathbf{\Delta}_{p,m\lambda_{\mathcal{Q}}}}\xi(\mathbf{\Delta}_{p,m\lambda_{\mathcal{Q}}},\mathbf{r})\xi^*(\mathbf{\Delta}_{p,m\lambda_{\mathcal{Q}}},\mathbf{r}') = \delta(\mathbf{r}'-\mathbf{r}),$$

the equation

$$\left(2\Delta^0_{p,m\lambda_{\mathcal{Q}}} - \widehat{H}_0\right)\xi(\mathbf{\Delta}_{p,m\lambda_{\mathcal{Q}}},\mathbf{r}) = 0, \qquad (17)$$

the transformations

$$\psi_{M_{\mathcal{Q}}}(\mathbf{r}) = \frac{1}{(2\pi)^3}\int d\Omega_{\mathbf{\Delta}_{p,m\lambda_{\mathcal{Q}}}}\xi(\mathbf{\Delta}_{p,m\lambda_{\mathcal{Q}}},\mathbf{r})\Psi_{M_{\mathcal{Q}}}(\mathbf{\Delta}_{p,m\lambda_{\mathcal{Q}}}),$$
$$\Psi_{M_{\mathcal{Q}}}(\mathbf{\Delta}_{p,m\lambda_{\mathcal{Q}}}) = \int d\mathbf{r}\xi^*(\mathbf{\Delta}_{p,m\lambda_{\mathcal{Q}}},\mathbf{r})\psi_{M_{\mathcal{Q}}}(\mathbf{r}),$$

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2) The function

$$\xi(\mathbf{\Delta}_{p,m\lambda_{\mathcal{Q}}},\mathbf{r}) = \left(\frac{\Delta^0_{p,m\lambda_{\mathcal{Q}}} - \mathbf{\Delta}_{p,m\lambda_{\mathcal{Q}}}\cdot\mathbf{n}}{m}\right)^{-1-ir/\lambda}$$

realize unitary irreducible representations of the Lorentz group – the group of motions of the mass hyperboloid (8) and in the nonrelativistic limit ($|\mathbf{\Delta}_{p,m\lambda_{\mathcal{Q}}}| \ll 1/\lambda$, $r \gg \lambda$) $\xi(\mathbf{\Delta}_{p,m\lambda_{\mathcal{Q}}},\mathbf{r}) \to \exp(i\mathbf{\Delta}_{p,m\lambda_{\mathcal{Q}}}\cdot\mathbf{r})$.

the addition theorem for them in the form

$$\int d\omega_n \xi\left(\mathbf{\Delta}_{k,m\lambda_Q}(-)\frac{m}{M_Q}\mathbf{\Delta}_{\mathcal{P},Q},\mathbf{r}\right) = \int d\omega_n \xi(\mathbf{\Delta}_{k,m\lambda_Q},\mathbf{r})\xi^*\left(\frac{m}{M_Q}\mathbf{\Delta}_{\mathcal{P},Q},\mathbf{r}\right),$$

and the fact that the free-Hamiltonian operator (2) is Hermitian, we can represent the form factor (11) as the relativistic Fourier transform of the covariant wave RQP-functions in the configuration representation:

$$F(t) = 8m^3(z_1+z_2)\int d\mathbf{r}\,\xi^*\left(\frac{m}{M}\mathbf{\Delta}_{\mathcal{P},Q},\mathbf{r}\right)\left\{2\tilde{a}\left|\frac{\widehat{H}_0}{2m}\psi_M(\mathbf{r})\right|^2 - \right. \tag{18}$$

$$\left. -2(\tilde{a}-\tilde{b})\mathrm{Re}\left[\psi_M^*(\mathbf{r})\left(\frac{\widehat{H}_0}{2m}\right)^2\psi_M(\mathbf{r})\right] + (\tilde{a}-\tilde{b})\left|\psi_M(\mathbf{r})\right|^2\right\}.$$

For s-state of the composite system the integrations in (18) respecting of angles gives ($\rho = rm$)

$$F_{\ell=0}(t) = 32m^2\pi(z_1+z_2)\frac{\chi_\Delta}{\sinh\chi_\Delta}\int\limits_0^\infty d\rho\frac{\sin(\rho\chi_\Delta)}{\rho\chi_\Delta}\left\{2\tilde{a}\left|\widehat{H}^{\mathrm{rad}}_{0,\ell=0}\varphi_0(\rho,\chi)\right|^2 - \right. \tag{19}$$

$$\left. -2(\tilde{a}-\tilde{b})\mathrm{Re}\left[\varphi_0^*(\rho,\chi)\left(\widehat{H}^{\mathrm{rad}}_{0,\ell=0}\right)^2\varphi_0(\rho,\chi)\right] + (\tilde{a}-\tilde{b})\left|\varphi_0(\rho,\chi)\right|^2\right\},$$

where the radial part $\widehat{H}^{\mathrm{rad}}_{0,\ell=0}$ of the free-Hamiltonian operator in (2) is defined in Eq. (6).

For the vector-current case, covariant expressions for the components of the elastic form factor for the composite system formed by two equal-mass relativistic fermions will have the form

$$F^{(+)}(t) = \frac{16m^4(z_1+z_2)(2M^2-t)}{M(4M^2-t)}\int d\mathbf{r}\xi^*\left(\frac{m}{M}\mathbf{\Delta}_{\mathcal{P},Q},\mathbf{r}\right)\mathrm{Re}\left\{(\tilde{a}-\tilde{b})\psi_M^*(\mathbf{r})\frac{\widehat{H}_0}{2m}\psi_M(\mathbf{r}) + \right. \tag{20}$$

$$\left. +(\tilde{a}+\tilde{b})\left[\frac{\widehat{H}_0}{2m}\psi_M(\mathbf{r})\right]^*\left(\frac{\widehat{H}_0}{2m}\right)^2\psi_M(\mathbf{r}) - (\tilde{a}-\tilde{b})\psi_M^*(\mathbf{r})\left(\frac{\widehat{H}_0}{2m}\right)^3\psi_M(\mathbf{r})\right\},$$

$$F^{(-)}(t) = \frac{16m^4(z_1+z_2)(2M^2-t)}{-Mt}\int d\mathbf{r}\xi^*\left(\frac{m}{M}\mathbf{\Delta}_{\mathcal{P},Q},\mathbf{r}\right)\times \tag{21}$$

$$\times\mathrm{Im}\left\{(\tilde{a}-\tilde{b})\psi_M(\mathbf{r})\left[\frac{\widehat{H}_0}{2m}\psi_M(\mathbf{r})\right]^* + (3\tilde{a}-\tilde{b})\left[\frac{\widehat{H}_0}{2m}\psi_M(\mathbf{r})\right]\left[\left(\frac{\widehat{H}_0}{2m}\right)^2\psi_M(\mathbf{r})\right]^* - \right.$$

$$\left. -(\tilde{a}-\tilde{b})\psi_M(\mathbf{r})\left[\left(\frac{\widehat{H}_0}{2m}\right)^3\psi_M(\mathbf{r})\right]^*\right\},$$

where the transverse component of the elastic form factorin (21) for real-valued potentials does not vanish even at equal masses ($m_1 = m_2 = m$).

For s-state of the composite system the integrations in (20) and (21) respecting of angles gives

$$F^{(+)}_{\ell=0}(t) = \frac{64\pi m^3(z_1+z_2)(2M^2-t)}{M(4M^2-t)}\frac{\chi_\Delta}{\sinh\chi_\Delta}\int\limits_0^\infty d\rho\frac{\sin(\rho\chi_\Delta)}{\rho\chi_\Delta}\times \tag{22}$$

$$\times\mathrm{Re}\left\{(\tilde{a}-\tilde{b})\varphi_0^*(\rho,\chi)\widehat{H}^{\mathrm{rad}}_{0,\ell=0}\varphi_0(\rho,\chi) + (\tilde{a}+\tilde{b})\left[\widehat{H}^{\mathrm{rad}}_{0,\ell=0}\varphi_0(\rho,\chi)\right]^*\left(\widehat{H}^{\mathrm{rad}}_{0,\ell=0}\right)^2\varphi_0(\rho,\chi) - \right.$$

$$\left. -(\tilde{a}-\tilde{b})\varphi_0^*(\rho,\chi)\left(\widehat{H}^{\mathrm{rad}}_{0,\ell=0}\right)^3\varphi_0(\rho,\chi)\right\},$$

$$F^{(-)}_{\ell=0}(t) = \frac{64\pi m^3(z_1+z_2)(2M^2-t)}{-Mt}\frac{\chi_\Delta}{\sinh\chi_\Delta}\int\limits_0^\infty d\rho\frac{\sin(\rho\chi_\Delta)}{\rho\chi_\Delta}\times \tag{23}$$

$$\times\mathrm{Im}\Big\{(\tilde a-\tilde b)\varphi_0(\rho,\chi)\big[\widehat H^{\rm rad}_{0,\ell=0}\varphi_0(\rho,\chi)\big]^* + (3\tilde a-\tilde b)\big[\big(\widehat H^{\rm rad}_{0,\ell=0}\big)^2\varphi_0(\rho,\chi)\big]^*\widehat H^{\rm rad}_{0,\ell=0}\varphi_0(\rho,\chi)-$$

$$-(\tilde a-\tilde b)\varphi_0(\rho,\chi)\big[\big(\widehat H^{\rm rad}_{0,\ell=0}\big)^3\varphi_0(\rho,\chi)\big]^*\Big\},$$

where the transverse component (23) of the form factor for real-valued potentials vanishes.

4. Root-Mean-Square Radius and Form Factor for the Coulomb Interaction

The expression for the invariant root-mean-square radius (r.m.s.) of a composite system in terms of wave function s-state according to (19) has form

$$< r^2_{0,S} >= \frac{6\partial F_{\ell=0}(t)/\partial t|_{t=0}}{F_{\ell=0}(0)} = \frac{1}{M^2}+\frac{1}{M^2}\int\limits_0^\infty d\rho\rho^2 R_S(\rho)\Big/\int\limits_0^\infty d\rho R_S(\rho), \tag{24}$$

$$R_S(\rho) = 2\tilde a\big|\widehat H^{\rm rad}_{0,\ell=0}\varphi_0(\rho,\chi)\big|^2 - 2(\tilde a-\tilde b)\mathrm{Re}\big[\varphi_0^*(\rho,\chi)(\widehat H^{\rm rad}_{0,\ell=0})^2\varphi_0(\rho,\chi)\big] + (\tilde a-\tilde b)\,|\varphi_0(\rho,\chi)|^2. \tag{25}$$

Thus, if function $R_S(\rho)$ has a constant sign then the wave function of s-state describes not all structure of the composite particle, but only the region which be upon distances that larger its of the Compton wavelength $1/M$. The central sphere with $< r^2_{0,S} >= 1/M^2$ will correspond function of spatial distribution in the form $R_S(\rho)=\delta(\rho)/4\pi$. This distribution brings about the value of contribution to form factor from this sphere that equal

$$F_{\ell=0}(Q^2)|_{r_{0,S}=1/M} = 8m^2(z_1+z_2)\frac{\chi_\Delta}{\sinh\chi_\Delta}.$$

But if function $R_S(\rho)$ in (25) has not a constant sign then negative importances

$$< r^2_S >= \frac{1}{M^2}\int\limits_0^\infty d\rho\rho^2 R_S(\rho)\Big/\int\limits_0^\infty d\rho R_S(\rho)$$

answer on this. So contribution negative $< r^2_S >$ brings, as this is seen from (24), to decrease of the value $< r^2_{0,S} >$ meson in contrast with its of the Compton wavelength $1/M$. This result conform with experimental importance for π-meson.

The result for the vector-current case obtained in terms of the longitudinal form factor component in (22) can be represented in the form

$$< r^2_{0,V} >= \frac{1}{M^2}\left[1+\int\limits_0^\infty d\rho\left(\rho^2-\frac{3}{2}\right)R_V(\rho)\Big/\int\limits_0^\infty d\rho R_V(\rho)\right], \tag{26}$$

$$R_V(\rho) = \mathrm{Re}\Big\{(\tilde a-\tilde b)\varphi_0^*(\rho,\chi)\widehat H^{\rm rad}_{0,\ell=0}\varphi_0(\rho,\chi)+ \tag{27}$$

$$+(\tilde a+\tilde b)\left[\widehat H^{\rm rad}_{0,\ell=0}\varphi_0(\rho,\chi)\right]^*\left(\widehat H^{\rm rad}_{0,\ell=0}\right)^2\varphi_0(\rho,\chi) - (\tilde a-\tilde b)\varphi_0^*(\rho,\chi)\left(\widehat H^{\rm rad}_{0,\ell=0}\right)^3\varphi_0(\rho,\chi)\Big\}.$$

Thus, if the function $R_V(\rho)$ is sign-constant and if $\int\limits_0^\infty d\rho\,(\rho^2-3/2)\,R_V(\rho) > 0$, then it describes only the region lying at distances longer than the Compton wavelength $1/M$ of the composite particle. In accordance with Eq. (22) the contribution of central sphere with $< r_{0,V}^2 >= 1/M^2$ to the form factor has the form

$$F_{\ell=0}^{(+)}(t)|_{r_{0,V}=1/M} = \frac{16m^3(z_1+z_2)(2M^2-t)}{M(4M^2-t)}\frac{\chi_\Delta}{\sinh\chi_\Delta}.$$

If the function $R_V(\rho)$ is sign-constant, but $\int\limits_0^\infty d\rho\,(\rho^2-3/2)\,R_V(\rho) < 0$, then negative values will correspond to it,

$$< r_V^2 >= \frac{1}{M^2}\int\limits_0^\infty d\rho\left(\rho^2-\frac{3}{2}\right)R_V(\rho)\Big/\int\limits_0^\infty d\rho R_V(\rho).$$

The quantity $< r_V^2 >$ also takes negative values if the function $R_V(\rho)$ in Eq. (27) is not sign-constant. In either case, the contribution of negative values of $< r_V^2 >$ leads, as can be seen from Eq. (26), to a decrease in the r.m.s. $< r_{0,V}^2 >$ of the meson in relation to its Compton wavelength $1/M$. This result agrees with the experimental value for the pion.

As example, we consider the form factor of meson in the case of the Coulomb field

$$V(r) = -\frac{\alpha_s}{r}, \alpha_s > 0. \tag{28}$$

The radial wave function of exact solution of the RQP-equation (5) with interaction (28) for the s-state and ground level $n=1$ with the energy M_1 has the form [16]

$$\varphi_0^{(1)}(\rho,\kappa_1) = C_0^{(1)}(\kappa_1)(\rho-\rho_{\kappa_1})e^{(\rho-\rho_{\kappa_1})\kappa_1}, \tag{29}$$

where

$$\rho_{\kappa_1} = \frac{a\tilde{\alpha}_s}{2}\cos\kappa_1, M_1 = 2m\cos\kappa_1, 0<\kappa_1<\pi/2, \tilde{\alpha}_s = m\alpha_s,$$

and κ_1 defines by the following quantization condition

$$\tilde{\alpha}_s(a\cos^2\kappa_1+b) = 4\sin\kappa_1. \tag{30}$$

The normalization factor $|C_0^{(1)}(\kappa_1)|^2 = m\kappa_1^3 e^{-2\kappa_1\rho_{\kappa_1}}/\pi(2\kappa_1^2\rho_{\kappa_1}^2-2\kappa_1\rho_{\kappa_1}+1)$ and it can be found from the normalization condition

$$4\pi\int\limits_0^\infty dr|\varphi_0^{(1)}(r,\kappa_1)|^2 = 1.$$

Then the form factor (19) and the r.m.s. (24) with interaction (28) for the ground level of bound s-state with the energy M_1 are given as

$$F_{\ell=0,n=1}^{\rm Coul}(Q^2) = \frac{64(z_1+z_2)m^3\kappa_1^3}{(2\kappa_1^2\rho_{\kappa_1}^2-2\kappa_1\rho_{\kappa_1}+1)}\frac{\chi_\Delta}{\sinh\chi_\Delta}\left\{\frac{2\kappa_1 A_S}{(\chi_\Delta^2+4\kappa_1^2)^2}-\right. \tag{31}$$

$$\left.-\frac{2B_S}{\chi_\Delta^2+4\kappa_1^2}+\frac{C_S}{\chi_\Delta}\arctan\frac{\chi_\Delta}{2\kappa_1}\right\},$$

$$< r_{0,S}^2 >_{\ell=0,n=1}^{\rm Coul} = \frac{1}{M_1^2}\left\{1+\frac{1}{2\kappa_1^2}\left[1+\frac{5A_S-8\kappa_1 B_S}{A_S-4\kappa_1 B_S+4\kappa_1^2 C_S}\right]\right\}; \tag{32}$$

$$A_S = \tilde{a}-\tilde{b}+2\tilde{b}\cos^2\kappa_1,\ B_S = \tilde{b}\cos\kappa_1\sin\kappa_1,\ C_S = \tilde{a}\sin^2\kappa_1.$$

For large Q^2 the rapidity behaves as $\chi_\Delta \approx \ln(Q^2/M_1^2)$ and, consequently, the leading behavior of form factor (31) gives by expression [18]

$$F^{\text{Coul}}_{\ell=0,n=1}(Q^2) \approx \frac{64\pi(z_1+z_2)m^3\kappa_1^3 C_S}{2\kappa_1^2\rho_{\kappa_1}^2 - 2\kappa_1\rho_{\kappa_1}+1}\frac{1}{(Q/M_1)^2}\Big\{1 + O\Big[\ln^{-1}(Q/M_1)^2\Big]\Big\}.$$

Such behavior of the form factor for large $t = -Q^2$ complies with the prediction of the dimensional quark counting rule [19, 20], which gives $F_\pi \sim |t|^{-1}$. In the case of relativistic spinless particles the decrease of the form factor occurs under the law $F_\pi \sim (|t|\ln^3|t|)^{-1}$ (see [11]).

The results obtained by calculating the longitudinal form factor component in (22) and the r.m.s. in (26) for the two-fermion composite system in the presence of the chromodynamic interaction (28) for the ground-state s-wave level at an energy M_1 can be represented in the form [21]

$$F^{(+)\text{Coul}}_{\ell=0,n=1}(t) = \frac{64(z_1+z_2)m^4\kappa_1^3\cos\kappa_1(2M_1^2-t)}{(2\kappa_1^2\rho_{\kappa_1}^2 - 2\kappa_1\rho_{\kappa_1}+1)M_1(4M_1^2-t)}\frac{\chi_\Delta}{\sinh\chi_\Delta}\Big\{\frac{4\kappa_1 A_S}{(\chi_\Delta^2+4\kappa_1^2)^2}+ \tag{33}$$

$$+\frac{B_V}{\chi_\Delta^2+4\kappa_1^2}+\frac{2C_V}{\chi_\Delta}\arctan\frac{\chi_\Delta}{2\kappa_1}\Big\},$$

$$< r^2_{0,V} >^{(+)\text{Coul}}_{\ell=0,n=1} = \frac{1}{M_1^2}\Big\{-\frac{1}{2}+\frac{1}{2\kappa_1^2}\Big[1+\frac{5A_S+2\kappa_1 B_V}{A_S+\kappa_1 B_V+4\kappa_1^2 C_V}\Big]\Big\}, \tag{34}$$

$$B_V = \tan\kappa_1(\tilde a - \tilde b - 2\tilde b\cos^2\kappa_1),\ C_V = (\tilde a - \tilde b)\sin^2\kappa_1.$$

We emphasize that, at reception of the expressions (33) and (34), we excluded the coupling constant $\tilde\alpha_s$ by means of not only the quantization condition in (30) but also the identity that we obtained for the spin parameters $a, b, \tilde a$, and $\tilde b$ in the process of the calculations and which has the form [21]

$$a(\tilde a + \tilde b) - 2\tilde b(a+b) \equiv 0. \tag{35}$$

Recall that the values of these parameters are given in (4) and (13).

For large Q^2, the behavior of the form factor in (33) gives by the expression

$$F^{(+)\text{Coul}}_{\ell=0,n=1}(t) \approx \frac{128(z_1+z_2)m^4\kappa_1^3\cos\kappa_1}{M_1(2\kappa_1^2\rho_{\kappa_1}^2 - 2\kappa_1\rho_{\kappa_1}+1)}\frac{1}{(Q/M_1)^2}\left[\pi C_V + B_V\ln^{-1}(Q/M_1)^2\right]. \tag{36}$$

From the asymptotic expression in (36), one can see that the behavior of the form factor for large $t = -Q^2$ also obeys the $F(t) \sim |t|^{-1}$ law (see [21]) predicted by the dimensional-quark-counting rule [19, 20].

Thus, exactly registration of the spin brings about such behaviour of the form factor the for large t which is predicted by the dimensional-quark-counting rule [19, 20].

By using the results that were obtained here, we calculate the r.m.s. in (32) for the ground level of bound s-state of pions $\pi^\pm$ (pseudoscalar) with $n = 1, \ell = 0, a = 1, b = 0, \tilde a = \tilde b = 1, \tilde\alpha_s\cos\kappa_1 = 4\tan\kappa_1$ (see Eqs. (4), (13) and (30)) and the energy $M_1 = M_{\pi^\pm} = 0.140\,\text{GeV}$ [22], but as theirs of wave function we take the Coulomb wave function in (29). Expression (32) can then be represented to the form

$$< r^2_{0,S} >^{\text{Coul},\pi^\pm}_{\ell=0,n=1} = \frac{0.0389}{M_1^2}\Big\{1+\frac{1}{2\kappa_1^2}\Big[1+\frac{6-4(2\kappa_1\tan\kappa_1-1)}{(2\kappa_1\tan\kappa_1-1)^2+1}\Big]\Big\}\,\text{fm}^2. \tag{37}$$

Calculation of the expression (37) with energy pions $M_1 = M_{\pi^\pm} = 0.140\,\text{GeV}$ and importance of the rapidity $\kappa_1 = 1.1414$ gives importance of the value scalar r.m.s.:

$< r_{0,S}^2 >_{\ell=0,n=1}^{\mathrm{Coul},\pi^\pm} = 2.30\,\mathrm{fm}^2$ that much more than $< r_0^2 >_{\mathrm{exp}}^{\pi^\pm} = 0.45 \pm 0.01\,\mathrm{fm}^2$ [22]. For this importance of the rapidity $\kappa_1 = 1.1414$ the parameters to model (the coupling constant and mass quark) have importances: $\tilde{\alpha}_s = 20.98, m = 0.168\,\mathrm{GeV}$. For kaons with energy $M_1 = M_{K^\pm} = 0.494\,\mathrm{GeV}$ and importance of the rapidity $\kappa_1 = 0.8902$ we have: $< r_{0,S}^2 >_{\ell=0,n=1}^{\mathrm{Coul},K^\pm} = 0.31\,\mathrm{fm}^2$ that complies with $< r_0^2 >_{\mathrm{exp}}^{K^\pm} = 0.31 \pm 0.03\,\mathrm{fm}^2$ [22]. For importance of the rapidity $\kappa_1 = 0.8902$ the parameters to model have importances: $\tilde{\alpha}_s = 7.85, m = 0.393\,\mathrm{GeV}$.

To complete our analysis, we calculate the r.m.s. for vector-current case in (34) corresponding of the ground-state s-wave level of $\pi^\pm$ mesons as pseudoscalar with $n = 1, \ell = 0, a = 1, b = 0, \tilde{a} = \tilde{b} = 1, \tilde{\alpha}_s \cos\kappa_1 = 4\tan\kappa_1$, the energy $M_1 = M_{\pi^\pm} = 0.140\,\mathrm{GeV}$ and theirs of the Coulomb wave function in (29). Expression in (34) then reduces to the form

$$< r_{0,V}^2 >_{\ell=0,n=1}^{(+)\mathrm{Coul},\pi^\pm} = \frac{0.0389}{M_1^2}\Big\{-\frac{1}{2} + \frac{3}{2\kappa_1^2}\Big[1 - \frac{1}{\kappa_1\tan\kappa_1 - 1}\Big]\Big\}\,\mathrm{fm}^2. \quad (38)$$

Calculation of the expression in (38) with energy pions $M_1 = M_{\pi^\pm} = 0.140\,\mathrm{GeV}$ and importance of the rapidity $\kappa_1 = 1.3687$ gives following importance of the value r.m.s. for vector-current case: $< r_{0,V}^2 >_{\ell=0,n=1}^{(+)\mathrm{Coul},\pi^\pm} = 0.32\,\mathrm{fm}^2$. For importance of the rapidity $\kappa_1 = 1.3687$ the parameters to model have importances: $\tilde{\alpha}_s = 97.26$ and $m = 0.349\,\mathrm{GeV}$. The value found for the r.m.s. of charged pions for vector-current case lies near the confidence interval of its experimental value, $< r_0^2 >_{\mathrm{exp}}^{\pi^\pm} = 0.45 \pm 0.01\,\mathrm{fm}^2$ [22]. This distinction means that the quark–quark potential for pseudoscalar mesons should include, in addition to Coulomb interaction, the confining and spin–spin components and take into account the difference in the quark masses. For kaons with energy $M_1 = M_{K^\pm} = 0.494\,\mathrm{GeV}$ and importance of the rapidity $\kappa_1 = 1.3687$ we have: $< r_{0,V}^2 >_{\ell=0,n=1}^{\mathrm{Coul},K^\pm} = 0.03\,\mathrm{fm}^2$ that much smaller than $< r_0^2 >_{\mathrm{exp}}^{K^\pm} = 0.31 \pm 0.03\,\mathrm{fm}^2$ [22]. For importance of the rapidity $\kappa_1 = 1.3687$ the parameters to model have importances: $\tilde{\alpha}_s = 97.26, m = 1.231\,\mathrm{GeV}$.

Thus, the value r.m.s. of charged pions that has been obtained here corresponds the case of vector-current, but the value r.m.s. of charged kaons corresponds the case of scalar-current.

It should be emphasized that expression (38) for the r.m.s. has a singularity at $\kappa_1 \tan\kappa_1 = 1$ that is, at $\kappa_1 \simeq 0.86034$. The same singularity arose in the scalar-current case [18].

5. Conclusions

For the scalar and the vector-current cases, new covariant expressions of the elastic form factor for the composite system formed by two relativistic spin 1/2 particles with equal masses has been found as functions of the invariant variable $\Delta_{\mathcal{P},\mathcal{Q}}^2$, which is the square of the 3-momentum transfer in LobachevskyŸs space. The expressions for the invariant root-mean-square radius of a composite system in terms of the wave function s-state were obtained. The pseudoscalar, vector, and pseudovector cases has been considered. The consideration is conducted within the framework of RQP approach on the basis of covariant Hamiltonian formulation of quantum field theory, by transition to the three-dimensional relativistic configurational representation in the case of bound system of two relativistic spin particles with equal masses.

The identity in the form (35) valid for the values of the spin parameters $a, b, \tilde{a}$, and $\tilde{b}$ in the pseudoscalar, vector, and pseudovector cases has been established.

The application of the three-dimensional relativistic configuration representation for a system of two relativistic spin particles with equal masses has allowed to install that if the function of spatial distribution has a constant sign then the wave function of s-state describes not all structure of the composite particle, but only the region which be upon distances that larger its of the Compton wavelength $1/M$. But if the function of spatial distribution has not a constant sign then this brings to decrease of the value $< r_0^2 >$

meson in contrast with its of the Compton wavelength $1/M$. This result conform with experimental importance for mesons.

The present analysis has shown that a dominant contribution to the structure of a composite system from the central sphere of radius $1/M$ is proportional to $\chi_\Delta/\sinh\chi_\Delta$. In the nonrelativistic limit, this relativistic geometric factor tends to unity.

As examples, for the scalar and the vector-current cases the expressions for the form factor of a bound system formed by two relativistic spin 1/2 particles with equal masses in the case of Coulomb potential were obtained. It is installed that the covariant wave RQP-function of Coulomb potential at $Q^2 = -t \gg 1$ give the decrease for these form factors under the law $F_\pi \sim |t|^{-1}$, which predicts the dimensional quark counting rule.

For the scalar and vector-current cases, we calculated the r.m.s. of the ground level of bound s-state of charged pions and kaons, and it have conducted comparison with its experimental values.

Acknowledgments

I an grateful to O. P. Solovtsova and V. V. Andreev for attention to this work, support, and enlightening comments, and to A. E. Dorokhov, Yu. A.Kurochkin, and I. S. Satsunkevich for their interest in this study and valuable discussions on its results. Partial support of the work by the International Program of Cooperation between Republic of Belarus and the Joint Institute of Nuclear Research (JINR, Dubna) and State Program of the scientific studies "Konvergentsiya-2020".

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