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Is there evidence for dimension-two correction in QCD from e^+e^--annihilation data?

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We report the result of re-evaluation of the lowest dimensional condensates from the high-precision fits of the e^+e^--annihilation data into pions: $\pi^+\pi^-$, $2\pi^+2\pi^-$, $\pi^+\pi^-2\pi^0$, $3\pi^+3\pi^-$, and $2\pi^+2\pi^-2\pi^0$. We use the method based on the Borel transform technique which is applied to the Adler function. We look for whether there is an operator with dimension 2 or not, and we discuss the difference that arises if an analytic invariant charge is used instead of the usual perturbative one. We obtain that the dimension-two correction is small, its value is negative and compatible with zero in the error range, and we confirm a strong (anti)correlation between the operator with dimension 2 and the local gluon condensate.

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1. Introduction

For many decades and up to now, the process of e^+e^- annihilation into hadrons serves as a laboratory for study of the strong interaction (see, e.g., [1, 2]). The emergence of increasingly accurate experimental e^+e^- data [3–16] opens up the possibility to study a more thin effects, such, for example, as an additional power corrections to the usual perturbative expansion that appear in the low Q^2 scales, $Q^2 \lesssim 1-2$ GeV^2. To go beyond a perturbative approximation, one usually applies the operator product expansion (OPE)

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approach for hadronic current-current correlators. In this approach the log-type perturbative contribution in perturbative sector accompanies with a nonperturbative part, which represents as a rule in the form of power corrections. These corrections are connected with a new parameters, the so-called vacuum condensates, that are vacuum expectation values of gauge invariant combinations quark and gluon fields. The QCD Lagrangian does not involve these parameters, and they are usually considered as phenomenological parameters which are extracted from processing experimental data. The vacuum condensates are responsible for the resonance structure observed at low energy and are an important point of the QCD sum rule method [17]. By using the analytic properties of the relevant Green functions combining with the conception of the quark-hadron duality, the parameters involving into the OPE-expansion are related with a dispersive integral containing the hadronic spectral function. In this fashion, a bridge between the hadronic and quark-gluon worlds is established. The Borel transform is the useful mathematical trick, which is widely used for QCD sum rule treatment [18, 19].

In this work, based on the experimental measurements of e^+e^--annihilation into pions cross-section, we reconsider the question on the the existence of an operator with dimension 2 whose contribution to the QCD sum rules is proportional to $1/Q^2$. It should be noted that well-known Cornell potential [20], $V(r) \approx -\dfrac{4\alpha_s(r)}{3r} + kr$, contains the term kr describing the string potential (connected to the phenomenon of confinement) at short distances and leading to the correction $\sim k/Q^2$. In the pioneering paper [21] the concept of short strings leading to corrections with dimension 2 was suggested. The search for relevant corrections of the OPE have already been studied for quite a long time, they are still of interest until now, see, e.g., [22–27]. The search of the contribution of the operator with dimension 2 is performed not only using the e^+e^- data, but also a rich experimental material coming from the tau lepton decay into hadrons data (see, e.g., [29–32]. It was shown that the operator with dimension 2 is compatible with zero but with large errors.

Obviously it is of interest to find the OPE-coefficients with less errors. I is the main purpose of this work in which we find the OPE-coefficients C_2 and C_4 from the experimental data on e^+e^--annihilation to pion channels $\pi^+\pi^-$, $2\pi^+2\pi^-$, $\pi^+\pi^-2\pi^0$, $3\pi^+3\pi^-$ and $2\pi^+2\pi^-2\pi^0$.

2. On experimental data and fittimg procedure

We will not dwell on the review of the experimental data used, but only note that in the present analysis instead of old data $e^+e^- \to \pi^+\pi^-$ [3], which we used earlier [25–27], now we use more modern data. For verification, we calculated the contribution channel $e^+e^- \to \pi^+\pi^-$ to the anomalous magnetic moment of the muon $a_\mu = (g-2)_\mu/2$, which is an important object for theoretical investigations (see, e.g., [35] for details). The $\pi^+\pi^-$ channel is by far the most important contribution to $a_\mu^{\mathrm{had,VP}}$, dominating both its mean value and uncertainty. The contribution of the $e^+e^- \to \pi^+\pi^-$ channel to the anomalous magnetic moment of the muon in the leading order in electromagnetic coupling constant reads as

$$a_\mu^{\pi^+\pi^-} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int\limits_0^\infty \frac{ds}{s} K(s)\, R_{\pi^+\pi^-}(s), \qquad (1)$$

where $K(s)$ is the vacuum polarization factor given by

$$K(s) = \frac{(1+x^2)(1+x)^2}{x^2}\ln(1+x) + \frac{x^2(1+x)}{1-x}\ln(x) + \frac{x^2}{2}(2-x^2) + \frac{(1+x^2)(1+x)^2}{x^2}\left(-x+\frac{x^2}{2}\right),$$

$x = \dfrac{1-v}{1+v}$, and $v = \sqrt{1 - 4m_\mu^2/s}$ and m_μ is muom mass.
We get

$$\begin{aligned} a_\mu^{\rm CMD2} &= (368.83 \pm 2.21) \cdot 10^{-10} \quad [9]\,, \\ a_\mu^{\rm KLOE12} &= (368.85 \pm 2.58) \cdot 10^{-10} \quad [15]\,, \\ a_\mu^{\rm BESIII} &= (368.11 \pm 3.31) \cdot 10^{-10} \quad [16]. \end{aligned}$$

Our result can be compared to the result from the paper [36], see Fig. 4 in this paper. Insignificant the difference in numbers can be explained by the fact that we did the data interpolation, as in [36] made a direct fitting of the $e^+e^- \to \pi^+\pi^-$ data. However, we can note a good match of the results.

3. Theoretical framework

By definition the quantity of interest to us, R, is the ratio of the cross-section of e^+e^--annihilation to hadrons to the cross-section of e^+e^--annihilation to a muon pair:

$$R(s) = \frac{\sigma_{e^+e^-\to\text{hadrons}}(s)}{\sigma_{e^+e^-\to\mu^+\mu^-}(s)}\,, \tag{2}$$

where $\sigma_{e^+e^-\to\mu^+\mu^-}(s) = \dfrac{4\,\pi\,\alpha_{\rm em}^2}{3\,s}$. The full R-ratio is the sum of R-ratios of particular processes, and here we consider the pion channels $\pi^+\pi^-$, $2\pi^+2\pi^-$, $\pi^+\pi^-2\pi^0$, $3\pi^+3\pi^-$, and $2\pi^+2\pi^-2\pi^0$.

The theoretical form of the full R-ratio in the PT and in the APT reads as

$$R_{\rm th}^{\rm PT/APT}(s) = N_c \sum_q e_q^2 \left(1 + \frac{\alpha_{\rm QCD}^{\rm PT/APT}(s)}{\pi}\right), \tag{3}$$

where $N_c = 3$, e_q is the charge of a quark with flavour q. According to considering the pion channels, we construct the isospin-1 part of the full electromagnetic current:

$$j_\mu = \frac{1}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right),$$

and therefore $\sum_q e_q^2 \to 1/4 + 1/4 = 1/2$.

In order to obtain the isospin-1 R-ratio in the full energy region, we take for $R(s)$ the following expression

$$R_{\text{exp-th}}(s) = R_{\rm exp}(s)\,\theta(s_0 - s) + R_{\rm th}(s)\,\theta(s - s_0)\,, \tag{4}$$

where s_0 is the continuum threshold. The summation result for R-ratio is presented in Fig. 1 by the blue solid curve. So, in the region of s from s_0 to ∞ where the R-ratio is described by the theoretical form (3) its error is neglected.

There exist two ways to write down the D-function. The first one corresponds to the dispersion form:

$$D_{\rm exp}(Q^2) = Q^2 \int_{4m_\pi^2}^{\infty} \frac{R_{\text{exp-th}}(s)\,ds}{(s + Q^2)^2}. \tag{5}$$

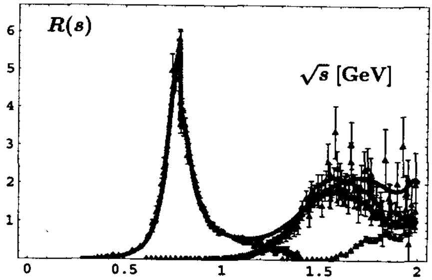

Figure 1: The full R-ratio ($R_{\rm exp}^{(I=1)}$) in dependence on $\sqrt{s}$ for $\sqrt{s} \leq 2$ GeV.

The second form corresponds to the OPE framework making use of the PT or the APT approach:

$$
\begin{aligned}
&D_{\rm PT+OPE}(Q^2) = \\
&= \frac{3}{2}\left[1 + \frac{\alpha_s(Q^2)}{\pi} + \sum_{n\geq 1}\Gamma(n)\,\frac{c_{2n}}{Q^{2n}}\right], \\
&D_{\rm APT+OPE}(Q^2) = \\
&= \frac{3}{2}\left[1 + \frac{\mathcal{A}_s(Q^2)}{\pi} + \sum_{n\geq 1}\Gamma(n)\,\frac{\tilde{c}_{2n}}{Q^{2n}}\right].
\end{aligned}
\tag{6}
$$

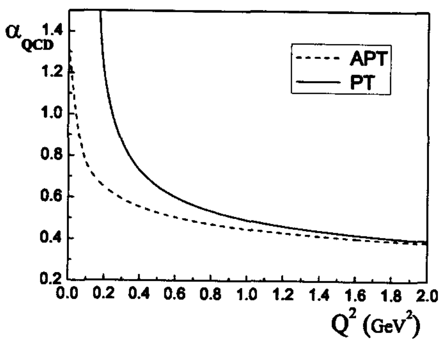

Figure 2: – The behavior of the ordinary (solid line) and the analytical (dashed line) running couplings.

The leading order (LO) PT running coupling reads

$$
\alpha_s(Q^2) = \frac{4\pi}{b_0}\frac{1}{\ln(Q^2/\Lambda^2)}. \tag{7}
$$

Here $b_0 = 11 - 2N_f/3$ is the first coefficient of the β-function expansion and Λ is the QCD scale parameter. Note that $\alpha_s(Q^2)$ has the unphysical singularity at $Q^2 = \Lambda^2$. In the framework of APT the running coupling has the form [33, 34]:

$$\mathcal{A}_s(Q^2) = \frac{4\pi}{b_0}\left[\frac{1}{\ln(Q^2/\Lambda^2_{\text{APT}})} - \frac{\Lambda^2_{\text{APT}}}{Q^2-\Lambda^2_{\text{APT}}}\right]. \tag{8}$$

One can see that the additional term in this expression removes the singularity. The behavior of the PT and the APT couplings in the low-energy region are shown in Fig. 2.

4. The Borel transform of $D(Q^2)$ and sum rules

The Borel transform of the function f by definition has the following form:

$$\hat{B}_{x\to y}[f(x)] = \lim_{n\to\infty}\frac{(-x)^n}{\Gamma(n)}\left[\frac{d^n}{dx^n}f(x)\right]_{x=ny}.$$

The Borel transformed forms of the D-function (5) and (6) are:

$$\begin{aligned}&\hat{B}_{Q^2\to M^2}[D_{\text{exp}}(Q^2)] = \Phi_{\text{exp}}(M^2) =\\ &= \int_0^\infty R_{\text{exp-th}}(s)\left(1-\frac{s}{M^2}\right)e^{-s/M^2}\frac{ds}{M^2},\end{aligned} \tag{9}$$

$$\begin{aligned}&\hat{B}_{Q^2\to M^2}[D_{\text{PT+OPE}}(Q^2)] = \Phi_{\text{PT+OPE}}(M^2) =\\ &= \frac{3}{2}\frac{\hat{B}_{Q^2\to M^2}[\alpha_s(Q^2)]}{\pi} + \frac{3}{2}\left(\frac{C_2}{M^2}+\frac{C_4}{M^4}+\frac{C_6}{M^6}\right),\\ &\hat{B}_{Q^2\to M^2}[D_{\text{APT+OPE}}(Q^2)] = \Phi_{\text{APT+OPE}}(M^2) =\\ &= \frac{3}{2}\frac{\hat{B}_{Q^2\to M^2}[\mathcal{A}_s(Q^2)]}{\pi} + \frac{3}{2}\left(\frac{\widetilde{C}_2}{M^2}+\frac{\widetilde{C}_4}{M^4}+\frac{\widetilde{C}_6}{M^6}\right),\end{aligned}$$

where $C_{2n} = \Gamma(n)\,c_{2n}$, $\widetilde{C}_{2n} = \Gamma(n)\,\tilde{c}_{2n}$. The Borel transform of the running couplings in PT and APT give the following results:

$$\begin{aligned}&\hat{B}_{Q^2\to M^2}\left[\alpha_s(Q^2)\right] =\\ &= \frac{4\pi}{b_0}\left[\frac{1}{M^2}\int\limits_0^\infty\frac{e^{-s/M^2}ds}{\ln^2(s/\Lambda^2)+\pi^2} + \frac{\Lambda^2}{M^2}e^{\Lambda^2/M^2}\right],\\ &\hat{B}_{Q^2\to M^2}\left[\mathcal{A}_s(Q^2)\right] =\\ &= \frac{4\pi}{b_0}\left[\frac{1}{M^2}\int\limits_0^\infty\frac{e^{-s/M^2}ds}{\ln^2(s/\Lambda^2)+\pi^2}\right].\end{aligned}$$

Equating the two forms of the Adler function, we get the sum rules for PT and APT:

$$\Phi_{\text{exp}}(M^2) = \begin{cases}\Phi_{\text{PT+OPE}}(M^2),\\ \Phi_{\text{APT+OPE}}(M^2).\end{cases} \tag{10}$$

The error of $\Phi(M^2)$, $\Delta\Phi(M^2)$ can be calculated using formula (9) through the error of the R-ratio and $\Delta R(s)$:

$$\Delta\Phi(M^2) = \int_0^{s_0}\Delta R(s)\left(1-\frac{s}{M^2}\right)e^{-s/M^2}\frac{ds}{M^2},\quad \Delta R(s) = \frac{\Delta\sigma_{e^+e^-\to\text{hadrons}}(s)}{\sigma_{e^+e^-\to\mu^+\mu^-}(s)}.$$

We compare both forms of the Borelized D-function by χ^2-minimization and find the coefficients C_2 and C_4. The coefficient C_4 can be expressed in terms of a gluon condensate

$$C_4 = \frac{2\pi^2}{3}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle.$$

The coefficient C_6 can be expressed in terms of a quark condensate [17] and in our analysis it is fixed as [38]

$$C_6 = -\frac{448\,\pi^3}{27}\,\alpha_s\,\langle\bar{q}q\rangle^2 \approx -0.116\,\mathrm{GeV}^6.$$

The continuum threshold s_0, we fixed from the global duality. The corresponding duality relation reads

$$I_{dual}(s_0) \equiv \int_0^{s_0} ds\, R_{\exp}(s)) = \int_0^{s_0} ds\, R_{\rm th}(s). \qquad (11)$$

Using the experimental data and the perturbative ansatz in the leading order (LO) and in the next to leading order (NLO), we find that the best value is $s_0 \simeq 1.51^2$ GeV^2: we show $I_{dual}(s_0)$ as a function of the upper limit of the integration in Fig. 3. As will be shown below, the values of coefficients C_2 and C_4 in the duality interval are stable (see Fig. 4 below).

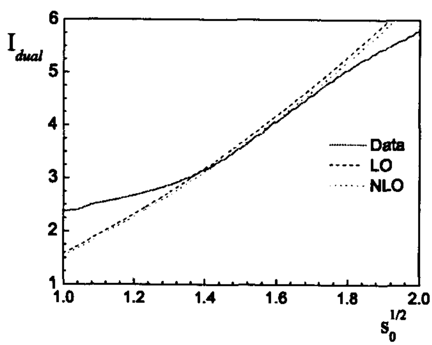

Figure 3: The integral (11) versus the upper integration limit, s_0. The integral for the experimental data corresponds to the solid line (Data). The theoretical curves are the dashed (LO) and dotted (NLO) lines.

5. Fitting result

Our analysis includes the research with different values of the parameter Λ: 0.25 GeV, 0.35 GeV and 0.45 GeV applying the PT and the APT approaches. The results for PT and APT are shown in Tables 1 and 2, correspondingly.

The dependence on the choose of the Borel interval M^2 in PT are shown in Tables 3 and 4, correspondingly for the left boundary, M^2_{min}, and for the right, M^2_{max}. These studies have shown that the interval $M^2 = 0.75 \div 3$ GeV^2 can be chosen as preferred.

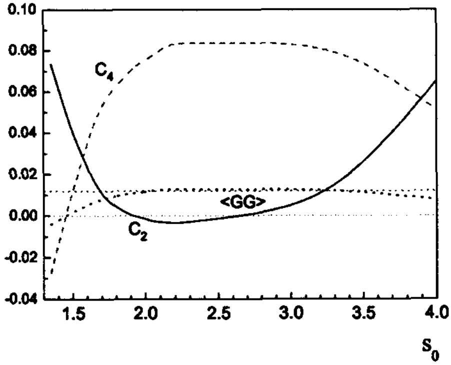

Figure 4: The coefficient C_2 (solid line) in units of GeV^2, C_4 (dashed line) in units of GeV^4, and the gluon condensate (dotted line) versus s_0. Horizontal lines correspond to the usual values.

Table 1: The extracted values for the interval of M^2 : [0.75, 3] GeV^2 in PT.

Λ_{PT}, GeV	C_2, GeV^2	C_4, GeV^4	$\frac{<\alpha_s GG>}{\pi}$, GeV^4
0.250	-0.003 ∓ 0.015	0.084 ± 0.012	0.013 ± 0.002
0.300	-0.032 ∓ 0.016	0.093 ± 0.013	0.014 ± 0.002
0.350	-0.062 ∓ 0.015	0.100 ± 0.012	0.015 ± 0.002

Table 2: The extracted values for the interval of M^2 : [0.75, 3] GeV^2 in APT.

Λ_{APT}, GeV	$\widetilde{C}_2$, GeV^2	$\widetilde{C}_4$, GeV^4	$\frac{<\alpha_s GG>}{\pi}$, GeV^4
0.277	0.022 ± 0.015	0.088 ± 0.012	0.013 ± 0.002
0.342	0.005 ± 0.016	0.099 ± 0.012	0.015 ± 0.002
0.412	-0.011 ∓ 0.016	0.110 ± 0.012	0.017 ± 0.002

Table 5 shows the dependency between the coefficient C_2 and C_4 and the best values for these condensates. The stability of the values found is demonstrated in Fig. 4.

6. Summary

Having done our analysis we again found that the dimension-two correction is small and its value is compatible with zero in the error range:

$$C_2 = -0.005 \mp 0.015\,\mathrm{GeV}^2, \quad \left\langle \frac{\alpha_s GG}{\pi} \right\rangle = 0.013 \pm 0.012\,\mathrm{GeV}^4.$$

Table 3: The dependence on the left boundary M^2_{min} at $s_0 = 1.51^2$ GeV^2.

M^2, GeV^2	C_2, GeV^2	C_4, GeV^4	$\frac{<\alpha_s GG>}{\pi}$, GeV^4
$[1.00, 3]$	-0.0143 ∓ 0.0175	0.1021 ± 0.0165	0.0155 ± 0.0025
$[0.95, 3]$	-0.0109 ∓ 0.0176	0.0963 ± 0.0160	0.0146 ± 0.0024
$[0.90, 3]$	-0.0082 ∓ 0.0171	0.0914 ± 0.0153	0.0139 ± 0.0023
$[0.85, 3]$	-0.0047 ∓ 0.0164	0.0865 ± 0.0140	0.0131 ± 0.0021
$[0.80, 3]$	-0.0031 ∓ 0.0161	0.0841 ± 0.0136	0.0127 ± 0.0020
$[0.75, 3]$	-0.003 ∓ 0.015	0.084 ± 0.012	0.013 ± 0.002
$[0.70, 3]$	-0.0041 ∓ 0.0147	0.0852 ± 0.0113	0.0129 ± 0.0017
$[0.65, 3]$	-0.0080 ∓ 0.0145	0.0901 ± 0.0107	0.0137 ± 0.0016
$[0.60, 3]$	-0.0145 ∓ 0.0128	0.0985 ± 0.0090	0.0149 ± 0.0013
$[0.55, 3]$	-0.0263 ∓ 0.0129	0.1122 ± 0.0086	0.0170 ± 0.0013
$[0.50, 3]$	-0.0445 ∓ 0.0120	0.1315 ± 0.0076	0.0199 ± 0.0011

Table 4: The dependence on the right boundary M^2_{max} at $s_0 = 1.51^2$ GeV^2.

M^2, GeV^2	C_2, GeV^2	C_4, GeV^4	$\frac{<\alpha_s GG>}{\pi}$, GeV^4
$[0.75, 2.0]$	0.0125 ± 0.0140	0.0688 ± 0.0105	0.0104 ± 0.0016
$[0.75, 2.5]$	0.0041 ± 0.0141	0.0768 ± 0.0112	0.0116 ± 0.0017
$[0.75, 3.0]$	-0.003 ∓ 0.015	0.084 ± 0.012	0.013 ± 0.002
$[0.75, 3.5]$	-0.0116 ∓ 0.0160	0.0918 ± 0.0132	0.0139 ± 0.0020
$[0.75, 4.0]$	-0.0174 ∓ 0.0161	0.0977 ± 0.0134	0.0148 ± 0.0020

This result is consistent with the results of other papers [17, 24, 29] and differs from what we have previously obtained [25, 26]

$$C_2 = -0.047 \mp 0.052\,\text{GeV}^2, \quad \left\langle \frac{\alpha_s GG}{\pi} \right\rangle = 0.018 \pm 0.010\,\text{GeV}^4 \quad (old).$$

That indicates a strong sensitivity to a set of experimental data. However, within the bounds of error, our results consistent with each other.

Table 5: The dependence between C_2 and C_4

C_4 (GeV^4)	C_2 (GeV^2)	χ^2	$\langle(\alpha_s/\pi)\,GG\rangle$
0.05929	0.019	2.45	0.009011
0.06776	0.011	1.51	0.010298
0.07623	0.003	0.94	0.011586
0.08047	-0.0007	0.78	0.012229
0.08470	-0.0046	0.75	0.012873
0.09317	-0.012	0.94	0.014160
0.10164	-0.020	1.51	0.015448
0.11011	-0.028	2.45	0.016735
0.11858	-0.035	3.78	0.018022
0.12705	-0.043	5.48	0.019309

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