Nonlinear Dynamics and Applications. Vol. 26 (2020) 100 - 106

Magnetic moment of ρ–meson in point form of Poincaré-invariant quantum mechanics

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In the framework of relativistic quark model based on the point form of Poincaré-invariant quantum mechanics the magnetic moment of the ρ–meson is calculated taking into account the internal structure of quarks. It's shown that the model parameters obtained from the condition of matching theoretical calculations with experimental data on lepton and radiative decays of hadrons lead to the results that correlate with calculations in models based on the front and instant forms of dynamics.

PACS numbers: 13.20.-v, 14.40.Cs, 14.65.-q
Keywords: Poincaré invariance, point form, quark, meson, anomalous magnetic moment.

1. Introduction

The study of the electromagnetic characteristics of mesons taking into account their inner structure is the most convenient way for testing various theoretical approaches and models for describing bounded quark systems. Updating the experimental data renewed interest in studying the mechanism of interaction of quarks inside hadrons. Of particular interest are light sector mesons consisting of u–, d– and s–quarks: such systems are purely relativistic, which requires the involvement of appropriate approaches and models for a phenomenological description of such systems. Among the variety of approaches and models devoted to the description of various characteristics of bound quark-antiquark states, we mark models based on the Poincaré group. Such models are purely relativistic, which makes their using for describing light sector mesons the the most natural approach.

Among of the three forms of Poincaré-invariant quantum mechanics (further PiQM) [1] for calculating the characteristics of light mesons widely used light-front form of dynamics. This circumstance is due to the fact that in this form there are no pair production from vacuum diagrams. The indicated advantage of this form makes it possible to use it successfully both for calculating lepton characteristics and for calculating semileptonic [2, 3] and radiative transitions of pseudoscalar and vector mesons [4, 5]. However, this form of PiQM has a serious disadvantage related to rotational invariance of dynamic group generators [6]. This feature leads to the so-called angular condition for matrix elements $I(\lambda', \lambda)$ (λ', λ – helicities of the final and initial and final state of the particle, respectively) which, in particular, complicates the calculation of the form factors of the

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ρ–meson. So, for example, in [7] for calculating the magnetic moment of ρ–meson used the matrix element $I(\lambda' = 1, \lambda = 0)$, the author of the work [3] used $I(\lambda' = 0, \lambda = 0)$. The indicated discrepancy is due to the restriction of the matrix elements $I(\lambda' = 1, \lambda = 0)$, $I(\lambda' = 0, \lambda = 0)$ and $I(\lambda' = 1, \lambda = -1)$ by angular condition, which in some works simply wasn't respected [6].

The instant form of dynamics is also used to calculate various characteristics of electroweak decays of pseudoscalar and vector mesons. The calculations of the magnetic moment μ_ρ in [8, 9] lead to the value $\mu_\rho = 2.16 \pm 0.03$. As features of the calculations note the taking into account the anomalous magnetic moments κ_u, κ_d of quarks.

In the framework of the point form of dynamics, calculations of the form factors of pseudoscalar and vector mesons are less common. The indicated fact connected with calculations both within the framework of this PiQM form and its various modifications lead to results that differ from experimental data. Attempts to apply the point form of PiQM to the calculation of form factors for various transmitted momentum for vector mesons were also unsuccessful.

Renewed interest in calculating the vector particle form factors is associated with updating experimental data on light mesons. So in [10] based on experimental data from the BaBar collaboration in the energy range from 0.9 to 2.2 GeV from reaction $e^+e^- \to \pi^+\pi^-2\pi^0$ the magnetic moment of the ρ–meson was calculated: the authors showed that indirect estimates give value $\mu_\rho = 2.1 \pm 0.5$ (in units $e/2m_\rho$). However, this calculation differs from theoretical predictions. It is known that the nonrelativistic $SU(6)$ - model, provided there is no interaction, gives the value $\mu_\rho = 2$ [3]. The indicated equality also holds for VMD - model, where ρ–meson supposed boson with a magnetic moment $\mu_\rho = 2$. In contrast to the listed numerical predictions, in the framework of light-front calculations in works [3, 7, 11] give an assessment $\mu_\rho < 2$ which also confirmed by the QCD-sum rules [12]. It's follows that further calculations of ρ–meson magnetic moment in different approaches and models are an important and relevant task.

In the work, the authors using relativistic quark model, based on the point form of PiQM carry out the calculation of ρ–meson magnetic moment taking in the internal structure of quarks. In section 2 the authors discuss the basic relation of the point form of PiQM. In section 3 briefly discusses the procedure for obtaining integral representations of decay constants $P(q\bar{Q}) \to \ell\nu_\ell$, $V(q\bar{q}) \to \ell^+\ell^-$ and $\ell \to V(q\bar{Q})\nu_\ell$ followed by calculation the basic parameters of the model. As a result of the work, in section 4 developed technique used to determine matrix elements for ρ–meson with following numerical calculation μ_ρ.

2. Basic features of the model

The scheme for obtaining the state vector of meson in point form of PiQM is as follows [1]: in the case of a two–particle system with the masses m_q, $m_{\bar{Q}}$ and, respectively, 4-momentums $p_1 = \left(\omega_{m_q}(\mathbf{p}_1), \mathbf{p}_1\right)$, $p_2 = \left(\omega_{m_{\bar{Q}}}(\mathbf{p}_2), \mathbf{p}_2\right)$ basis of direct product of two noninteracting particles

$$|\mathbf{p}_1, \lambda_1\rangle \otimes |\mathbf{p}_2, \lambda_2\rangle \equiv |\mathbf{p}_1, \lambda_1, \mathbf{p}_2, \lambda_2\rangle \quad (1)$$

defines a reducible representation of the Poincaré group. For irreducible representation, that characterizes the entire system, we introduce a full momentum

$$\mathbf{P}_{12} = \mathbf{p}_1 + \mathbf{p}_2 \quad (2)$$

and the relative momentum $\mathbf{k}$ of particles [1].

The requirement, that the operators for a free and bound $q\bar{Q}$ systems satisfy the algebra of the Poincaré group, leads to the fact, that the state vector of meson with mass M, total angular momentum J and its projection μ, are defined as the direct product of the state vectors of free particles (quarks) (4) with the wave function (WF) $\Phi^{J}_{\ell S}\left(\mathrm{k},\beta_{q\bar{Q}}\right)$:

$$|\mathbf{Q},J\mu,M\rangle=\sum_{\lambda_1,\lambda_2}\sum_{\nu_1,\nu_2}\int d\mathrm{k}\sqrt{\frac{\omega_{m_q}(\mathrm{p}_1)\,\omega_{m_{\bar{Q}}}(\mathrm{p}_2)}{\omega_{m_q}(\mathrm{k})\,\omega_{m_{\bar{Q}}}(\mathrm{k})\,V_0}}\,\Phi^{J}_{\ell S}\left(\mathrm{k},\beta_{q\bar{Q}}\right)\Omega\left\{\begin{smallmatrix}\ell & S & J\\ \nu_1,\nu_2,\mu\end{smallmatrix}\right\}(\theta_k,\phi_k)\times$$

$$\times\, D^{1/2}_{\lambda_1,\nu_1}\left(\mathbf{n}_{W_1}\right)\,D^{1/2}_{\lambda_2,\nu_2}\left(\mathbf{n}_{W_2}\right)|\mathbf{p}_1,\lambda_1,\mathbf{p}_2,\lambda_2\rangle\,. \qquad (3)$$

Note, that in eq. (3) the functions $\mathbf{C}\left\{\begin{smallmatrix}s_1 & s_2 & S\\ \nu_1,\nu_2,\lambda\end{smallmatrix}\right\}$, $\mathbf{C}\left\{\begin{smallmatrix}\ell & S & J\\ m,\lambda,\mu\end{smallmatrix}\right\}$ are Clebsh-Gordan coefficients of $SU(2)$ group, $Y_{\ell,m}(\theta_k,\phi_k)$ is the spherical harmonic and $D_{\lambda,\nu}(\mathbf{n}_W)$ – Wigner function. For brevity, we use following notations [15]:

$$M_0(\mathrm{k})=\omega_{m_q}(\mathrm{k})+\omega_{m_{\bar{Q}}}(\mathrm{k})\,,\;\;\omega_m(\mathrm{k})=\sqrt{\mathrm{k}^2+m^2},\;\;\mathrm{k}=|\mathbf{k}|\,, \qquad (4)$$

$$\Omega\left\{\begin{smallmatrix}\ell & S & J\\ \nu_1,\nu_2,\mu\end{smallmatrix}\right\}(\theta_k,\phi_k)=\mathbf{C}\left\{\begin{smallmatrix}\ell & S & J\\ \mu-(\nu_1+\nu_2),\,\nu_1+\nu_2,\,\mu\end{smallmatrix}\right\}Y_{\ell,\mu-(\nu_1+\nu_2)}(\theta_k,\phi_k) \qquad (5)$$

and WF $\Phi^{J}_{\ell S}\left(\mathrm{k},\beta_{q\bar{Q}}\right)$ in (3) is subject of the normalization condition

$$\sum_{\ell,S}\int d\mathrm{k}\;\mathrm{k}^2\;\left|\Phi^{J}_{\ell S}\left(\mathrm{k},\beta_{q\bar{Q}}\right)\right|^2=1. \qquad (6)$$

Note that for the further calculation WF of pseudoscalar and vector mesons chosen as [13, 15]

$$\Phi(\mathrm{k},\beta)=\frac{2}{\pi^{1/4}\,\beta^{3/2}}\;\exp\left[-\frac{\mathrm{k}^2}{2\beta^2}\right]. \qquad (7)$$

3. Calculation of model parameters

The basic parameters of the model are obtained with the integral representations of the lepton decay constant [14]

$$f_I(m_q,m_{\bar{Q}},\beta^{I}_{q\bar{Q}})=\sqrt{\frac{3}{2}\frac{1}{\pi}}\int_0^{\infty}d\mathrm{k}\;\mathrm{k}^2\;\Phi(\mathrm{k},\beta^{I}_{q\bar{Q}})\sqrt{\frac{W^{+}_{m_q}(\mathrm{k})\;W^{+}_{m_{\bar{Q}}}(\mathrm{k})}{M_0(\mathrm{k})\;\omega_{m_q}(\mathrm{k})\,\omega_{m_{\bar{Q}}}(\mathrm{k})}}\times$$

$$\times\left(1+a_I\frac{\mathrm{k}^2}{W^{+}_{m_q}(\mathrm{k})\,W^{+}_{m_{\bar{Q}}}(\mathrm{k})}\right);\;\;W^{\pm}_{m}(\mathrm{k})=\omega_m(\mathrm{k})\pm m,\,a_P=-1,\;\;a_V=1/3 \qquad (8)$$

for pseudoscalar ($I=P$) and vector ($I=V$) mesons. Using the pseudoscalar density constant

$$g_P\left(m_q,m_{\bar{Q}},\beta^{P}_{q\bar{Q}}\right)=\sqrt{\frac{3}{2}\frac{1}{\pi}}\int d\mathrm{k}\;\mathrm{k}^2\;\Phi\left(\mathrm{k},\beta^{P}_{q\bar{Q}}\right)\sqrt{\frac{M_0(\mathrm{k})}{\omega_{m_q}(\mathrm{k})\,\omega_{m_{\bar{Q}}}(\mathrm{k})}}\left(\sqrt{W^{+}_{m_q}(\mathrm{k})\,W^{+}_{m_{\bar{Q}}}(\mathrm{k})}+\right.$$

$$\left.+\sqrt{W^{-}_{m_q}(\mathrm{k})\,W^{-}_{m_{\bar{Q}}}(\mathrm{k})}\right) \qquad (9)$$

from the equations

$$\begin{cases} m_d - m_u = \hat{m}_d - \hat{m}_u = (2.5 \pm 0.4)\ \text{MeV} \\ f_P(m_q, m_{\bar{Q}}, \beta^P_{q\bar{Q}}) = f_P^{(\text{exp})}, \\ f_V(m_q, m_{\bar{Q}}, \beta^P_{q\bar{Q}}) = f_V^{(\text{exp})}, \\ (\hat{m}_q + \hat{m}_{\bar{Q}})\ g_P(m_q, m_{\bar{Q}}, \beta^P_{q\bar{Q}}) = f_P^{(\text{exp})} M_P^2, \end{cases} \tag{10}$$

where $\hat{m}_{q,\bar{Q}}$ – current quark masses and $f^{(\text{exp})}_{P,V}$ – experimental values of decay constants [16], one can obtain following values of quark masses and β–parameters of the WF [15]:

$$m_u = (219.48 \pm 9.69)\ \text{MeV},\ m_d = (221.97 \pm 9.69)\ \text{MeV},$$
$$\beta^P_{u\bar{d}} = (367.93 \pm 25.10)\ \text{MeV},\ \beta^V_{u\bar{d}} = (311.95 \pm 2.14)\ \text{MeV}. \tag{11}$$

4. Form-factors of ρ–meson in point form of PiQM

Calculation form-factors of vector ρ–meson will hold with matrix element [3]

$$I^\mu(\lambda', \lambda) = \langle \mathbf{Q}', M', \lambda' | J^\mu | \mathbf{Q}, M, \lambda \rangle = \frac{1}{(2\pi)^3} \frac{1}{\sqrt{2\ \omega_M(\mathbf{Q})}} \frac{1}{\sqrt{2\ \omega_{M'}(\mathbf{Q}')}} \times \tag{12}$$

$$\times \Bigg(F_1(q^2)\Big(\epsilon^*(\lambda') \cdot \epsilon(\lambda)\Big) P^\mu + F_2(q^2) \Big(\Big(P \cdot \epsilon^*(\lambda')\Big)\ \epsilon^\mu(\lambda) + \Big(P \cdot \epsilon(\lambda)\Big)\ \epsilon^{*\mu}(\lambda') \Big) +$$
$$+ F_3(q^2) \frac{\Big(P \cdot \epsilon^*(\lambda')\Big)\Big(P \cdot \epsilon(\lambda)\Big)}{2\ M\ M'}\ P^\mu \Bigg).$$

In relation (12)

$$P = Q + Q',\ q = Q - Q', \tag{13}$$

and form-factors F_1, F_2 and F_3 when $q^2 \to 0$ defined as charge e, magnetic μ_ρ and quadrupole moment Q_ρ of ρ–meson [3]:

$$F_1(0) = e,\ F_2(0) = -\mu_\rho,\ F_3(0) = Q_\rho. \tag{14}$$

Since polarization 4-vectors $\epsilon(\lambda)$ and $\epsilon^*(\lambda')$ determined by the relations

$$\Big(\epsilon(\lambda) \cdot \epsilon^*(\lambda)\Big) = -1,\ \Big(\epsilon(\lambda) \cdot Q\Big) = 0, \tag{15}$$

for generalized Breit system $(\mathbf{V_Q} + \mathbf{V'_{Q'}} = 0)$ we define 4-momentum vectors by

$$Q = \frac{1}{\sqrt{2}}\{M\sqrt{1+\varpi}, 0, 0, M\sqrt{-1+\varpi}\},\ Q' = \frac{1}{\sqrt{2}}\{M'\sqrt{1+\varpi}, 0, 0, -M'\sqrt{-1+\varpi}\}, \tag{16}$$

where $(V_Q \cdot V'_{Q'}) = \varpi$; in this case

$$\epsilon(\lambda) = \frac{1}{\sqrt{2}}\{(\lambda^2-1)^2\sqrt{-1+\varpi}, \lambda^2, -\mathrm{i}\lambda, (\lambda^2-1)^2\sqrt{1+\varpi}\}, \tag{17}$$

Table 1: Matrix element for different pairs of ρ–meson helicities

	$\lambda=1$	$\lambda=0$	$\lambda=-1$
$\lambda'=1$	$F_1(q^2)\times \ell_{F_1}$	$F_2(q^2)\times \ell^{(+)}_{F_2}$	0
$\lambda'=0$	$F_2(q^2)\times \ell^{(-)}_{F_2}$	linear combination of $F_1(q^2)$, $F_2(q^2)$ and $F_3(q^2)$	$F_2(q^2)\times \ell^{(+)}_{F_2}$
$\lambda'=-1$	0	$F_2(q^2)\times \ell^{(-)}_{F_2}$	$F_1(q^2)\times \ell_{F_1}$

$$\epsilon(\lambda')=\frac{1}{\sqrt{2}}\{(\lambda'^2-1)^2\sqrt{-1+\varpi},\lambda'^2,-i\lambda',-(\lambda'^2-1)^2\sqrt{1+\varpi}\},$$

so relations (15) fulfilled automatically. Using (13) - (17) one can get the following table (see table 1) for the matrix elements of form-factors for different helicities of the initial and final states. Note, that in 1 we use following notations:

$$\ell_{F_1}=\frac{M+M'}{2\sqrt{M\ M'}}\left\{-1,0,0,\frac{M'\sqrt{-1+\varpi_{1,2}}-M}{M+M'\sqrt{1+\varpi_{1,2}}}\right\},\ \ell^{(\pm)}_{F_2}=\frac{1}{2}\sqrt{\frac{M'(-1+\varpi_{1,2})}{M}}\{0,1,\pm i,0\}. \tag{18}$$

From the table one can see that form-factor $F_1(q^2)$ can be calculated directly from a matrix element $I^{\mu}_{\pm1,\pm1}$. Using that in point form of PiQM

$$M_0=\omega_{m_q}(\mathrm{k})+\omega_{m_{\bar{Q}}}(\mathrm{k}),\quad M'_0=\omega_{m_q}(\mathrm{k}_{1,2})+\omega_{m_{\bar{Q}}}(\mathrm{k}_{1,2}) \tag{19}$$

where

$$\mathbf{k}_{1,2}=\mathbf{k}\ \pm\ \boldsymbol{v}_{\mathbf{Q}}\left((\varpi_{1,2}+1)\omega_{m_{q,\bar{Q}}}(\mathrm{k})-\mathrm{k}\sqrt{\varpi^2_{1,2}-1}\cos\theta_{\mathrm{k}}\right), \tag{20}$$

after some calculations of the spinor part the expression

$$I^{\mu}(\lambda',\lambda)=\frac{1}{4\pi}\frac{1}{(2\pi)^3}\frac{1}{\sqrt{V_0\ V'_0}}\sum_{\nu_1,\nu'_1}\sum_{\nu_2,\nu'_2}\int \mathrm{dk}\,\Phi\left(\mathrm{k},\beta^V_{q\bar{Q}}\right)\mathrm{C}\left\{\begin{smallmatrix}\frac{1}{2}&\frac{1}{2}&1\\ \nu_1&\nu_2&\lambda\end{smallmatrix}\right\}\mathrm{C}\left\{\begin{smallmatrix}\frac{1}{2}&\frac{1}{2}&1\\ \nu'_1&\nu'_2&\lambda'\end{smallmatrix}\right\}\times \tag{21}$$
$$\times\left(\frac{\bar{u}_{\nu'_1}(\mathbf{k}_2,m_q)B(\boldsymbol{v}_{\mathbf{Q}})\Gamma^{\mu}_{q}u_{\nu_1}(\mathbf{k},m_q)}{\sqrt{2\ \omega_{m_q}(\mathrm{k}_2)\ 2\ \omega_{m_q}(\mathrm{k})}}\Phi\left(\mathrm{k}_2,\beta^V_{q\bar{Q}}\right)\sqrt{\frac{\omega_{m_{\bar{Q}}}(\mathrm{k}_2)}{\omega_{m_{\bar{Q}}}(\mathrm{k})}}D_{\nu'_2,\nu_2}(\mathbf{n}_{W_2})+\right.$$
$$\left.+\frac{\bar{v}_{\nu_2}(\mathbf{k},m_{\bar{Q}})B(-\boldsymbol{v}_{\mathbf{Q}})\Gamma^{\mu}_{\bar{Q}}v_{\nu'_2}(\mathbf{k}_1,m_{\bar{Q}})}{\sqrt{2\ \omega_{m_{\bar{Q}}}(\mathrm{k})2\ \omega_{m_{\bar{Q}}}(\mathrm{k}_1)}}\Phi\left(\mathrm{k}_1,\beta^V_{q\bar{Q}}\right)\sqrt{\frac{\omega_{m_q}(\mathrm{k}_1)}{\omega_{m_q}(\mathrm{k})}}D_{\nu'_1,\nu_1}(\mathbf{n}_{W_1})\right)$$

with

$$\Gamma^{\mu}_{q,\bar{Q}}=F_1(q^2)\gamma^{\mu}+F_2(q^2)\frac{i\ \sigma^{\mu\nu}}{2\ m_{q,\bar{Q}}}q_{\nu},\ F_1(0)=e_{q,\bar{Q}},\ F_1(0)+F_2(0)=\mu_{q,\bar{Q}} \tag{22}$$

one can obtain

$$G_C(0)=F_1(0)=\int \mathrm{dk}\ \mathrm{k}^2\left|\Phi\left(\mathrm{k},\beta^V_{q\bar{Q}}\right)\right|^2(e_q+e_{\bar{Q}})\,. \tag{23}$$

Since charge ρ-meson consists of $e_q=e_u=2/3$ and $e_{\bar{Q}}=e_{\bar{d}}=1/3$ (in units e) taking account relation (5) from (23) one can obtain $G_C(0)=1$.

Calculating magnetic moment of ρ-meson could be performed directly from $I^{1,2}_{\pm 1,0}$ or $I^{1,2}_{0,\pm 1}$ (calculation equivalents). Similar procedure for calculating quark transition currents and limiting $q^2 \to 0$ from (12) – (21) gives

$$F_2(0) = \int \mathrm{dk}\ \mathrm{k}^2 \left|\Phi\left(\mathrm{k}, \beta^V_{q\bar{Q}}\right)\right|^2 \left(e_q\ \eta_1(k, m_q, m_{\bar{Q}}) + \right. \tag{24}$$

$$\left. + \frac{e_q \kappa_q}{2m_q}\ \eta_2(k, m_q, m_{\bar{Q}}) - e_{\bar{Q}}\ \eta_1(k; m_{\bar{Q}}, m_q) - \frac{e_{\bar{Q}} \kappa_{\bar{Q}}}{2m_{\bar{Q}}}\ \eta_2(k; m_{\bar{Q}}, m_q)\right),$$

where

$$\eta_1(k, m_q, m_{\bar{Q}}) = \frac{1}{3} \frac{\omega_{m_q}(\mathrm{k})\left(2\ m_q - m_{\bar{Q}} + \omega_{m_q}(\mathrm{k})\right) + \omega_{m_{\bar{Q}}}(\mathrm{k})\left(m_q + 3\ \omega_{m_q}(\mathrm{k})\right)}{\omega^2_{m_q}(\mathrm{k})} \tag{25}$$

and

$$\eta_2(k, m_q, m_{\bar{Q}}) = \frac{2}{3} \frac{\mathrm{k}^2\left(\mathrm{k}^2 + \left(W^+_{m_q}(k) + m_q\right)\left(\omega_{m_{\bar{Q}}}(\mathrm{k}) + 2\ m_q\right)\right) + 3\ m_q^2\ W^+_{m_q}(k)\left(\omega_{m_{\bar{Q}}}(\mathrm{k}) + m_q\right)}{\omega^2_{m_q}(\mathrm{k})\ W^+_{m_q}(k)}. \tag{26}$$

Using values of u, d-quark masses, β-parameters of the WF (see (11)) and anomalous magnetic moments of quarks, obtained from radiative $V(P) \to P(V)\gamma$ decays [14, 15]

$$\kappa_u = -0.123, \quad \kappa_d = -0.088 \tag{27}$$

after numerical calculation with oscillator wave function (7) one can obtain $\mu_\rho = 2.29 \pm 0.02$ with anomalous magnetic moments and $\mu_\rho = 2.91 \pm 0.02$ without κ_u, κ_d (in $e/2m_\rho$ units). Comparing with experimental data $\mu_\rho = 2.1 \pm 0.5$ we can conclude that the proposed model describes both leptonic decays and hadronic form-factors of pseudoscalar and vector mesons.

5. Conclusion

The work is devoted to the calculation of the form factors of vector particles in a model based on the point form of PiQM. As a result of calculations, it was shown that the developed model for calculating the leptonic and radiative decays can be successfully used to analyze the observed of ρ-meson. The calculation results for μ_ρ are compared and in a good agreement with experimental data. As the result of the work one can note the obtainment of a self-consistent model for describing leptonic and hadronic meson decays, based on point form of Poincaré-invariant quantum mechanics.

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