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Solution of quasipotential equation with linear quasipotential: the case of arbitrary masses

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Solution of integral equation with linear quasipotential for s-wave is received. Consideration is conducted within the framework of relativistic quasipotential approach in quantum field theory, formulated in the relativistic configurational representation in the case of two particles with arbitrary masses.

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1. Introduction

Logunov and Tavkhelidze's relativistic quasipotential (RQP) approach [1] is constructed based on a covariant single-time formulation of the two-body problem in quantum field theory (QFT). The RQP approach in QFT remains one of the efficient methods for description of the bounded system two particles and has been widely used to describe properties of atoms, hadrons, and nuclei as bound states. Exact solutions of RQP equations with quasipotentials of specific forms (see, e.g., [2]– [21]) are as interesting as the corresponding solutions of the nonrelativistic Schrödinger equation.

Our aim here is to derive within the framework of RQP formalism in QFT [22] the solution of integral equation formulated in the relativistic configuration representation for the interaction of two relativistic particles of unequal masses m_1, m_2 [23] with linear quasipotential in the case s-wave ($\ell = 0$).

2. The regular and irregular solutions

Our consideration is based on the full covariant RQP integral form of the relativistic Schrödinger equation in the configurational representation (r-representation), constructed in [23] for the RQP wave function $\psi_{q'}(\mathbf{r})$ for the interaction of two relativistic particles with arbitrary masses m_1 and m_2. This equation for the RQP partial wave function $\varphi_\ell(\rho, \chi')$ has the form (we use the system of units where $c = \hbar = 1$) [24]

$$\begin{aligned}&\frac{2}{\pi}\int\limits_0^\infty d\chi \frac{(\sinh\chi)^{2\ell+2}(-1)^{\ell+1}}{\rho^{(\ell+1)}}\,(2\cosh\chi' - 2\cosh\chi)\left(\frac{d}{d\cosh\chi}\right)^\ell\left(\frac{\sin\rho\chi}{\sinh\chi}\right)\times\\ &\times\left(\frac{d}{d\cosh\chi}\right)^\ell\frac{1}{\sinh\chi}\int\limits_0^\infty d\rho'\,\frac{\rho'\sin\rho'\chi}{(-\rho')^{(\ell+1)}}\varphi_\ell(\rho',\chi') = \frac{2\mu}{m'^2}\frac{V(\rho/m')\varphi_\ell(\rho,\chi')}{\rho}.\end{aligned} \tag{1}$$

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ere $\mu = m_1 m_2/(m_1+m_2)$ is the usual reduced mass, χ' is the rapidity of an effective relativistic article which concept appears in RQP approach [23], and that plays the role of a two-particle ystem, has a mass $m' = \sqrt{m_1 m_2}$ and a relative three-momentum $\mathbf{q}'$, and carries the total c.i.s. nergy $\sqrt{s}$ of the interacting particles, proportional to the energy $E_{q'}$ of one effective relativistic article of the mass m' [6], [23]):

$$\sqrt{s} = \sqrt{{m_1}^2 + \mathbf{q}^2} + \sqrt{{m_2}^2 + \mathbf{q}^2} = \frac{m'}{\mu} E_{q'}, \quad E_{q'} = \sqrt{m'^2 + \mathbf{q}'^2} = m' \cosh \chi'; \tag{2}$$

$V(\rho/m')$ is a local in $\mathbf{r}$-representation a spherically symmetric potential, the function $(-\rho)^{(l)} = i^l\Gamma(l+i\rho)/\Gamma(i\rho)$ is called the generalized power [25] where $\Gamma(z)$ is the gamma-function, and the modulus of the radius vector $\boldsymbol{\rho} = m'\mathbf{r}$ $(\boldsymbol{\rho} = \rho\mathbf{n}, |\mathbf{n}| = 1)$ is a relativistic invariant [6].

We will seek a solution of RQP equation (1) with linear quasipotential

$$V(\rho/m') = \frac{\beta\rho}{m'}, \ \beta > 0, \tag{3}$$

for the case s-wave $(\ell = 0)$ in the form [20]

$$\varphi_0(\rho, \chi') = \int_{\alpha_-}^{\alpha_+} d\zeta e^{i\rho\zeta} R_0(\zeta, \chi'). \tag{4}$$

where the integration over ζ is performed in the complex plane along the contour with the endpoints α_- and α_+.

Substituting representation (4) in Eq. (1) with $\ell = 0$ and integrating on variables ρ' and χ, we arrive at the equation

$$\int_{\alpha_-}^{\alpha_+} d\zeta e^{i\rho\zeta} (\cosh \chi' - \cosh \zeta) R_0(\zeta, \chi') = \frac{\rho}{\tilde{\beta}} \int_{\alpha_-}^{\alpha_+} d\zeta e^{i\rho\zeta} R_0(\zeta, \chi'), \ \tilde{\beta} = \frac{m'^3}{\mu\beta}, \ \rho \geq 0. \tag{5}$$

Integrating Eq. (5) by parts leads to the equation

$$\frac{dR_0(\zeta, \chi')}{d\zeta} - i\tilde{\beta} (\cosh \zeta - \cosh \chi') R_0(\zeta, \chi') = 0 \tag{6}$$

with the boundary condition

$$e^{i\rho\zeta} R_0(\zeta, \chi') \Big|_{\alpha_-}^{\alpha_+} = 0, \ \forall \rho \geq 0. \tag{7}$$

The solution of Eq. (6) is

$$R_0(\zeta, \chi') = C_0(\chi') \exp\left[i\tilde{\beta} (\sinh \zeta - \zeta \cosh \chi')\right], \tag{8}$$

where $C_0(\chi')$ is an arbitrary function of χ'.

Substituting solution (8) into representation (4), we obtain the expression for the RQP partial wave function $\varphi_0(\rho, \chi')$

$$\varphi_0(\rho, \chi') = C_0(\chi') \int_{\alpha_-}^{\alpha_+} d\zeta e^{i\tilde{\beta} \sinh \zeta + i\zeta(\rho - \tilde{\beta} \cosh \chi')}. \tag{9}$$

Sommerfeld's representations for the Bessel functions are given by expressions [26]

$$H_{\nu}^{(1,2)}(z) = -\frac{1}{i\pi}\int\limits_{\gamma_{1,2}} d\zeta e^{z\sinh\zeta-\nu\zeta}, \tag{10}$$

$$J_{\nu}(z) = -\frac{1}{2i\pi}\int\limits_{\gamma_{3}} d\zeta e^{z\sinh\zeta-\nu\zeta}, \tag{11}$$

where

$$-\eta < \arg z < \pi - \eta,\ 0 \le \eta \le \pi, \tag{12}$$

and the integration contours $\gamma_k\,(k=1,2,3)$ in Eqs. (10) and (11) are given by

$$\begin{cases} \gamma_1 : \zeta \in \left(\infty + i\eta + \dfrac{i\pi}{2}; -\infty - i\eta + \dfrac{i\pi}{2}\right); \\ \gamma_2 : \zeta \in \left(-\infty - i\eta + \dfrac{i\pi}{2}; \infty + i\eta - \dfrac{3i\pi}{2}\right); \\ \gamma_3 : \zeta \in \left(\infty + i\eta + \dfrac{i\pi}{2}; \infty + i\eta - \dfrac{3i\pi}{2}\right). \end{cases} \tag{13}$$

By using solution (9) and the representations in Eqs. (10) and (11), we find that

$$z = i\tilde{\beta},\ \nu = -i(\rho - \tilde{\beta}\cosh\chi'), \tag{14}$$

but as the integration contours with the endpoints α_- and α_+ necessary to choose one of the curves $\gamma_k\,(k=1,2,3)$ in Eqs. (13).

The values of the endpoints α_- and α_+ in solution (9) and parameters η and z in Eqs. (10)–(13) must satisfy the boundary condition (7) and conditions (12) and (14), that leads to the system inequality

$$\begin{cases} \operatorname{Re}\alpha_\pm = \pm\infty,\ 0 < \operatorname{Im}\alpha_\pm < \pi,\ 0 \le \eta < \dfrac{\pi}{2}, \\ -\eta < \arg z < \pi - \eta,\ \arg z = \dfrac{\pi}{2}. \end{cases} \tag{15}$$

Hence, we find $\eta = 0$, but then the integration curves $\gamma_k\,(k=1,2,3)$ in (13) takes the form

$$\begin{cases} \gamma_1 : \zeta \in \left(\infty + \dfrac{i\pi}{2}; -\infty + \dfrac{i\pi}{2}\right);\ \gamma_2 : \zeta \in \left(-\infty + \dfrac{i\pi}{2}; \infty - \dfrac{3i\pi}{2}\right); \\ \gamma_3 : \zeta \in \left(\infty + \dfrac{i\pi}{2}; \infty - \dfrac{3i\pi}{2}\right). \end{cases} \tag{16}$$

Therefore, the linear independent solutions of the equation (1) with linear quasipotential (3) for case s-wave are given by expressions

$$\varphi_0^{\mathrm{R}}(\rho,\chi') = \frac{i\pi C_0(\chi')}{2} H^{(1)}_{-i(\rho-\tilde{\beta}\cosh\chi')}\left(i\tilde{\beta}\right) = \frac{C_0(\chi')}{2}\int\limits_{-\infty+i\pi/2}^{\infty+i\pi/2} d\zeta e^{i\tilde{\beta}\sinh\zeta + i(\rho-\tilde{\beta}\cosh\chi')\zeta}, \tag{17}$$

$$\varphi_0^{(+)}(\rho,\chi') = C_0^{(+)}(\chi') J_{-i(\rho-\tilde{\beta}\cosh\chi')}\left(i\tilde{\beta}\right) = \frac{C_0^{(+)}(\chi')}{2i\pi}\int\limits_{\infty-3i\pi/2}^{\infty+i\pi/2} d\zeta e^{i\tilde{\beta}\sinh\zeta + i(\rho-\tilde{\beta}\cosh\chi')\zeta}, \tag{18}$$

$$\varphi_0^{(2)}(\rho,\chi') = C_0^{(2)}(\chi')H^{(2)}_{-i(\rho-\tilde{\beta}\cosh\chi')}\left(i\tilde{\beta}\right) = -\frac{C_0^{(2)}(\chi')}{i\pi}\int\limits_{-\infty+i\pi/2}^{\infty-3i\pi/2} d\zeta e^{i\tilde{\beta}\sinh\zeta+i(\rho-\tilde{\beta}\cosh\chi')\zeta}. \tag{19}$$

Since the potential (3) is the potential of confining then in the region of $\rho \leq 0$ the relativistic regular solution must be identical is a zero, but in the region of $\rho \geq 0$ *it on the strength of* oscillations must apply to zero n once [27]. Therefore, solution (17) is the relativistic regular solution and satisfies as the boundary condition at zero

$$\varphi_0^{\mathrm{R}}(0,\chi') = 0, \tag{20}$$

so and condition of the complex conjugacy

$$[\varphi_0^{\mathrm{R}}(\rho,\chi')]^* = \varphi_0^{\mathrm{R}}(\rho,\chi'). \tag{21}$$

The real normalization factor $C_0(\chi')$ is defined from the condition of normalization

$$4\pi\int\limits_0^\infty d\rho[\varphi_0^{\mathrm{R}}(\rho,\chi')]^2 = 1. \tag{22}$$

The regular solution (17) possible express through modified Bessel function (the MacDonald function) $K_\nu(z)$, connected with the Hankel function of the first kind $H_\nu^{(1)}(z)$ by formula [28]

$$K_\nu(z) = \frac{i\pi}{2}e^{i\pi\nu/2}H_\nu^{(1)}\left(ze^{i\pi/2}\right), \; -\pi < \arg z \leq \frac{\pi}{2}. \tag{23}$$

It then follows from expression (17) and (23) that

$$\varphi_0^{\mathrm{R}}(\rho,\chi') = C_0(\chi')e^{-\pi(\rho-\tilde{\beta}\cosh\chi')/2}K_{-i(\rho-\tilde{\beta}\cosh\chi')}\left(\tilde{\beta}\right). \tag{24}$$

The solving of Logunov-Tavkhelidze quasipotential integral equation in the space of moments (or its finite-difference of the analogue in r-representation) with linear quasipotential in the case of two interacting relativistic particles with equal masses [16] leads to the similar result as in Eq. (24).

3. Bound states and analogues of the Jost solutions

The regular solution in Eq. (24) under $\rho = 0$ satisfies the boundary condition (20). Thence follows the exact quantization condition for the energies $E_n = (m'^2/\mu)\cosh\chi'_n$ of bound states of an effective relativistic particle of mass m' that emerges instead of the system of interacting relativistic particles with masses m_1 and m_2

$$K_{i\tilde{\beta}\cosh\chi'_n}\left(\tilde{\beta}\right) = 0. \tag{25}$$

Exactly such the condition as in Eq. (25) but in the case of two interacting relativistic particles with equal masses was received in [16]. The MacDonald function $K_\nu(z)$ considered as function from ν has infinite number of zeroes and all of these are pure imaginary [28].

To research the behavior of regular solution $\varphi_0^R(\rho,\chi')$ in Eq. (24) at different ranges of the relativistic relative coordinate ρ, we consider large values of $\tilde{\beta}$ (small of β) and use the asymptotic expression for the MacDonald function [28]

$$K_{ip}(x) = \frac{\sqrt{2}}{\sqrt[4]{p^2 - x^2}} e^{-p\pi/2} \left[\sum_{m=0}^{N-1} 2^m b_m \Gamma\left(m + \frac{1}{2}\right) \sqrt{(p^2 - x^2)^{-m}} \times \right.$$
$$\left. \times \sin\left(\frac{m\pi}{2} + p \mathrm{Arcosh} \frac{p}{x} - \sqrt{p^2 - x^2} + \frac{\pi}{4}\right) + O\left(x^{-N}\right) \right], p > x > 0; \tag{26}$$

$$K_{ip}(x) = \frac{1}{\sqrt[4]{4\left(x^2 - p^2\right)}} \exp\left[-\sqrt{x^2 - p^2} - p \arcsin \frac{p}{x}\right] \times$$
$$\times \left[\sum_{m=0}^{N-1} (-2)^m b_m \Gamma\left(m + \frac{1}{2}\right) \sqrt{(x^2 - p^2)^{-m}} + O\left(x^{-N}\right)\right], \tag{27}$$
$$x > p > 0,\ b_0 = 1,\ b_1 = \frac{1}{8} - \frac{5}{24}\left(1 - \frac{x^2}{p^2}\right)^{-1}, \ldots;$$

$$K_{ip}(x) \sim \frac{\pi}{3} e^{-p\pi/2} \sum_{m=0}^{\infty} (-1)^m C_m(\epsilon x) \sin\left[(m+1)\frac{\pi}{3}\right] \Gamma\left(\frac{m+1}{3}\right) \left(\frac{x}{6}\right)^{-(m+1)/3},$$
$$p \approx x,\ p, x > 0,\ \epsilon = 1 - \frac{p}{x},\ \epsilon = o\left(x^{-2/3}\right),\ C_0(\epsilon x) = 1,\ C_1(\epsilon x) = \epsilon x, \ldots. \tag{28}$$

By using solution in (24) and the asymptotic expression in (26), we then find that the region $p = |\rho - \tilde{\beta} \cosh \chi'| > \tilde{\beta} > 0$ answer two regions: classical available the region $0 \leq \rho < \tilde{\beta} \cosh \chi' - \tilde{\beta}$ (the region I) and the region of the quark pair production $\rho > \tilde{\beta} \cosh \chi' + \tilde{\beta}$ (the region III). In the region I and III wave function $\varphi_0^{\mathrm{R}}(\rho, \chi')$ has the form ($|\rho - \tilde{\beta} \cosh \chi'| > \tilde{\beta} > 0$)

$$\varphi_0^{\mathrm{R}}(\rho, \chi') \sim \frac{C_0(\chi') \sqrt{2\pi}}{\sqrt[4]{\left(\rho - \tilde{\beta} \cosh \chi'\right)^2 - \tilde{\beta}^2}} \exp\left[-\frac{\pi}{2}\left(\rho - \tilde{\beta} \cosh \chi' + |\rho - \tilde{\beta} \cosh \chi'|\right)\right] \times$$
$$\times \left[\sin\left(|\rho - \tilde{\beta} \cosh \chi'| \mathrm{Arcosh} \frac{|\rho - \tilde{\beta} \cosh \chi'|}{\tilde{\beta}} - \sqrt{\left(\rho - \tilde{\beta} \cosh \chi'\right)^2 - \tilde{\beta}^2} + \frac{\pi}{4}\right) + O\left(\tilde{\beta}^{-1}\right)\right]. \tag{29}$$

The RQP partial wave function $\varphi_0^{\mathrm{R}}(\rho, \chi')$ on the strength of oscillations in the region I applies in this the region to zero n once [27], but its expression under $\rho \ll \tilde{\beta} \cosh \chi' - \tilde{\beta}$ in the case of large values of the parameter $\tilde{\beta}$ takes more simple asymptotic the form

$$\varphi_0^{\mathrm{R}}(\rho, \chi') \sim C_0(\chi') \sqrt{\frac{2\pi}{\tilde{\beta} \sinh \chi'}} \sin\left[\rho \chi' + \tilde{\beta}\left(\sinh \chi' - \chi' \cosh \chi'\right) - \frac{\pi}{4} - \frac{\rho^2}{2\tilde{\beta} \sinh \chi'}\right]. \tag{30}$$

From the boundary condition (20) and the asymptotic expression (30) follows the approximate quantization condition for the energies $E_n = (m'^2/\mu) \cosh \chi'_n$ of bound states of an effective relativistic particle of mass m'

$$\chi'_n \cosh \chi'_n - \sinh \chi'_n = \frac{\pi}{\tilde{\beta}}\left(n - \frac{1}{4}\right),\ n = 1, 2, \ldots, \tag{31}$$

which in the nonrelativistic limit go to the approximate quantization condition of the energy levels for Schrödinger equation with linear potential. The approximate quantization condition of the energy levels in (31) with linear potential in case of two interacting relativistic particles with equal masses was received in [16]. To exactly such the approximate quantization condition of the energy levels as in Eq. (31) leads the using of JWKB approximation to solve the finite-difference equation in r-representation in the form of Kadyshevsky with linear potential in the case of two interacting relativistic particles with equal masses [27].

The value of relativistic relative coordinate $\rho = \tilde{\beta}\cosh\chi'_n = m'^3\cosh\chi'_n/(\mu\beta)$ makes sense "size" an effective relativistic particle of the mass m' with the total energy $E_n = (m'^2/\mu)\cosh\chi'_n$.

The region $0 < p = |\rho - \tilde{\beta}\cosh\chi'| < \tilde{\beta}$ (the region II) answer two regions: $\tilde{\beta}\cosh\chi' - \tilde{\beta} < \rho < \tilde{\beta}\cosh\chi'$ and $\tilde{\beta}\cosh\chi' < \rho < \tilde{\beta}\cosh\chi' + \tilde{\beta}$. The behavior of the wave function $\varphi_0^{\mathrm{R}}(\rho,\chi')$ in accordance with expressions in (24) and (27) has decreasing nature:

$$\varphi_0^{\mathrm{R}}(\rho,\chi') \sim \frac{C_0(\chi')\sqrt{\pi/2}}{\sqrt[4]{\tilde{\beta}^2 - \left(\rho - \tilde{\beta}\cosh\chi'\right)^2}} \exp\left[-\frac{\pi}{2}\left(\rho - \tilde{\beta}\cosh\chi'\right) - \sqrt{\tilde{\beta}^2 - \left(\rho - \tilde{\beta}\cosh\chi'\right)^2} - \right.$$
$$\left. -|\rho - \tilde{\beta}\cosh\chi'| \arcsin\left(\frac{|\rho - \tilde{\beta}\cosh\chi'|}{\tilde{\beta}}\right)\right]\left[1 + O\left(\tilde{\beta}^{-1}\right)\right],\ 0 < |\rho - \tilde{\beta}\cosh\chi'| < \tilde{\beta}. \tag{32}$$

At the same time, under $p = |\rho - \tilde{\beta}\cosh\chi'| \approx \tilde{\beta}$, that is in the region of verging to region II, where $\rho \approx \tilde{\beta}\cosh\chi' \pm \tilde{\beta}$, the wave function $\varphi_0^{\mathrm{R}}(\rho,\chi')$ in accordance with expressions in (24) and (28) behaves as follows

$$\varphi_0^{\mathrm{R}}(\rho,\chi') \sim \frac{C_0(\chi')\pi\sqrt{3}}{6}\Gamma\left(\frac{1}{3}\right)\exp\left[-\frac{\pi}{2}\left(\rho - \tilde{\beta}\cosh\chi' + |\rho - \tilde{\beta}\cosh\chi'|\right)\right] \times$$
$$\times\left\{\left(\frac{\tilde{\beta}}{6}\right)^{-1/3} + O\left[\left(\tilde{\beta} - |\rho - \tilde{\beta}\cosh\chi'|\right)\tilde{\beta}^{-2/3}\right]\right\},\ \rho \approx \tilde{\beta}\cosh\chi' \pm \tilde{\beta}, \tag{33}$$

that is, the wave function is decreasing in the vicinities of the points $\rho = \tilde{\beta}\cosh\chi' \pm \tilde{\beta}$.

Analogues of the Jost solutions $f_0^{(\pm)}(\rho,\chi')$ in the absence of interaction (at values of $\beta \to 0$) and at fixed values of ρ must go to the corresponding free waves functions $e_0^{(1,2)}(\rho,\chi') = e^{\pm i\rho\chi'}$ [6]. Consequently, the boundary conditions for analogues of the Jost solutions $f_0^{(\pm)}(\rho,\chi')$ we choose in the form

$$\lim_{\beta\to 0} e^{\mp i\rho\chi'} f_0^{(\pm)}(\rho,\chi') = 1. \tag{34}$$

Analogues of the Jost solutions $f_0^{(\pm)}(\rho,\chi')$ must be connected with the functions $\varphi_0^{(+)}(\rho,\chi')$ and $\varphi_0^{(-)}(\rho,\chi') = \left[\varphi_0^{(+)}(\rho,\chi')\right]^*$. Consequently, the regular solution $\varphi_0^{\mathrm{R}}(\rho,\chi')$ can be represented in the form of a linear combination of two Jost solutions with constant (independent of ρ) coefficients [29]:

$$\varphi_0^{\mathrm{R}}(\rho,\chi') = \frac{\pi C_0(\chi')\left|J_{i\tilde{\beta}\cosh\chi'}\left(i\tilde{\beta}\right)\right|}{i}\left[F_0^{(-)}(\chi')f_0^{(+)}(\rho,\chi') - F_0^{(+)}(\chi')f_0^{(-)}(\rho,\chi')\right], \tag{35}$$

where analogues of the Jost solutions $f_0^{(\pm)}(\rho,\chi')$ and the Jost functions $F_0^{(\pm)}(\chi')$ are given by expressions

$$f_0^{(\pm)}(\rho,\chi') = -\frac{\sinh(\pi\tilde{\beta}\cosh\chi')e^{-\pi\tilde{\beta}\cosh\chi'}J_{\mp i\tilde{\beta}\cosh\chi'}\left(\mp i\tilde{\beta}\right)}{\sinh[\pi(\rho - \tilde{\beta}\cosh\chi')]e^{\pi(\rho - \tilde{\beta}\cosh\chi')}\left|J_{i\tilde{\beta}\cosh\chi'}\left(i\tilde{\beta}\right)\right|^2}J_{\mp i(\rho - \tilde{\beta}\cosh\chi')}\left(\pm i\tilde{\beta}\right), \tag{36}$$

$$F_0^{(\pm)}(\chi') = -\frac{e^{\pi\tilde{\beta}\cosh\chi'}J_{\mp i\tilde{\beta}\cosh\chi'}\left(\mp i\tilde{\beta}\right)}{2\sinh(\pi\tilde{\beta}\cosh\chi')\left|J_{i\tilde{\beta}\cosh\chi'}\left(i\tilde{\beta}\right)\right|}. \tag{37}$$

Analogues of the Jost solutions $f_0^{(\pm)}(\rho,\chi')$ in (36) satisfy both the equation in (1) with linear quasipotential in (3), and the boundary conditions in (34). By using the asymptotic expression

$$\varphi_0^{(+)}(\rho,\chi') \sim \frac{C_0^{(+)}(\chi')}{\sqrt{2\pi\tilde{\beta}\sinh\chi'}}\Big\{\exp\Big[i\Big(\rho\chi' + \tilde{\beta}(\sinh\chi' - \chi'\cosh\chi') + \frac{3\pi}{4} - \frac{\rho^2}{2\tilde{\beta}\sinh\chi'}\Big)\Big]\Big\}, \ \rho \ll \tilde{\beta}\cosh\chi', \tag{38}$$

easy make sure that for the Jost functions $F_0^{(\pm)}(\chi')$ in (37) limiting equality is executed

$$\lim_{\tilde{\beta}\to\infty}\left|F_0^{(\pm)}(\chi')\right| = 1.$$

4. Conclusion

Within the framework of considered RQP approach in quantum field theory formulated in the relativistic configurational representation in the case of two relativistic spinless particles with arbitrary masses m_1 and m_2, a method has been constructed for the finding of regular and irregular solutions of integral quasipotential equation in the case s-wave ($\ell = 0$) with local linear quasipotential.

Exact and approximate quantization condition for the energies of bound states of an effective relativistic particle of mass m' that emerges instead of the system of two interacting relativistic particles with arbitrary masses are received. The behavior of regular solution at different ranges of the relativistic relative coordinate ρ in the case of large values of $\tilde{\beta}$ (small of β) was investigated. Analogues of the Jost solutions and the Jost functions were defined.

Considered here method is directly connected with the possibility of representing the total c.m. energy of two relativistic spinless particles of the unequal masses as a quantity that is proportional to the energy of an one efficient relativistic particle of mass m'.

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