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The method for analysis of target mass effects on the structure functions

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We analyze the target mass correction to the structure functions of the deep-inelastic scattering by using Solovtsov's approach. According this approach the structure functions contain a dependence on a target mass and are in the consent with the spectral property. We demonstrate, that target mass corrections to structure functions calculated by using the new method noticeably differ that the traditional Georgi-Politzer method gives as well as from the Steffens-Melnitchouk approach.

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1. Introduction

The operator product expansion is one of the most general formalisms to analyze the properties of the structure functions in deep-inelastic scattering. This method was first used to study target mass effects by Georgi and Politzer in Ref. [1]. Such an approach for considering mass corrections, which are of purely kinematic origin, became known as the Georgi and Politzer (GP) approach or ξ-scaling method because, was formulated through the Nachtmann ξ variable [2]. The target mass corrections play a somewhat special role become larger and larger at low Q^2 and approaching to the kinematic limit as the Bjorken variable x tends to unity. However, the expressions for the structure functions obtained by ξ-scaling method have a difficulty arising from the violation of the spectral condition. It hence became a problem to describe the structure functions as the Bjorken variable $x \to 1$. This problem has been widely discussed in the literature ever since its appearance [3–12].

It was shown by Solovtsov [13] that this problem is similar to the problem that appears for an invariant charge in quantum chromodynamics, when the violation of the general principles of the theory, which are reflected in the Källén–Lehmann representation, leads to unphysical singularities. A solution of this problem was proposed in the Shirkov–Solovtsov analytic approach [14], which was later generalized to more complicated objects, such as structure functions. For the inelastic lepton-hadron scattering process, the general principles of the theory are accumulated in the Jost–Lehmann–Dyson (JLD) integral representation. This representation was proposed for the symmetric case in [15] and for the general case in [16]. The proof of the JLD representation is based on the most general principles of the theory, such as the covariance, Hermiticity, spectrality, and causality. As it was shown in [13] that by using the JLD representation, the natural scaling variable is a new variable ξ_S, which leads to the moments $\mathcal{M}_n(Q^2)$ that are analytic functions. In this case, the spectral property for the structure functions is satisfied automatically, and no problem arises in the limit as the Bjorken variable x tends to unity (see [17] for more details).

Our purpose here is to use the Solovtsov's approach to study both the QCD and target mass corrections to the proton and neutron structure functions and compare results with other approaches.

2. The GP method

According to the GP approach, which is founded on the operator product expansion, in the leading twist for massless quarks, the structure functions are given by (see, e.g., [18, 19])

$$F_2(x,Q^2) = \frac{x^2}{\xi^2\cdot r^3}F_2^0(\xi,Q^2) + \frac{6\varepsilon x^3}{r^4}\int_\xi^1 \frac{F_2^0(y,Q^2)}{y^2}dy + \frac{12\varepsilon^2 x^4}{r^5}\cdot\int_\xi^1 dy\int_y^1 \frac{F_2^0(z,Q^2)}{z^2}dz, \quad (1)$$

$$F_1(x,Q^2) = \frac{x}{\xi\cdot r}F_1^0(\xi,Q^2) + \frac{\varepsilon x^2}{r^2}\int_\xi^1 \frac{F_2^0(y,Q^2)}{y^2}dy + \frac{2\varepsilon^2 x^3}{r^3}\cdot\int_\xi^1 dy\int_y^1 \frac{F_2^0(z,Q^2)}{z^2}dz, \quad (2)$$

$$F_L(x,Q^2) = \frac{x^2}{\xi^2\cdot r}\left[F_2^0(\xi,Q^2) - 2\xi\, F_1^0(\xi,Q^2)\right] + \frac{4\varepsilon x^3}{r^2}\int_\xi^1 \frac{F_2^0(y,Q^2)}{y^2}dy$$
$$+ \frac{8\varepsilon^2 x^4}{r^3}\int_\xi^1 dy\int_y^1 \frac{F_2^0(z,Q^2)}{z^2}dz. \quad (3)$$

Here $x = Q^2/2\nu = Q^2/2(q\cdot P)$ is the Bjorken scaling variable, $r = (1+4\varepsilon x^2)^{\frac{1}{2}}$, $\varepsilon = M^2/Q^2$, M is the target mass, and ξ is the Nachtmann variable

$$\xi = \frac{2x}{1+\sqrt{1+4x^2\varepsilon}},$$

$F_1^0(\xi,Q^2) = \lim_{M\to 0} F_1(x,Q^2)_{x=\xi}$, $F_2^0(\xi,Q^2) = \lim_{M\to 0} F_2(x,Q^2)_{x=\xi}$, $F_L^0(\xi,Q^2) = \lim_{M\to 0} F_L(x,Q^2)_{x=\xi}$.

At the next-to-leading order (NLO) the function $F_2^0(x,Q^2)$ reads as (see., e.g, [20])

$$\frac{1}{x}F_2^0(x,Q^2) = \sum_q e_q^2\{q(x,Q^2) + \bar{q}(x,Q^2) + \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}[C_{2,q}^1(x/y)[q(y,Q^2)+\bar{q}(y,Q^2)]$$
$$+C_{2,g}^1(x/y)\cdot G(y,Q^2)]\}, \quad (4)$$

where

$$C_{2,q}^1(z) = \frac{4}{3}\left[\frac{1+z^2}{1-z}(ln(1-z)-\frac{3}{4}) + \frac{1}{4}(9+5z)\right]_+,$$

$$C_{2,g}^1(z) = \left[z^2+(1-z)^2\right]\cdot \ln\frac{1-z}{z} - 1 + 8z(1-z),$$

$$\int_x^1 f(x/y)_+\cdot\frac{dy}{y}q(y) = \int_x^1 \frac{dy}{y}\cdot f(x/y)[q(y) - \frac{x}{y}q(x)] - q(x)\int_0^x f(y)dy,$$

and the functions $q(x,Q^2)$, $\bar{q}(x,Q^2)$ are quark, anti-quark distributions, $G(x,Q^2)$ is the gluon distribution, and e_q is the quark charge.

Corresponding expression for the longitudinal structure function $F_L^0(x,Q^2)$ has the form

$$\frac{1}{x}F_L^0(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\cdot\sum_q e_q^2\{\int_x^1 \frac{dy}{y}[C_{L,q}^1(x/y)(q(y,Q^2)+\bar{q}(y,Q^2)) + 2\,C_{L,g}^1(x/y)\cdot G(y,Q^2)]\} \quad (5)$$

with $C_{L,q}^1(z) = \frac{8}{3}z$, $C_{L,g}^1(z) = 2z(1-z)$.

3. New approach

To include the target mass effects we apply the method proposed by Solovtsov [13]. As it was shown in [13] that by using the JLD representation, the natural scaling variable is a new variable ξ_S,

$$\xi_S = x\sqrt{\frac{1+4\varepsilon x^2}{1+4\varepsilon}},$$

which leads to the moments $\mathcal{M}_n(Q^2)$ that are analytic functions. In this case, the spectral property for the structure functions is satisfied automatically, and no problem arises in the limit as the Bjorken variable x tends to unity. According to the Solovtsov's method, instead of the distribution function $F(\xi)$, which is used in the Georgi–Politzer formalism, we consider the function $F(x, Q^2)$. This function can be presented as

$$F(x,Q^2) = \begin{cases} F(\beta_-) - F(1), & 0 \leqslant x < \overline{x}, \\ F(\beta_-) - F(\beta_+), & \overline{x} \leqslant x \leqslant 1, \end{cases} \qquad (6)$$

where $\overline{x} = 1/\sqrt{1+4\varepsilon^2}$,

$$\beta_\pm = \frac{x\sqrt{1+4\varepsilon x^2}}{1+4\varepsilon x^2+4\varepsilon^2 x^2}\left(1+2\varepsilon \pm 2\varepsilon\sqrt{\frac{1-x^2}{1+4\varepsilon x^2}}\right).$$

Note, the function (6) is in the consent with the spectral property: $F(x, Q^2) \to 0$ as $x \to 1$.

First of all, in Fig. 1 we present the behavior of the structure functions xF_1 and F_2 at Q^2=2 GeV^2 as functions of the Bjorken variable x (without QCD corrections). The solid line corresponds to our result obtained by using the Solovtsov's approach, the dashed curve reflects the result obtained by standard the GP method [1]. Dash-dotted line corresponds to the result got by using the method suggested by Steffens-Melnitchouk (SM) [11]. The parton distribution, $F(x) = \sqrt{x}(1-x)^3$, is shown as a dotted line. This figure shows that there is a difference in results got by different methods. It is necessary to emphasize that both functions calculated by us on base of Solovtsov's approach and according to SM method in the limit $x \to 1$ go to zero, but the function, corresponding to SM method goes to zero much faster, than function, got by using the Solovtsov's approach (see [17] for more details).

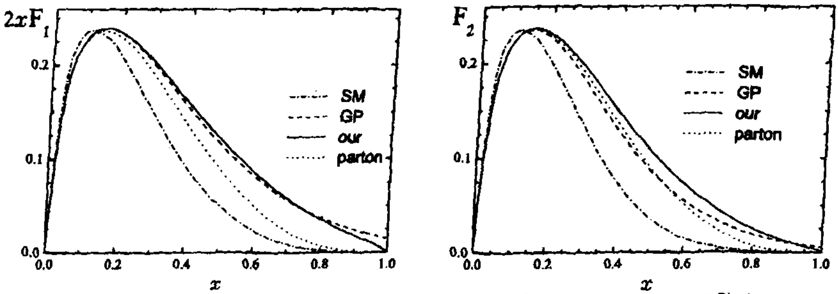

FIG. 1: The behavior of the structure functions $xF_1(x, Q^2)$ (left) and $F_2(x, Q^2)$ (right) vs x (without QCD corrections) at Q^2=2 GeV^2.

Next, according to (6), we modify expressions for structured functions (1) – (3). As a result we get:

$$F_2(x,Q^2) = \frac{x^2}{\beta_-^2\cdot r^3}F_2^0(\beta_-,Q^2) + \frac{6\varepsilon x^3}{r^4}\int\limits_{\beta_-}^{1}\frac{F_2^0(y,Q^2)}{y^2}dy + \frac{12\varepsilon^2 x^4}{r^5}\cdot\int\limits_{\beta_-}^{1}dy\int\limits_{y}^{1}\frac{F_2^0(z,Q^2)}{z^2}dz,$$

$$F_1(x,Q^2) = \frac{x}{\beta_-^{\tau}}F_1^0(\beta_-,Q^2) + \frac{\varepsilon x^2}{r^2}\int\limits_{\beta_-}^{1}\frac{F_2^0(y,Q^2)}{y^2}dy + \frac{2\varepsilon^2 x^3}{r^3}\cdot\int\limits_{\beta_-}^{1}dy\int\limits_{y}^{1}\frac{F_2^0(z,Q^2)}{z^2}dz, \qquad (7)$$

$$F_L(x,Q^2) = \frac{x^2}{\beta_-^2 r}F_L^0(\beta_-,Q^2) + \frac{4\varepsilon x^3}{r^2}\int\limits_{\beta_-}^{1}\frac{F_2^0(y,Q^2)}{y^2}dy + \frac{8\varepsilon^2 x^4}{r^3}\cdot\int\limits_{\beta_-}^{1}dy\int\limits_{y}^{1}\frac{F_2^0(z,Q^2)}{z^2}dz$$

if $0 \leqslant x \leqslant \overline{x}$, and

$$F_2(x,Q^2) = \frac{x^2}{r^3}\left[\frac{F_2^0(\beta_-,Q^2)}{\beta_-^2} - \frac{F_2^0(\beta_+,Q^2)}{\beta_+^2}\right] + \frac{6\varepsilon x^3}{r^4}\int\limits_{\beta_-}^{\beta_+}\frac{F_2^0(y,Q^2)}{y^2}dy + \frac{12\varepsilon^2 x^4}{r^5}\cdot\int\limits_{\beta_-}^{\beta_+}dy\int\limits_{y}^{1}\frac{F_2^0(z,Q^2)}{z^2}dz.$$

$$F_1(x,Q^2) = \frac{x}{r}\left[\frac{F_1^0(\beta_-,Q^2)}{\beta_-} - \frac{F_1^0(\beta_+,Q^2)}{\beta_+}\right] + \frac{\varepsilon x^2}{r^2}\int\limits_{\beta_-}^{1}\frac{F_2^0(y,Q^2)}{y^2}dy + \frac{2\varepsilon^2 x^3}{r^3}\cdot\int\limits_{\beta_-}^{1}dy\int\limits_{y}^{1}\frac{F_2^0(z,Q^2)}{z^2}dz, \qquad (8)$$

$$F_L(x,Q^2) = \frac{x^2}{r}\left[\frac{F_L^0(\beta_-,Q^2)}{\beta_-^2} - \frac{F_L^0(\beta_+,Q^2)}{\beta_+^2}\right] + \frac{4\varepsilon x^3}{r^2}\int\limits_{\beta_-}^{\beta_+}\frac{F_2^0(y,Q^2)}{y^2}dy + \frac{8\varepsilon^2 x^4}{r^3}\cdot\int\limits_{\beta_-}^{\beta_+}dy\int\limits_{y}^{1}\frac{F_2^0(z,Q^2)}{z^2}dz,$$

if $\overline{x} \leqslant x \leqslant 1$.

Since target mass corrections most essential on energy scale of order of nucleon mass then in our analysis we were limited the contributions from light quarks. We take the distribution of light u, d and s quarks and anti-quarks, as well as distribution of gluons from Ref. [20], where was also fixed $\Lambda_{QCD} = 0.248$ GeV. Note that distribution, provided in other papers, in the region of $x > 0.2$, for which become essential the target mass corrections, close to distribution given in [20] as differences appear under small x values.

The QCD running coupling in the NLO order for three active quarks has the well-known form

$$\frac{\alpha_s(Q^2)}{2\pi} = \frac{2}{9\ln(L)} - \frac{128}{9^3}\cdot\frac{\ln(L)}{L^2}, \quad L \equiv \ln\left(\frac{Q^2}{\Lambda^2{}_{QCD}}\right).$$

In Fig. 2 we present the results of our calculations for structure functions of the proton and the neutron computable with use the expressions (1) – (5), (7) and (8) at Q^2=1 GeV^2. One can see that target mass corrections to structure functions calculated by using the new method may be noticeably differ that the traditional Georgi–Politzer method gives.

4. Conclusion

We have reported the result for the proton and the neutron structure functions F_1, F_2 and F_L which was considered by using the Solovtsov approach including both the target mass and the NLO QCD corrections. As precision of experimental deep inelastic scattering data raises and the problem of study fine effects becomes actual, it is important to base on theoretical

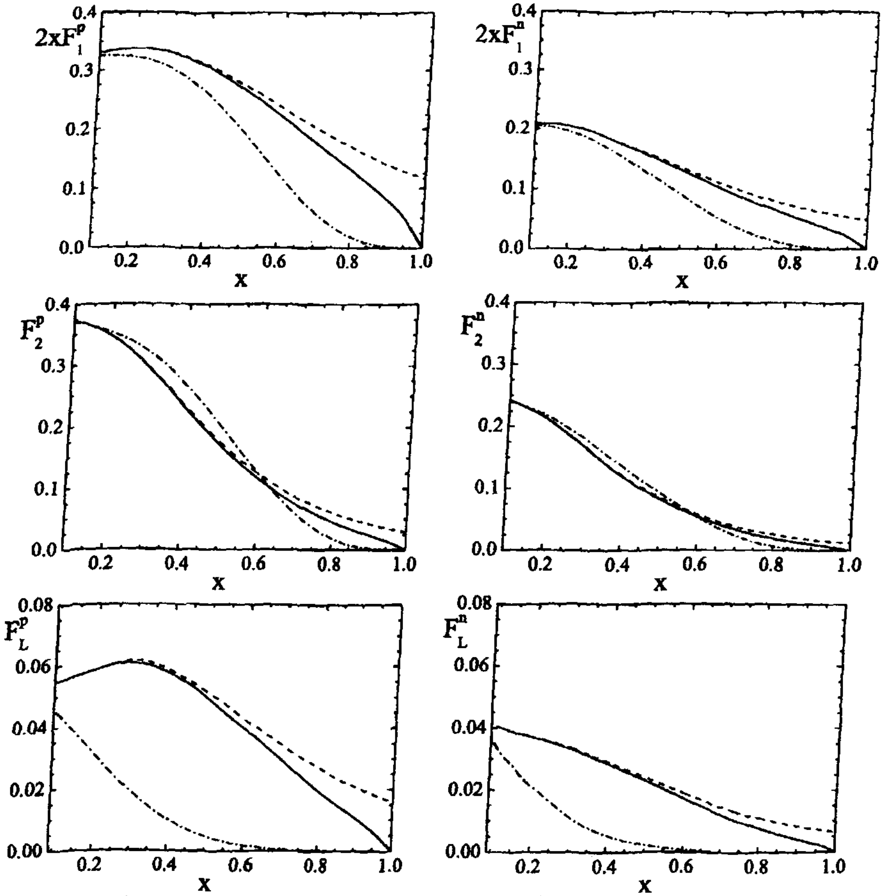

FIG. 2: The behavior of the proton structure functions (left) and the neutron structure functions (right) vs x at Q^2=1 GeV^2. The solid line corresponds to our result obtained by using the Solovtsov's approach, the dashed curve reflects the result obtained by standard GP method [1]. Dash-dotted line corresponds to the result without target mass corrections [20].

approaches which are in the consent with the general principles of quantum field theory. We have used the method which is in the consent with the spectral property. It has allowed to resolve a conflict with general spectral condition at $x = 1$. We have found that target mass corrections for different approaches may be essentially differ at low transfer momenta, $Q^2 \sim 1 \div 2$ GeV^2, however the distinction smooths out with growth Q^2.

We believe that the new method including target mass effects will be useful in extracting the magnitude of the structure functions from the experimental data. It is known that target mass corrections (relating to kinematic contributions) influence on the dependence of the higher-twist contribution, which is connected with the process dynamics and is extracted from experimental data. Processing experimental data (see, e.g., [21–23]) implies that the higher-twist contributions can increases sharply for large values of the Bjorken variable x.

Therefore, it is reasonable to expect that using new method will significantly affect the dependence of the higher-twist contribution on x obtained from experimental data.

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