PP-Scattering

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Abstract

Scattering of pseudoscalar mesons is considered in the framework of Quark Confinement Model. Analytical expressions of the contributions from box graph and graphs with intermediate vector and scalar mesons have been obtained.The results are used to calculate the $\pi\pi$ scattering lengths.It is found that the numerical values a_0^0 and a_0^2 depend strongly on the mass of the light intermediate scalar meson.It is shown that the best agreement with the experimental data obtained at $m_{f_0} = 500 \div 515$ MeV. The numerical values of the scattering lengths are found to be $a_0^0 = (0.25 \pm 0.3) \times m_\pi^{-1}$ and $a_0^2 = (-0.038 \pm 0.06) \times m_\pi^{-1}$

1 Introduction

The study of processes involving pseudoscalar mesons provides a unique opportunity to explore the strong interactions of hadrons at low energies.This is so for several reasons.The properties of the pions are intimately related to an approximate symmetry of QCD. In the chiral limit, where quark masses vanish, this symmetry becomes exact, the Lagrangian being invariant under the group $SU(2)_R \times SU(2)_L$ of chiral rotations. The symmetry is spontaneously broken to the isospin subgroup $SU(2)_V$. The pions represent the corresponding Goldstone bosons.As such, $\pi\pi$ reactions provide a direct link between the theoretical formalism of chiral symmetry and experiment. This is exemplified in the many existing studies of $\pi\pi$ scattering using chiral Lagrangians. that endeavour to calculate scattering

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amplitudes and scattering lengths (see, for example [1]).To date, the study of the scattering within the in chiral perturbation theory (χPT) have already reached the two loop order and low-energy constants entering in the $\pi\pi$ scattering were estimated up to($\mathcal{O}(p^6)$) [3].The next-to-leading order (NLO) and next-to-next-to-leading order (NNLO) corrections in the chiral expansion lead to perturbative deviations from the LO and involve both computable nonanalytic contributions and analytic terms with some unknown low-energy constants (LECs),which can be obtained from lattice simulations or experimental measurements.Lattice studies on the $\pi\pi$ scattering have been conducted in quenched QCD by various groups. The full lattice study of the s-wave $I = 2\pi\pi$ scattering length was first carried out by CP-PACS [4]. In this paper, we study the PP-scattering in the QCM [6].

2 Notations

Meson - meson scattering amplitude

$$M(s,t,u) = \langle P^c(q_1)P^d(q_2)|P^a(p_1)P^b(p_2)\rangle \tag{1}$$

is determined by the graph shown in Figure 1.

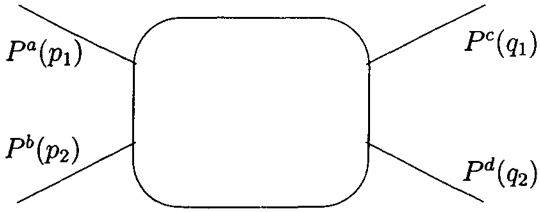

Fig.1

In general, it can be represented as

$$\begin{aligned} M(s,t,u) = &[Tr\{\lambda^a\lambda^c\lambda^d\lambda^b\} + Tr\{\lambda^a\lambda^d\lambda^b\lambda^c\}]B(s,t,u) + \\ &+[Tr\{\lambda^a\lambda^b\lambda^d\lambda^c\} + Tr\{\lambda^a\lambda^c\lambda^d\lambda^b\}]B(t,u,s) + \end{aligned}$$

$$+[Tr\{\lambda^a\lambda^d\lambda^c\lambda^b\} + Tr\{\lambda^a\lambda^b\lambda^c\lambda^d\}]B(u,s,t)$$
$$+\delta^{ac}\delta^{bd}C(s,t,u) + \delta^{ab}\delta^{cd}C(t,u,s) + \delta^{ad}\delta^{bc}C(u,s,t) \qquad (2)$$

where

$$s = (p_1+p_2)^2, t = (p_1-q_1)^2, u = (p_1-q_2)^2 \qquad (3)$$

λ^a-combination of Gell-Mann matrices that determine the kind of scattered mesons ,$B(s,t,u)$, $C(s,t,u)$ - invariant amplitudes. Generally invariant amplitude $A(s,t,u)(A = B,C)$ can be represented as

$$A(s,t,u) = Box(s,t,u) + V(s,t,u) + S(s,t,u) \qquad (4)$$

where $Box(s,t,u)$,$V(s,t,u)$, $S(s,t,u)$ - denotes contributions from respectively box and graphs with intermediate vector and scalar mesons respectively.

3 Box Contribution

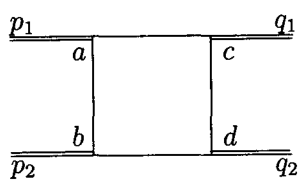

Fig.2

Box- diagram describing the scattering of pseudoscalar mesons is shown in Figure 2. and can be written as

$$Box = \frac{3(g_1g_2g_3g_4)}{4\pi^2}\int d\sigma \int \frac{d^4k}{4\pi^2 i} Tr\{i\gamma^5 S_{\Lambda_1\sigma}(\hat{k}+\hat{q}_1)\cdot$$
$$\cdot i\gamma^5 S_{\Lambda_2\sigma}(\hat{k}+\hat{q}_1+\hat{q}_2)\cdot i\gamma^5 S_{\Lambda_3\sigma}(\hat{k}+\hat{\imath}) i\gamma^5 S_{\Lambda_4\sigma}(\hat{k})\} \qquad (5)$$

The following analytical expression have been obtained in the framework of the QCM:

$$Box(s,t,u,p_1^2,p_2^2,q_1^2,q_2^2,\Lambda_1,\Lambda_2,\Lambda_3,\Lambda_4) =$$

$$= \frac{4\pi^2\sqrt{h_1h_2h_3h_4}}{3} I_{box}(s,t,u,p_1^2,p_2^2,q_1^2,q_2^2,\Lambda_1,\Lambda_2,\Lambda_3,\Lambda_4) \quad (6)$$

where

$$I_{box}(\ldots) = I_1(\ldots) + I_2(\ldots) + I_3(\ldots)$$
$$I_1(\ldots) = \Gamma(4)\int_0^1 \{d^4\alpha\} \int_0^\infty du b(u - Q(\ldots))$$
$$I_2(\ldots) = \Gamma(4)\int_0^1 \{d^4\alpha\} K(\ldots) b(-Q(\ldots))$$
$$I_3(\ldots) = \Gamma(4)\int_0^1 \{d^4\alpha\} L(\ldots) b'(-Q(\ldots))$$
$$K = \frac{1}{3}Q\frac{\Lambda_{1234}}{\Lambda^4(\alpha)} + \frac{1}{3}Q\frac{M}{\Lambda^2(\alpha)} + \frac{1}{6}\frac{D}{\Lambda^2(\alpha)} + \frac{1}{6}\frac{A}{\Lambda^4(\alpha)}$$
$$L = -\frac{1}{6}\frac{Q^2\Lambda_{1234}}{\Lambda^4(\alpha)} + \frac{F}{\Lambda^4(\alpha)} + Q\frac{A}{\Lambda^4(\alpha)} \quad (7)$$

where we use the following notation:

$$Q = q_1^2\alpha_3(1-\alpha_3) + (q_1+q_2)^2\alpha_2(1-\alpha_2) + p^2\alpha_1(1-\alpha_1) -$$
$$2q_1(q_1+q_2)\alpha_2\alpha_3 - 2q_1p\alpha_1\alpha_3 - 2p(q_1+q_2)\alpha_1\alpha_2 \quad (8)$$

$$A = -MB^2 + (Bq_1)M_{q_1} + (Bq_2)M_{q_2} + (Bp)M_p -$$
$$(pq_1)\Lambda_4(\Lambda_3 - \Lambda_2) - (pq_2)\Lambda_3\Lambda_4 - (q_1^2 + q_1q_2)_{14}$$
$$B = p\alpha_1 + (q_1+q_2)\alpha_2 + q_1\alpha_3$$
$$C = pq_2 + q_1^2 + q_1q_2$$
$$D = 3(Bp + Bq_1 + B(q_1+q_2)) - 2C - pq_1 - 6B^2$$
$$F = B^2(B^2 - Bp - B)(q_1+q_2) - Bq_1 + C) + 2(Bp)(Bq_1) -$$
$$(Bq_1)(pq_2) + (Bq_2)(pq_1) - (Bp)(q_1^2 + q_1q_2) \quad (9)$$

$$M = (\Lambda_1 - \Lambda_4)(\Lambda_2 - \Lambda_3) + \Lambda_1\Lambda_4 + \Lambda_2\Lambda_3$$
$$M_{q_1} = (\Lambda_1 - \Lambda_4)(\Lambda_2 - \Lambda_3) + 2\Lambda_1\Lambda_2$$
$$M_{q_2} = (\Lambda_1 - \Lambda_4)\Lambda_3 + \Lambda_1\Lambda_4$$
$$M_p = \Lambda_4(\Lambda_3 - \Lambda_2) + \Lambda_2\Lambda_3 \quad (10)$$

$$\Lambda_{1234} = \Lambda_1\Lambda_2\Lambda_3\Lambda_4$$
$$\Lambda^2(\alpha) = \Lambda_1^2\alpha_1 + \Lambda_2^2\alpha_2 + \Lambda_3^2\alpha_3 + \Lambda_4^2\alpha_4 \quad (11)$$

Products q_1q_2, pq_1, pq_2 incoming (8)-(11) are expressed in terms of $s, t, u, p_1^2, p_2^2, q_1^2, q_2^2$ by (3).

4 Intermediate Vector Meson Contribution

The contribution of the intermediate vector mesons is determined by the diagram shown in Figure 3 and can be written as:

$$V = T^{\mu}(P_{+}^{2}, p_{1}^{2}, p_{2}^{2}) h_{V} G^{\mu\nu}(P_{+}^{2}) T^{\nu}(Q_{+}^{2}, q_{1}^{2}, q_{2}^{2}) \tag{12}$$

where

$$P_{+} = p_{1} + p_{2}; P_{-} = p_{1} - p_{2}; Q_{+} = q_{1} + q_{2}; Q_{-} = q_{1} - q_{2} \tag{13}$$

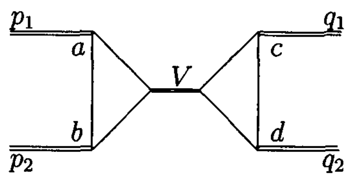

Fig.3

Form factor $T^{\mu}(P_{+}^{2}, p_{1}^{2}, p_{2}^{2})$, that is defined by triangular diagram ,was obtained as

$$T^{\mu}(P_{+}^{2}, p_{1}^{2}, p_{2}^{2}) = \int d\sigma \int \frac{d^4 k}{4\pi^2} Tr\{\gamma^{\mu} S_{\sigma}(\hat{k}) i\gamma^5 S_{\sigma}(\hat{k} + \hat{p_1}) i\gamma^5 S_{\sigma}(\hat{k} + \hat{p_{+}})\} =$$
$$= F^{+}(P_{+}^{2}, p_{1}^{2}, p_{2}^{2}) P_{+}^{\mu} + F^{-}(P_{+}^{2}, p_{1}^{2}, p_{2}^{2}) P_{-}^{\mu} \tag{14}$$

The product of the intermediate vector meson propagator by a constant of its interaction with the quarks in chain approximation has the form:

$$h_{V} G^{\mu\nu}(p^2) = \frac{1}{\Pi_1(p^2) - \Pi_1(m_V^2)} \{-g^{\mu\nu} + \frac{p^{\mu} p^{\nu} \Pi_2(p^2)}{\Pi_1(p^2) - \Pi_1(m_V^2) + p^2 \Pi_2(p^2)}\} \tag{15}$$

Where $\Pi_1(p^2)$,$\Pi_2(p^2)$ - transverse and longitudinal parts of the polarization operator.

In general, the contribution of the intermediate vector meson is obtained in the form of:

$$V = \frac{1}{\Pi_1(m_V^2} [F_p^{+} F_q^{+}(P_{+}Q_{+}) + F_p^{+} F_q^{-}(P_{+}Q_{-}) +$$
$$F_p^{-} F_q^{+}(P_{-}Q_{+}) + F_p^{-} F_q^{-}(P_{-}Q_{-})] +$$

$$+\frac{1}{\Pi_1(P_+^2)-\Pi_1(m_V^2)}\frac{\Pi_2(P_+^2)}{\Pi_1(m_V^2)}[P_+^2(F_p^-F_q^-(P_-Q_-)+$$

$$F_p^-F_q^+(P_-Q_+))+(P_+P_-)(F_p^-F_q^+(P_+Q_+)+F_p^-F_q^-(P_-Q_+))] \qquad (16)$$

5 Intermediate Scalar Contribution

The contribution of the intermediate scalar meson is described by the diagram shown in Figure 4, and can be written as follows:

$$S=F_{SPP}(P_+^2,p_1^2,p_2^2)h_SG_S(P_+^2)F_{SPP}(Q_+^2,q_1^2,q_2^2) \qquad (17)$$

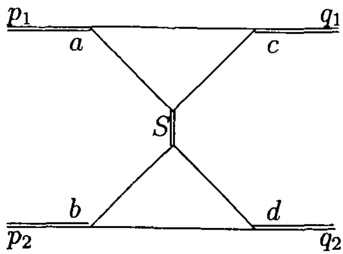

Fig.4

Form Factor $F_{SPP}(P_+^2,p_1^2,p_2^2)$ is determined by the triangular diagram $S\rightarrow PP$.In the QCM scalar meson interaction with quarks is determined by the Lagrangian:

$$L_S=\frac{g_S}{\sqrt{2}}S(x)\overline{q}(x)(I-i\frac{H}{\Lambda}(\overleftarrow{\partial}-\overrightarrow{\partial}))\lambda_s q(x) \qquad (18)$$

where

$$\lambda_S=\{\begin{array}{c} diag(1,-1,0)\Rightarrow a_0 \\ diag(\cos\delta_S,\cos\delta_S,-\sqrt{2}\sin\delta_S)\Rightarrow f_0(600) \\ diag(-\sin\delta_S,-\sin\delta_S,-\sqrt{2}\cos\delta_S)\Rightarrow f_0(980)\end{array} \qquad (19)$$

The parameter H and the mixing angle δ_S in (18),(19)have been determined from the condition of consistency and have values:

$$H=0,54;\delta_s=17^\circ \qquad (20)$$

The product of the coupling constants and scalar meson propagator has the form:

$$h_SG_S(p^2)=\frac{1}{\Pi_s(p^2)-\Pi_s(m_s^2)} \qquad (21)$$

6 $\pi\pi$ Scattering

Matrix element of $\pi\pi$ scattering has the form:

$$M_{\pi\pi}(s,t,u) = \delta^{ab}\delta^{cd}A(s,t,u) + \delta^{ac}\delta^{bd}A(t,u,s) + \delta^{ad}\delta^{bc}A(u,s,t) \qquad (22)$$

where a, b, c, d-isotopic indexes.

The amplitude $A(s,t,u)$ looks like:

$$A(s,t,u) = I^{\pi\pi}_{box}(s,t,u) + S^{\pi\pi}(s,t,u) + V^{\pi\pi}(s,t,u) \qquad (23)$$

where $I^{\pi\pi}_{box}(s,t,u), S^{\pi\pi}(s,t,u), V^{\pi\pi}(s,t,u)$ are defined by (7)-(11),(16),(17):

$$I^{\pi\pi}_{box}(s,t,u) = I_{box}(s,t,u,m_\pi^2,m_\pi^2,m_\pi^2,m_\pi^2,\Lambda_n,\Lambda_n,\Lambda_n,\Lambda_n) \qquad (24)$$

$$S^{\pi\pi}(s,t,u) = F^2_{S\pi\pi}(s)\left(\frac{cos^2\delta_s}{\Pi_s(s)-\Pi_s(m_1^2)} + \frac{sin^2\delta_s}{\Pi_s(s)-\Pi_s(m_2^2)}\right) + $$
$$+F^2_{S\pi\pi}(t)\left(\frac{cos^2\delta_s}{\Pi_s(t)-\Pi_s(m_1^2)} + \frac{sin^2\delta_s}{\Pi_s(t)-\Pi_s(m_2^2)}\right) \qquad (25)$$

$$F_{S\pi\pi}(x) = F_{S\pi\pi}(x, m_\pi^2, m_\pi^2) \qquad (26)$$

m_1- $f_0(500)$ mass , m_2- $f_0(980)$ mass.

$$V^{\pi\pi}(s,t,u) = \frac{1}{\Pi_1(m_\rho^2)}((F^-(s))^2(t-u) + (F^-(t))^2(s-u)) \qquad (27)$$

Scattering of π meson to π meson is possible via three channels $I = 0,1,2$. The scattering amplitudes for different channels T^I can be expressed by $A(s,t,u), A(t,s,u), A(u,t,s)$ as follows:

$$T^0(s,t,u) = 3A(s,t,u) + A(t,s,u) + A(u,t,s)$$
$$T^1(s,t,u) = A(t,s,u) - A(u,s,t)$$
$$T^2(s,t,u) = A(t,s,u) + A(u,s,t) \qquad (28)$$

Because of the symmetry between the final mesons we have the equality $A(s,t,u) = A(s,u,t)$, therefore, are different from zero only $T^0(s,t,u)$ and $T^2(s,t,u)$.

The scattering lengthes a^I are calculated by the formula

$$a^I = \frac{1}{32\pi}T^I(4m_\pi^2,0,0) \qquad (29)$$

From (25) one can see the numerical value of the contribution of diagrams with intermediate scalar meson to be dependent on the mass of $f_0(500)$ meson, the value of which is unknown at present and has been intensively discussed in the literature [7]-[11]. It turned out that in order to the obtained the numerical values of $\pi\pi$-scattering lengths a_0^0 anda_0^2 does not contradict the experimental data [12]-[14], the mass of the intermediate $f_0(500)$ meson has to be chosen in the range of $500 \div 515$ MeV.

7 Conclusion

The obtained numerical values of the $\pi\pi$-scattering lengths a_0^0 and a_0^2 are shown in the Table. The table also shows the results obtained in in chiral models,in quark models and by lattice calculation.

	$a_0^0 \times m_\pi^{-1}$	$a_0^2 \times m_\pi^{-1}$
Experiment [12]	0.26 ± 0.05	-0.028 ± 0.012
Experiment [13]	$0.216 \pm 0.013 \pm 0.02$	-0.04540 ± 0.0031
Experiment [14]	$0.2210 \pm 0.0047 \pm 0.015$	-0.04240 ± 0.0044
Tree level prediction [2]	0.1595 ± 0.005	-0.04557 ± 0.00014
$\chi PT(\mathcal{O}(p^6))$ [3]	0.22 ± 0.005	-0.044 ± 0.0001
Unitary chiral perturbation theory[11]	0.219 ± 0.005	-0.04240 ± 0.0012
QMSCT[15]	0.29	−0.025
PNJL model[16]	0.173	−0.045
Lattice[5]	−	−0.0410.0069
Lattice[17]	$0.214 \pm 0.004 \pm 0.007$	−0.044300.00025
Our result	0.25 ± 0.3	-0.038 ± 0.06

The obtained numerical values within the accuracy of the model being used are in good agreement with the experimental data.

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