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Comparative Analysis to the Bjorken Sum Rule in the Low Energy Domain

O. P. Solovtsova,* V. G. Teplyakov, and V. L. Khandramai
International Center for Advanced Studies,
Gomel State Technical University, Belarus

We apply the analytic approach which modernizes perturbative expansions so that the new approximations reflect basic principles of the theory: renormalization invariance and spectrality, to the Bjorken sum rule. The comparison is given for the results derived within the framework of the standard perturbation theory and the analytic approach. It is shown that the usage of the analytic perturbation theory allows us to get a reliable conclusion about value of higher-twist contributions and achieve a good quantitative description of the recent precise Jefferson Lab data for the Bjorken sum rule down to $Q \simeq 280$ MeV.

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1. Introduction

The most important and fundamental sum rule for polarized deep inelastic scattering is the Bjorken sum rule which was derived in 1966 from current algebra [1]. This sum rule is so very fundamental because it relies only on isospin invariance, i.e. on a SU(2) symmetry between u and d quarks. The Bjorken integral has been measured in polarized deep inelastic lepton scattering at SLAC, CERN, DESY [2–7], and recently at low Q^2, $0.05 < Q^2 < 3\,\text{GeV}^2$, in the CEBAF at Jefferson Lab (JLab) [8, 9].

The Bjorken sum rule is an asymptotic result which relates low and high Q^2 domains. In low-energy domain, non-perturbative effects are of paramount importance and must be taken into account along with the perturbative contribution. A theoretical analysis therefore includes the perturbative and non-perturbative components related to each other.

Studying the low-energy domain requires developing methods that allow modifying the perturbative expansions to reduce the theoretical uncertainties related. It is well known that the renormalization-group (RG) method allows one to modify a perturbative expansion in accordance with the general principle of renormalization invariance, thus improving the properties of the series in the ultraviolet region. As to the infrared region, where the perturbative invariant charge possesses unphysical singularities (a ghost pole in the one-loop approximation), the RG-modified perturbative series remains unstable.

In the late 1950s, in the context of quantum electrodynamics, a method of elimination of the unphysical singularity from the invariant charge was proposed [10, 11]. In Ref. [12], this idea was applied to the case of QCD. The analytic approach formulated there combines the RG method and Q^2-analyticity (reflecting the general principles of local quantum field theory such as spectrality and causality) and results in a number of new interesting properties of the expansion. Further developments and applications of the the analytic method, called analytic perturbation theory (APT) [13], have been considered in many works (see [14] for a review). In

*E-mail: solovtsova@gstu.gomel.by

the framework of APT, the theoretical ambiguity associated with higher-loop corrections and with the choice of renormalization scheme is diminished. Using the APT method to describe the perturbative component of the QCD description can change the values of nonperturbative parameters extracted from experimental data [15–18]. It is motivation to using the APT for the recent precision JLab data.

The first APT analysis for the Bjorken sum rule has been performed in [15] and has not been consider higher-twist effects because for that period the values of higher-twist parameters have not been well determined neither experimentally nor theoretically. In this paper we continue the researches begun in [15] (see also [18]) and extend the analysis to include higher-twist effects. We compare APT result with standard PT description (see, e.g., [19] as review) and extract higher twist parameters for the Bjorken sum rule.

2. QCD corrections

The polarized Bjorken sum rule refers to the integral over all x (the Bjorken scaling variable $x = Q^2/(2M\nu)$ in the lab frame, M is the nucleon mass and ν is the energy transferred from the electron to the target nucleon) at fixed Q^2 of the difference between polarized structure functions of the proton $g_1^{\rm p}$ and the neutron $g_1^{\rm n}$,

$$\Gamma_1^{\rm p-n}(Q^2) = \int_0^1 \left[g_1^{\rm p}(x,Q^2) - g_1^{\rm n}(x,Q^2)\right] dx\,. \tag{1}$$

For $Q^2 \to \infty$, the Bjorken sum rule should equal $g_A/6$, where $g_A = 1.267 \pm 0.004$ [20] is the nucleon axial charge that controls the strength of neutron β-decay.

It is generally believed that the Bjorken sum rule gets only moderate QCD corrections. Away from the large Q^2 limit, the Bjorken integral (1) can be written as the perturbative QCD part (the leading twist term), $\Delta_{\rm Bj}$, and the other nonperturbative power corrections (higher twists terms)

$$\Gamma_1^{\rm p-n}(Q^2) = \frac{g_A}{6}\left[1 - \Delta_{\rm Bj}(Q^2)\right] + \sum_{i=2}^{\infty}\frac{\mu_{2i}}{Q^{2i-2}}\,. \tag{2}$$

To begin, we concentrate on the perturbative contribution, considering in turn standard PT and APT methods. This contribution in the PT three-loop approximation with the use of the $\overline{\rm MS}$ renormalization scheme and in the massless quark limit, has the form

$$\Delta_{\rm Bj}^{\rm PT}(Q^2) = \frac{\alpha_{\rm PT}(Q^2)}{\pi} + d_1\left[\frac{\alpha_{\rm PT}(Q^2)}{\pi}\right]^2 + d_2\left[\frac{\alpha_{\rm PT}(Q^2)}{\pi}\right]^3, \tag{3}$$

where for three active quarks the coefficients are $d_1^{\overline{\rm MS}} = 3.5833$ and $d_2^{\overline{\rm MS}} = 20.2153$ [21]. The perturbative running coupling $\alpha_{\rm PT}(Q^2)$ is obtained by integration of the RG equation with the three-loop β-function. It is well known that the PT approximation violates the Q^2-analyticity of the running coupling and does not allow one to describe low energy scales – the perturbative series diverges in the infrared region. The APT method removes this difficulty and leads to a quite stable result for the entire interval of momentum. The difference between the shapes of the PT and APT running couplings becomes very significant at low Q^2-scales.

As it has been demonstrated in [22] the moments of the structure functions are analytic functions of Q^2 in the complex Q^2-plane with a cut along the negative part of the real axis. It is clear that the perturbative representation (3) violates these analytic properties due to the unphysical singularities of the PT running coupling. To avoid this problem we apply the APT method, which gives the possibility of combining the renormalization group resummation with correct analytical properties of the QCD correction to the Bjorken sum rule.

Let us write down the QCD correction in the form of a spectral representation

$$\Delta_{\rm Bj}(Q^2) = \frac{1}{\pi}\int_0^\infty \frac{d\sigma}{\sigma + Q^2}\,\varrho(\sigma)\,, \tag{4}$$

where we have introduced the spectral function, which is defined as the discontinuity of $\Delta_{\rm Bj}(Q^2)$: $\varrho(\sigma) = {\rm Disc}\left\{\Delta_{\rm Bj}(-\sigma - i\epsilon)\right\}/2i$. By calculating the spectral function $\varrho(\sigma)$ perturbatively we get a expression for $\Delta_{\rm Bj}(Q^2)$ which has the correct analytic properties and therefore has no unphysical singularities. The three-loop APT approximation to $\Delta_{\rm Bj}(Q^2)$ is

$$\Delta_{\rm Bj}^{\rm APT}(Q^2) = \delta_{\rm APT}^{(1)}(Q^2) + d_1\delta_{\rm APT}^{(2)}(Q^2) + d_2\delta_{\rm APT}^{(3)}(Q^2)\,, \tag{5}$$

where the coefficients d_1 and d_2 are the same as in Eq. (3) and the functions $\delta_{\rm APT}^{(k)}(Q^2)$ are derived from the spectral representation and correspond to the discontinuity of the k-th power of the PT running coupling

$$\delta_{\rm APT}^{(k)}(Q^2) = \frac{1}{\pi^k}\int_0^\infty \frac{d\sigma}{\sigma + Q^2}\,{\rm Im}\,\left\{\alpha_{\rm PT}^k(-\sigma - i\epsilon)\right\}\,. \tag{6}$$

The function $\delta_{\rm APT}^{(1)}(Q^2)$ defines the APT running coupling, $\alpha_{\rm APT}(Q^2) = \pi\delta_{\rm APT}^{(1)}(Q^2)$.

In the case of PT the QCD correction to the Bjorken sum rule is represented in the form of a power series in $\alpha_{\rm PT}$ [see Eq. (3)], but in the case of APT the same QCD correction is not a polynomial in the APT running coupling. As can be seen from (5), the first term of the expansion is $\alpha_{\rm APT}/\pi$, but the following terms are not representable as powers of $\alpha_{\rm APT}$ unlike in the PT case. For instance, it is possible to get

$$\delta^{(2)}(z) \simeq \left[\frac{\alpha_{\rm PT}(z)}{\pi}\right]^2 + \frac{4}{\beta_0^2}\left[-\frac{1}{(1-z)^2} + \left(1 + \frac{\beta_0^2}{2\beta_1}\right)\frac{1}{1-z} + \frac{C_1}{z}\right]\,,\; z = Q^2/\Lambda^2\,, \tag{7}$$

where $C_1 = 0.02288$ for $n_f = 3$ active flavors and $\beta_1 = 102 - 38n_f/3$. Since the difference between the PT and APT forms of the perturbative QCD correction is of order Λ^2/Q^2, both these functions will coincide in the asymptotic region, where the perturbative approximation is valid. However, at low Q^2 scales this difference become important.

As it has been shown in [15], the convergence properties of the APT series (5) are much better than are those of the PT expansion (3), and the APT results have extraordinary stability with respect to the choice of the renormalization scheme. Thus, calculations in the framework of APT considerably reduce the theoretical uncertainty of the results. Use of APT to describe the perturbative component of the process should increase the reliability in obtaining information about nonperturbative effects. Also in [15] it has been noticed, that the Q^2-evolution obtained by using PT without taking higher-twist contributions into account practically coincides with the result of the APT, but with the $1/Q^2$-correction.

Note that the Bjorken integral value is quite stable with respect to small variations of $\Lambda_{\rm QCD}$ in the APT case: in contrast with huge instability in the PT, it changes by about $2-3\%$ within the interval $\Lambda_{\rm QCD} = 300 - 400\,{\rm MeV}$. As before [18], we take the value $\Lambda_{\rm QCD} = 380\,{\rm MeV}$ for three-loop approximation and $\Lambda_{\rm QCD} = 340\,{\rm MeV}$ for two-loop one.

3. Numerical result

To compare theoretical predictions with experimental data we plot in Fig. 1 the full contribution to the QCD correction, $\Delta_{\rm tot}$, with the perturbative part calculated in the PT and APT approaches, and the HT part taken with extracted values of higher twist coefficients: taking into account μ_4 (solid lines), μ_4 and μ_6 (dashed lines), μ_4, μ_6 and μ_8 (dotted lines), and without HT (short-dashed lines). In this fugue the CLAS Hall B'06 JLab data is denoted by stars, the CLAS Hall B'03 data by a downward pointing triangles, the CLAS Hall A'02 data by squares, and the SLAC E143 data [26] by a circle. The dash-dotted line is the Deur's fit result [27]. Usually, it is assumed [28] that the perturbative effects are less important low Q^2 scales than the power ones due to a nontrivial structure in QCD vacuum. This figure demonstrates that at low Q^2 scales, the difference between the PT and APT results becomes significant and it influences on the nonperturbative contribution. One can see that the best description of the low energy data on Bjorken sum rule down to can be achieved by using APT and taking into account three higher twist terms ($\mu_{4,6,8}$).

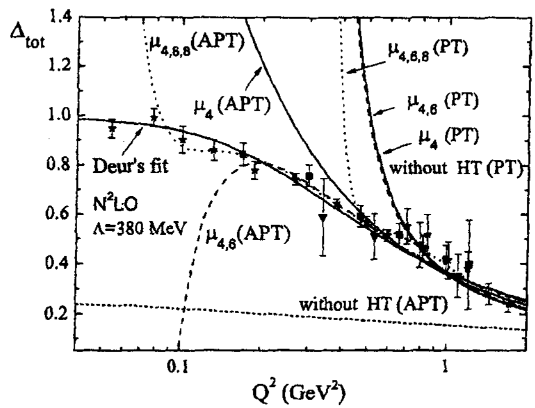

FIG. 1: The APT and PT results for the total QCD correction together with experimental data.

The same may be seen in Table 1, where the best fits of the combined data set for the Bjorken sum rule is shown. Fits in APT give the HT values indicating a better convergence of the operator product expansion due to decreasing magnitudes and alternating signs of consecutive terms, in contrast to the usual PT fit results. As can be seen from this table, including only twist-4 term, μ_4/M^2, the APT method allowed us to get its value with higher accuracy then one in the standard PT approach shifting applicability of perturbative QCD down to $Q^2_{min} = 0.47$ GeV^2.

4. Conclusion

We have considered the Bjorken sum rule by using the standard PT and APT approaches. From the theoretical point of view, the remarkable properties of the APT create a basis for the its preferable application. At high Q^2 scales, the PT and APT results close to each other, both including the HT terms and without them, whereas at low Q^2 scales, the difference between the PT and APT behaviours becomes to be very significant.

Application of the APT has allowed to obtain stable estimations for higher twist parameters and achieve a good quantitative description of the JLab data down to $Q \simeq 280$ MeV.

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Table 1: The fit results for the HT terms in APT and PT.

Method	Q^2_{min}, GeV^2	μ_4/M^2	μ_6/M^4	μ_8/M^6
NLO PT	0.47	−0.034(3)	0	0
	0.27	−0.054(6)	0.014(2)	0
	0.17	−0.034(1)	−0.005(6)	0.004(1)
N^{2}LO PT	0.66	−0.00043(6)	0	0
	0.66	−0.005(15)	0.005(16)	0
	0.47	0.027(15)	−0.10(3)	0.064(12)
NLO APT	0.47	−0.050(3)	0	0
	0.27	−0.059(4)	0.007(1)	0
	0.078	−0.062(4)	0.009(1)	−0.0004(1)
N^{2}LO APT	0.47	−0.049(3)	0	0
	0.27	−0.058(6)	0.007(2)	0
	0.078	−0.060(4)	0.009(1)	−0.0004(1)

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