

# Model Independent Constraints on Anomalous Gauge Couplings from LEP2 Experiments

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Model independent analysis of anomalous trilinear gauge boson interactions was performed. Model independent bounds on anomalous gauge couplings were derived using LEP2 experimental data on the production of two longitudinally polarized charged gauge bosons  $e^+ + e^- \rightarrow W_L^+ + W_L^- \rightarrow q\bar{q} l\nu$  ( $l = e, \mu$ ). Obtained upper bounds on anomalous gauge couplings at 95% C.L. are  $-0.33 \leq \Delta g_1^Z \leq 0.20$ ,  $-0.28 \leq \Delta k_\gamma \leq 0.78$ ,  $-0.43 \leq \Delta k_Z \leq 0.27$ .

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## 1. Introduction

Experimental tests of the Standard Model (SM) at electron-positron collider LEP2 were performed for processes caused by fermion-boson and boson-boson interactions [1]. The results of these experiments turned out to be in good agreement with SM predictions. Investigation of triple gauge boson interactions is one of the most important tests of SM because they are directly connected with non-Abelian structure of electroweak symmetry group. Therefore in physics research programmes of active (Tevatron) and future (LHC, ILC) colliders a big attention is traditionally paid to the measurement of triple boson gauge couplings  $W^+W^-\gamma$  and  $W^+W^-Z$ . Such measurements can either prove the accuracy of the SM or disprove it, having discovered effects induced by anomalous gauge couplings (AGC). This phenomenon can be interpreted as a manifestation of existence of "new physics" beyond the SM.

Measurements of triple boson interactions at LEP2 were performed, in particular, in the process of  $W^\pm$ -bosons pair birth

$$e^+ + e^- \rightarrow W^+ + W^- \quad (1)$$

The main feature of the process (1) is its high sensitivity to anomalous triple boson interactions, especially at high energies, when  $\sqrt{s} \gg 2M_W$  [2]. Such interactions violate gauge cancellation mechanism which plays an important role in SM. The effect of gauge cancellation mechanism consists in that fact that it provides "proper" behaviour of the cross section of the process (1) with energy increase. This behaviour does not violate unitary limit though separate contributions to the cross section unrestrictedly increase with energy increasing. At the same time,

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anomalous triple boson interactions violate this mechanism, which causes energy behaviour of cross section, which is essentially different from that of the SM.

In general case  $C$ - and  $P$ - conserving part of the effective lagrangian of triple boson interactions depends on five triple boson couplings. The task of experimental differentiation of contributions of AGC to the cross section is very important and at the same time very difficult. This is determined not only by a big number of independent parameters, but also by the probable mutual cancellation of separate cross section contributions, which can lead to decrease in the sensitivity of variables to AGC. However, the possibility of measuring the processes of longitudinally polarized  $W_L^\pm$ -bosons production at LEP2 collider leads to significant simplification of the problem. The determination of model independent constraints on AGC can be in this case simplified due to the fact that polarized process  $e^+ + e^- \rightarrow W_L^+ + W_L^-$  depends on three (out of five) AGC only.

In this paper a model independent analysis of anomalous triple boson interactions is performed. Constraints on AGC were derived using LEP2 experimental data on the production of two longitudinally-polarized charged gauge bosons  $e^+ + e^- \rightarrow W_L^+ + W_L^- \rightarrow q\bar{q}l\nu$  ( $l = e, \mu$ ).

The article has the following structure. In the second section a standard parametrization of  $W^+W^-\gamma$  and  $W^+W^-Z$  vertices is given. A brief review of contemporary constraints on AGC is provided. In the third section expressions for helical amplitudes and differential cross sections of the process (1) are given. A significance of polarization of final  $W_L^\pm$ -bosons for obtaining the most stringent constraints on AGC is determined. The fourth section provides the numerical results of the obtained constraints, both model dependent (for a set of specific models) and model independent. Finally, in the last section one can find conclusions and remarks.

## 2. Triple boson couplings $\gamma WW$ and $ZWW$

Effective lagrangian which is Lorentz-invariant, invariant relatively gradient transformations  $U(1)_{em}$  and  $C$ -,  $P$ -symmetry transformations, can be written as [3, 4]:

$$L_{eff} = -i e [A_\mu (W^{-\mu\nu}W_\nu^+ - W^{+\mu\nu}W_\nu^-) + k_\gamma F_{\mu\nu}W^{+\mu}W^{-\nu} + \frac{\lambda_\gamma}{M_W^2} F^{\nu\lambda}W_{\lambda\mu}^-W_\nu^{+\mu}] - \\ -i e \cot\theta_W [g_1^Z Z_\mu (W^{-\mu\nu}W_\nu^+ - W^{+\mu\nu}W_\nu^-) + k_Z Z_{\mu\nu}W^{+\mu}W^{-\nu} + \frac{\lambda_Z}{M_W^2} Z^{\nu\lambda}W_{\lambda\mu}^-W_\nu^{+\mu}]. \quad (2)$$

Here  $W_{\mu\nu}^\pm = \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm$ ,  $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ ,  $e = \sqrt{4\pi\alpha}$  and  $\theta_W$  - weak-mixing angle. Lagrangian (2) consists of 2 SM terms and 5 terms which contain anomalous gauge couplings (AGC)  $\Delta g_1^Z, \Delta k_\gamma, \Delta k_Z, \lambda_\gamma, \lambda_Z$ . These couplings are expressed by means of their deviations from SM values:

$$\Delta g_1^Z = (g_1^Z - 1) \equiv \tan\theta_W \delta_Z, \quad \Delta k_Z = (k_Z - 1) \equiv \tan\theta_W x_Z + \Delta g_1^Z = \tan\theta_W (x_Z + \delta_Z), \\ \Delta k_\gamma = (k_\gamma - 1) \equiv x_\gamma, \quad \lambda_\gamma \equiv y_\gamma, \quad \lambda_Z \equiv \tan\theta_W y_Z. \quad (3)$$

It should be noted that couplings  $k_\gamma$  and  $\lambda_\gamma$  are connected with static characteristics of  $W^\pm$ -bosons such as dipole ( $\mu_W$ ) and electric quadrupole moment ( $Q_W$ ):

$$\mu_W = \frac{e}{2M_W} (1 + k_\gamma + \lambda_\gamma), \quad Q_W = -\frac{e}{M_W^2} (k_\gamma - \lambda_\gamma). \quad (4)$$

In this paper couplings  $\Delta g_1^Z, \Delta k_\gamma, \Delta k_Z, \lambda_\gamma, \lambda_Z$  are used as a set of free dynamical parameters. The most stringent contemporary constraints on AGC were obtained from LEP2 and Tevatron experiments [5]. In both cases only model boundary values are provided. They were obtained

using simplifying assumptions about one (or few) non-zero AGC, all other couplings were supposed to be equal to their SM values. These constraints can be given as a set of inequalities:

$$-0.63 < \Delta k_\gamma < 0.99, \quad -0.15 < \Delta k_Z < 0.19, \quad -0.14 < \Delta g_1^2 < 0.34.$$

### 3. Cross section of the process

The process (1) in Born approximation consists of two s-channel diagrams with  $\gamma$  and  $Z^0$ -boson exchange and a t-channel diagram with neutrino  $\nu$  exchange (Fig.1).

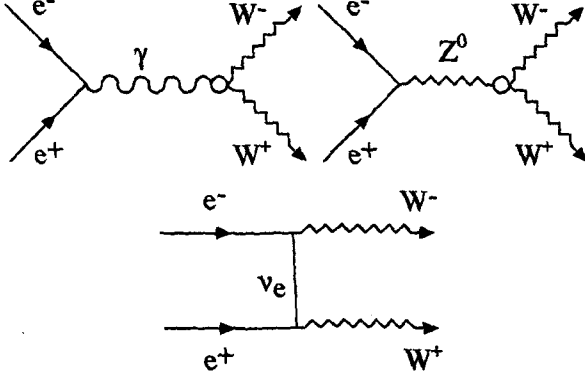


FIG. 1: Feynman diagrams for the process  $e^- e^+ \rightarrow W^- W^+$  in Born approximation.

With the help of method of basis spinors [6, 7], one can obtain expressions for Feynman diagrams with neutrino exchange in the case of massless fermions:

$$M_{0,0}^{\lambda,\lambda'}(\nu) = 2 \delta_{\lambda',-\lambda} \delta_{\lambda,-1} n_s \left( \frac{t_W(\theta)}{\gamma_W^2} - \gamma_W^2 \right) \sin \theta,$$

$$M_{\tau,\tau'}^{\lambda,\lambda'}(\nu) = \delta_{\lambda',-\lambda} \delta_{\lambda,-1} n_s [1 - t_W(\theta)(1 + \tau\beta_W)(1 - \tau'\beta_W)] \sin \theta,$$

$$M_{0,\tau}^{\lambda,\lambda'}(\nu) = -M_{-\tau,0}^{\lambda,\lambda'}(\nu) = \delta_{\lambda',-\lambda} \delta_{\lambda,-1} n_s \sqrt{2} \gamma_W \tau (1 + \tau \cos \theta) \left[ 1 - \frac{(1 - \tau\beta_W) t_W(\theta)}{\gamma_W^2} \right], \quad (5)$$

where  $t_W(\theta) = (1 + \beta_W^2 - 2\beta_W \cos \theta)^{-1}$ ,  $n_s = (2\pi\alpha) / (\beta_W s_W^2)$ ,  $\theta$  - the angle between a  $W^-$ -boson and an electron. For s-channel diagrams with photon and  $Z^0$ -boson exchange we obtain:

$$M_{\tau,\tau'}^{\lambda,\lambda'}(\gamma) + M_{\tau,\tau'}^{\lambda,\lambda'}(Z) = 4\pi\alpha \lambda \delta_{\lambda,-\lambda'} \beta_W \left( A_{\tau,\tau'}^\lambda(\gamma) - \frac{g_\lambda^2 \chi(s)}{s_W^2} A_{\tau,\tau'}^\lambda(Z) \right), \quad (6)$$

where  $\chi(s) = s / (s - m_Z^2 + im_Z \Gamma_Z)$ ,  $g_\lambda^2 = -1/2(1 + \lambda) + 2s_W^2$ , and helical structures  $A_{\tau,\tau'}^\lambda$  are defined as:

$$A_{0,0}^\lambda(V) = -(1 + 2\gamma_W^2 [1 + f_0^V]) \sin \theta, \quad (\tau, \tau' = 0), \quad (7)$$

$$A_{\tau,\tau'}^\lambda(V) = (-1) f_{1,\tau,\tau'}^V \sin \theta, \quad (\tau, \tau' = \pm 1), \quad (8)$$

$$A_{0,\tau}^{\lambda}(V) = -A_{-\tau,0}^{\lambda}(V) = f_3^V \gamma_W \frac{(\tau \lambda - \cos \theta)}{\sqrt{2}}, \quad (\lambda, \tau = \pm 1), \quad (9)$$

Here

$$f_0^{\gamma} = \Delta k_{\gamma}, \quad f_0^Z = \cot \theta_W (\Delta k_Z + \frac{1 - \beta_W}{2} \Delta g_1^Z). \quad (10)$$

Differential cross section has the following form:

$$\frac{d\sigma_{\tau\tau'}^{\lambda\lambda'}}{d \cos \theta} = \frac{\beta_W}{32\pi s} |M_{\tau\tau'}^{\lambda\lambda'}|^2, \quad (11)$$

where  $\beta_W = \sqrt{1 - 1/\gamma_W^2}$  and  $\gamma_W = \sqrt{s}/M_W^*$ . Index  $\lambda(\lambda') = \pm 1$  denotes the helicity of electrons (positrons), and  $\tau(\tau') = \pm 1 (L), 0 (L)$  denotes the spin states of  $W^{\pm}$ -bosons,  $\tau(\tau') = \pm 1$  - for transversal and  $\tau(\tau') = 0$  - for longitudinal states.

Let us consider triple boson vertices, which correspond to lagrangian (2). It should be noted that introduction of anomalous parameters, which characterize the deviation of triple boson couplings from their SM values, modifies only s-channel diagrams with photon and  $Z^0$ -boson exchange. In momentum space they have the form:

$$\Gamma_V^{\mu\alpha\beta}(P, k_1, k_2) = -i g_{VWW}^{SM} \times \\ \times \left( \left[ f_1^V g^{\alpha\beta} - \frac{f_2^V}{M_W^2} P^{\alpha} P^{\beta} \right] (k_1 - k_2)^{\mu} + f_3^V (P^{\alpha} g^{\mu\beta} - P^{\beta} g^{\mu\alpha}) \right), \quad (12)$$

$$f_1^{\gamma} = 1 + 2\gamma_W^2 \lambda_{\gamma}, \quad f_2^{\gamma} = \lambda_{\gamma}, \quad f_3^{\gamma} = 2 + \Delta k_{\gamma} + \lambda_{\gamma}, \quad f_2^Z = \lambda_Z, \quad (13)$$

$$f_1^Z = 1 + \Delta g_1^Z + 2\gamma_W^2 \lambda_Z, \quad f_3^Z = 2 + \Delta g_1^Z + \Delta k_Z + \lambda_Z. \quad (14)$$

Triple boson couplings in SM have the following form:  $g_{\gamma WW}^{SM} = e$ ,  $g_{Z WW}^{SM} = e \cot \theta_W$ .

Differential cross section of the process (1) in the case of unpolarized initial beams can be expressed as:

$$\frac{d\sigma}{dz} = \frac{1}{4} \left[ \frac{d\sigma^+}{dz} + \frac{d\sigma^-}{dz} \right], \quad (15)$$

where  $\frac{d\sigma^{\lambda}}{dz} = \frac{\beta_W}{32\pi s} |M^{\lambda-\lambda}|^2$ ,  $z = \cos \theta$ ,  $(\lambda = \pm 1)$ . From formulae (7) and (10) one can see that differential cross-section of longitudinally polarized  $W^{\pm}$ -bosons production depends on three AGC ( $\Delta g_1^Z, \Delta k_{\gamma}, \Delta k_Z$ ), while unpolarized cross section of the process (1) depends on the whole set of five AGC.

#### 4. Model independent constraints on AGC

For AGC constraints estimation on the basis of LEP2 data in the case of longitudinally polarized  $W^{\pm}$ -bosons  $\chi^2$  method is used:

$$\chi^2(\Omega) = \sum_{i=1}^{\text{bins}} \left[ \frac{N_i^{\text{anom}}(\Omega) - N_i^{SM}}{\delta N_i^{SM}} \right]^2, \quad (16)$$

where  $N_i^{SM}$  is a number of events hitting the angle interval limited by the size of the  $i$ -th bin,  $N_i^{\text{anom}}(\Omega)$  - a number of events induced by anomalous gauge couplings  $\Omega = \{\Delta k_{\gamma, Z}, \Delta g_1^Z\}$ . In formula (16) there is a summation over 10 bins, which cover the whole interval of scattering angle  $\theta$  ( $-0.98 \leq \cos \theta \leq 0.98$ ). The number of events in  $i$ -th bin is evaluated as:

$$N_i = L_{\text{int}} \epsilon \sigma_i, \quad (17)$$

where scattering cross section  $\sigma_i$  in  $i$ -th bin is equal to:

$$\sigma_i \equiv \sigma(z_i, z_{i+1}) = \int_{z_i}^{z_{i+1}} \left( \frac{d\sigma}{dz} \right) dz. \quad (18)$$

$L_{int}$  in formula (17) is the integrated luminosity of the detector for the whole period of experiment conduction. The efficiency of events registration is denoted as  $\varepsilon$ . Registration of  $W^\pm$ -bosons is usually made by the detection of their decay products into lepton and quark pairs. The total error in the determination of cross section in equation (16) consists of two parts. statistical and systematic errors:

$$\delta N_i^{SM} = \sqrt{N_i^{SM} + (\delta_{sys} N_i^{SM})^2}. \quad (19)$$

When obtaining constraints on anomalous parameters of  $W^\pm$ -bosons we use the assumption that the results of future cross section measurements for the process (1) are in agreement with SM within the limits of expected precision. In such a case one can write:

$$\chi^2(\Theta) \leq \Delta\chi_{crit}^2, \quad (20)$$

Here  $\Delta\chi_{crit}^2$  is determined by the confidence level (C.L.) and a number of parameters in a set  $\Omega$ . For C.L. = 95%,  $\Delta\chi_{crit}^2 = 3.84, 5.99, 7.82$  for 1, 2 and 3 parameters respectively [8].

Experimental values of efficiency  $\varepsilon$  in different energy points, as well as the corresponding values of integrated luminosity  $L_{int}$  and mean energy of the beam  $E = \sqrt{s}$  were taken from the recent data of LEP2 collaborations [9,10]. The total efficiency is determined as a product of events registration efficiency and efficiencies that are specified by the relative probabilities of  $W$ -bosons decay into a pair of leptons ( $\Gamma_{\ell\nu_\ell}/\Gamma_W$ ) and two hadron jets ( $\Gamma_{q\bar{q}}/\Gamma_W$ ).

Systematic error of the experiment is  $\delta_{sys} = 2\%$ . This error includes both uncertainties in luminosity measurements and uncertainties due to background effects and radiation corrections.

As mentioned above, differential cross section of the process of longitudinally polarized  $W^\pm$ -bosons production depends only on three AGC  $\Theta = \{\Delta g_1^2, \Delta k_\gamma, \Delta k_Z\}$ . Thus, model independent analysis of the given process is performed for three AGC, considering them as free parameters.

To obtain model independent constraints, data from OPAL and DELPHI experiments was used [9, 10]. The values of experimental efficiencies of events registration  $\varepsilon$ , integrated luminosities  $L_{int}$  and mean energies of the beam  $E = \sqrt{s}$  are given in Table 1.

On Fig.2 and Fig.3 one can see the results of the combined  $\chi^2$  analysis of experimental data. In particular, on Fig.2 and Fig.3 three different cases are depicted (for specific number of free parameters). The first case (gray area) corresponds to three free parameters. Gray area here is

Table 1. Experimental efficiency of  $(e/\mu, \bar{\nu}) + (q, \bar{q})$  events registration, integrated luminosities and beam energies in OPAL and DELPHI experiments.

Energy $E$ , GeV	Luminosity $L_{int}$ , $pb^{-1}$	Efficiency $\varepsilon$ , %
172	10.4	84.2
182	57.4	84.2
189	153.8	72.1
198	218.0	69.9
206	148.6	68.5

the projection of 3D surface (20) onto the plane of two parameters  $(\Delta k_\gamma, \Delta g_1^2)$  or  $(\Delta k_z, \Delta k_\gamma)$ . The second case (dashed line) corresponds to a pair of free parameters. Dashed line defines the allowed region for AGC with 95% C.L. Finally, solid lines define the allowed intervals (95% C.L) for AGC in

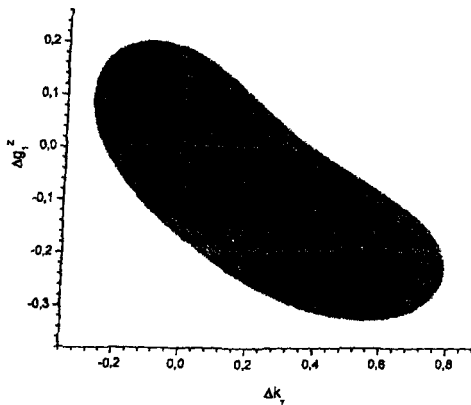


FIG. 2. Model independent (gray area) and model (dashed and solid lines) constraints on AGC.  $\Delta g_1^2 - \Delta k_\gamma$  plane.

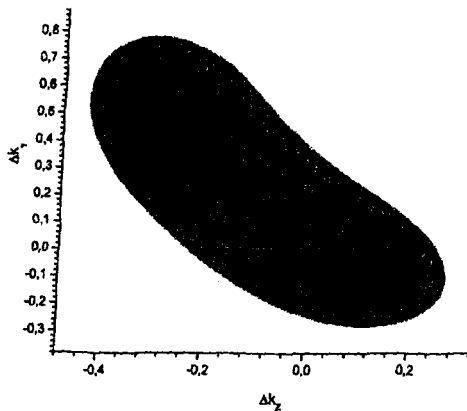


FIG. 3. Model independent (gray area) and model (dashed and solid lines) constraints on AGC.  $\Delta k_\gamma - \Delta k_z$  plane.

## 5. Conclusion

Model independent analysis of anomalous triple gauge boson interactions was performed. Model independent constraints on anomalous gauge couplings were derived using LEP2 experimental data for the production of two longitudinally polarized charged gauge bosons  $e^+ + e^- \rightarrow W_L^+ + W_L^- \rightarrow q\bar{q}l\nu (l = e, \mu)$ . Obtained upper bounds on anomalous gauge couplings at 95% C.L are:

$$\begin{aligned} -0.33 &\leq \Delta g_1^Z \leq 0.20, \\ -0.28 &\leq \Delta k_\gamma \leq 0.78, \\ -0.43 &\leq \Delta k_Z \leq 0.27. \end{aligned}$$

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