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THE BJORKEN SUM RULE: HOW FAR CAN PERTURBATIVE THEORY PENETRATE TO NONPERTURBATIVE REGION?

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We study the polarized Bjorken sum rule, $\Gamma_1^{p-n}(Q^2)$, using recent experimental data of the Jefferson Lab on this sum rule at low momentum transfers in the range $0.05 < Q^2 < 3\,\mathrm{GeV}^2$ and the four-loop expression for the coefficient function $C_{\mathrm{Bj}}(\alpha_{\mathrm{s}})$ available now both in the framework of the standard QCD perturbation theory (PT) and the singularity-free analytic perturbation theory (APT). The analysis of the standard perturbative series for $C_{\mathrm{Bj}}(\alpha_{\mathrm{s}})$ up to N^{3}LO level exposes its asymptotic character in the region of low $Q \leq 1$ GeV, and the precision of the theoretical PT predictions can not be reached to better than 10%. Our analysis shows that the usage of APT allows to describe the precise low energy JLab data down to $Q \sim 300$ MeV and gives a possibility for reliable extraction of the higher twist coefficients.

1 Introduction

The spin-dependent structure function g_1 for the deep-inelastic lepton-nucleon scattering is one of the main sources of information about the nucleon structure. The g_1 structure function depends both on the Bjorken variable x, the fractional momentum carried by a parton, and the four-momentum squared of the exchanged virtual photon, Q^2. A fundamental sum rule originally derived from the current algebra by J.D. Bjorken in Ref. [1] predicts that the integral of the difference of the proton and neutron structure functions over all possible values of x (or its first moment),

$$\Gamma_1^{p-n}(Q^2) \;=\; \int_0^1 \; \left[g_1^p(x,Q^2) - g_1^n(x,Q^2)\right] dx\,, \tag{1}$$

in the limit $Q^2 \to \infty$ is equal to $g_A/6$, where $g_A = 1.267 \pm 0.004$ [2] is the nucleon axial charge defined from the neutron β-decay data.

Among the moments of the nucleon structure functions, the Bjorken sum rule (BjSR) is one of the convenient tests of the perturbative QCD (pQCD), especially, at low momentum transfers Q. Since this sum rule relates the difference of the proton and neutron first moments, only flavor non-singlet quark operators appear in the operator product expansion (OPE), which is the basis for the theoretical QCD analysis.

The Q^2-evolution of the BjSR is given by a double series in powers of $1/Q^2$ (OPE higher twists corrections) and in powers of the QCD running coupling $\alpha_{\mathrm{s}}(Q^2)$ (pQCD radiative corrections). Until very recently, the pQCD contribution to BjSR has been known up to a third order in perturbative α_{s} expansion [3]. So far, the corresponding expression have been used in many studies aimed, in particular, to extraction of the α_{s} values at low momentum scales (see, e.g. [4, 5, 6, 7]).

New high precision data on the BjSR at low Q^2 in the wide range $0.05 < Q^2 < 3$ GeV^2 was obtained recently at Jefferson Lab (JLab) [8]. Using this data, in our previous works [9, 10] we have shown that the satisfactory description of the data down to $Q_{min} \sim \Lambda_{QCD}$ can be achieved by using the ghost-free Analytic Perturbation Theory (APT) [11].

The APT approach is based on the causality principle implemented as the analyticity imperative in the complex Q^2-plane for the QCD coupling $\alpha_{\mathrm{s}}(Q^2)$ in the form of the Källen-Lehmann spectral representation (for a review on APT concepts, see e.g. Ref. [12]). It is important to notice that in the framework of APT, the theoretical ambiguity associated with pQCD higher-loop corrections is diminished (see Ref. [13]). This allows to increase stability of the pQCD results at low Q^2 and reliability of the information about the non-perturbative corrections extracted from the data.

A comparison of the theoretical description with experimental data on the Bjorken integral (1) supplies us with the important information both on the pQCD contributions and the higher twists (HT) effects. One of the important theoretical questions is the interplay between the higher twists and higher order pQCD corrections at low Q^2, which has previously been investigated in Refs. [9, 10]. The question we now rise in the current

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work is what the best theoretical uncertainty inherent to pQCD series could be reached at low energies by taking into account the higher order QCD and higher OPE corrections. Answer to this question is very critical for understanding the limitations of the pQCD description. The most precise ever experimental data from the Jefferson lab on $\Gamma_1^{p-n}(Q^2)$ is the challenge to the accuracy of the pQCD expansions, such that we approach and might even be able to see how the asymptotic character of the perturbative series imposes constraints on pQCD capabilities at low energies.

The four-loop expression for the pQCD contribution to the BjSR, which became recently available in Ref. [14], gives us a reasonable motivation for a new extended QCD analysis of the precise low energy combined data on $\Gamma_1^{p-n}(Q^2)$ [8, 15, 16] accounting for up to α_s^4-order of the Perturbation Theory in both the standard PT and APT approaches. The goal of the present work is to reveal features of the four-loop PT and APT expansions in the analysis of the BjSR.

2 The perturbative QCD contribution

It is convenient to write down the Bjorken integral (1) as a sum of the perturbative part and the higher twist contributions

$$\Gamma_1^{p-n}(Q^2) = \frac{g_A}{6}\left[1 - \Delta_{\mathrm{Bj}}(Q^2)\right] + \sum_{i=2}^{\infty} \frac{\mu_{2i}}{Q^{2i-2}}\,. \tag{2}$$

The contribution $\Delta_{\mathrm{Bj}}(Q^2)$ is defined by the coefficient function $C_{\mathrm{Bj}}(\alpha_{\mathrm{s}})$,

$$\Delta_{\mathrm{Bj}}(Q^2) \equiv 1 - C_{\mathrm{Bj}}(\alpha_{\mathrm{s}}),$$

which is proportional to the flavor-nonsinglet axial vector current in the corresponding correction of the short distance Wilson expansion. In the framework of the standard pQCD, the approximation for $\Delta_{\mathrm{Bj}}(Q^2)$ has a form of the power series in the running coupling α_{s}. At the up-to-date four-loop (N^{3}LO) level in the massless case, the standard PT expansion reads

$$\Delta_{\mathrm{Bj}}^{\mathrm{PT}}(Q^2) = c_1\, a(Q^2) + c_2\, a^2(Q^2) + c_3\, a^3(Q^2) + \; c_4\, a^4(Q^2)\,, \quad a(Q^2) \equiv \alpha_{\mathrm{PT}}(Q^2)/\pi\,, \tag{3}$$

where the expansion coefficients c_i in the modified minimal subtraction scheme, $\overline{\mathrm{MS}}$, for three active flavors are $c_1 = 1$, $c_2 = 3.5833$ [17], $c_3 = 20.2153$ [3] and $c_4 = 175.7$ [14].

The evolution of the pQCD running coupling can be obtained by integration of the renormalization group equation

$$\mu^2 \frac{\partial\, a}{\partial\, \mu^2} = -\beta_0\, a^2\, (1 + b_1\, a + b_2\, a^2 + b_3\, a^3 + \ldots)\,, \tag{4}$$

where $b_i = \beta_i/\beta_0$ are related to the coefficients of the β-function. For the four-loop $\overline{\mathrm{MS}}$-coupling normalized at the scale $\mu = Q$ the equation can be rewritten as

$$\beta_0 \ln\left(\frac{Q^2}{\Lambda^2_{\overline{\mathrm{MS}}}}\right) = \frac{1}{a} + b_1 \ln\frac{\beta_0\, a}{1 + b_1 a} + b_2 \int_0^a dx \frac{b_2 + b_3 x}{(1 + b_1 x)(1 + b_1 x + b_2 x^2 + b_3 x^3)}\,, \tag{5}$$

where $\beta_0 = 9/4 = 2.25$, $b_1 = 1.778$, $b_2 = 4.471$ [18], $b_3 = 20.990$ [19] [1] (see Ref. [20] for details).

The moments of the structure functions are analytic functions of Q^2 in the complex Q^2-plane with a cut along the negative part of the real axis (see Refs. [21, 22, 23]). The perturbative representation (3) violates these analytic properties due to the unphysical singularities of the PT running coupling for $Q^2 > 0$. To avoid this problem we apply the APT method [11, 12], which gives the possibility for combining the renormalization group resummation with correct analytical properties of the QCD correction to the BjSR. In the framework of the APT, the correct analytic properties of the perturbative expansions are preserved at any fixed PT order including the four-loop one where expression for $\Delta_{\mathrm{Bj}}(Q^2)$ is given by

$$\Delta_{\mathrm{Bj}}^{\mathrm{APT}}(Q^2) = c_1\, \mathcal{A}_1(Q^2) + c_2\, \mathcal{A}_2(Q^2) + c_3\, \mathcal{A}_3(Q^2) + c_4\, \mathcal{A}_4(Q^2)\,. \tag{6}$$

Here coefficients c_1, c_2, c_3 c_4 are the same as in Eq. (3), and functions $\mathcal{A}_{k)}(Q^2)$ can be expressed through the spectral functions $\varrho_k(\sigma) \equiv \mathrm{Im}\left[a^k(-\sigma - i\epsilon)\right]$ by the Källen-Lehman representation as

$$\mathcal{A}_k(Q^2) = \frac{1}{\pi} \int\limits_0^{\infty} d\sigma\, \frac{\varrho_k(\sigma)}{\sigma + Q^2}\,.$$

[1] All numerical coefficients are given here for three flavors that is adequate for the low energy scale.

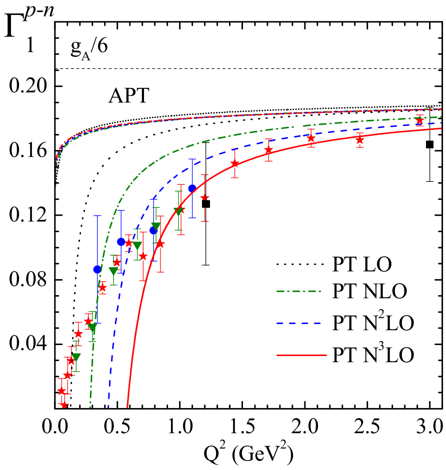

Figure 1. Perturbative part of the BjSR in different orders in the standard PT.

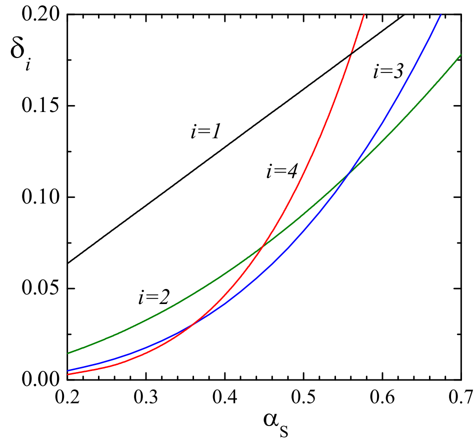

Figure 2. Contributions to the pQCD correction $\Delta_{\rm Bj}$ vs the PT running coupling $\alpha_{\rm s}$.

Note, the first function in Eq. (6), i.e. $\mathcal{A}_1(Q^2)$, is related with the analytic coupling: $\alpha_{\rm APT}(Q^2) = \pi\mathcal{A}_1(Q^2)$. Relation (6) shows that in the framework of the APT the pQCD contribution is expressed not in terms of the expansion in power series of the strong coupling, as it is done in the standard PT, but as an expansion in analytic functions $\mathcal{A}_k(Q^2)$. At large momentum transfers, these functions become proportional to k-th power of the usual perturbative coupling, $a^k(Q^2)$, and the expansion reduces to the power series. However, at small Q^2 the properties of the non-power expansion (6) become considerably different from the PT power series (3). More details about applications of the APT method can be found in Refs. [10, 12, 24, 25].

2.1 The Q^2-dependence

Now we analyze the Q^2-dependence of the BjSR in the framework of both PT and APT approaches in different orders (LO, NLO, N^{2}LO and N^{3}LO) of the perturbative expansions (3) and (6), respectively. As a normalization point, we use the most accurate $\alpha_{\rm s}$-value at $Q = M_Z$, $\alpha_{\rm s}(M_Z) = 0.1184 \pm 0.0007$ [2, 7]. In order to take into account flavor thresholds, we apply the matching conditions for the values of $\alpha_{\rm s}$ which are rather nontrivial in higher PT orders (see Refs. [20, 26, 27]). Following to analysis in Ref. [28], our matched calculation for the four-loop $\overline{\rm MS}$-coupling gives $\Lambda^{(n_f=3)} = 340 \pm 10$ MeV. Note, we obtain practically the same results, but with larger errors, if we choose the pseudo-observable value $R(M_Z^2) = 1.03904 \pm 0.00087$ as a normalization point [29], which leads to the four-loop running coupling equal to $\alpha_{\rm s}(M_Z) = 0.1190 \pm 0.0026$.

In Fig. 1, we illustrate the behavior of the perturbative part of the BjSR in different orders in $\alpha_{\rm s}$ in both PT and APT approaches. For completeness, we also show here the combined SLAC and JLab data on $\Gamma_1^{p-n}(Q^2)$ which are used in our analysis. The SLAC data points [15] are denoted by squares, the JLab CLAS Hall A 2002 data – by downward pointing triangles, the JLab CLAS Hall B 2003 data – by circles [16], and the most recent JLab data [8] – by stars.

As it follows from this figures, in the framework of the standard PT, the low energy behavior of $\Gamma_1^{p-n}(Q^2)$ is strongly dependent on the order of the initial expansion, and the lower border of satisfactory description of the JLab data shifts towards the larger Q^2 values when increasing the number of loops in the pQCD expansion (3). At the same time, in the framework of the APT, we observe the higher-loop stability given the fact that curves corresponding to different PT orders in APT are very close to each other (see Fig. 1). The reason for such infrared stability is absence of the unphysical singularities in APT expansions and, additionally, the analytic coupling in all orders has stable infrared-finite value, $\alpha_{\rm APT}(0) = \pi/\beta_0$.

2.2 Convergence of the PT and APT expansions

At low Q^2 a value of the strong coupling is quite large, questioning the convergence of pQCD series. We first illustrate the problem of convergence of the PT power series truncated after four-loop order (c.f. Eq. (3))

$$\Delta_{\rm Bj}^{\rm PT}(\alpha_{\rm s}) = 0.3183\,\alpha_{\rm s} + 0.3631\,\alpha_{\rm s}^2 + 0.6520\,\alpha_{\rm s}^3 + 1.804\,\alpha_{\rm s}^4 = \sum_{i=1}^{4}\delta_i(\alpha_{\rm s}),$$

by drawing δ_i (i-th term series) vs. $\alpha_{\rm s}$ in Fig. 2. This figure demonstrates that at $\alpha_{\rm s} \sim 0.36$ forth term becomes larger than the third one, at $\alpha_{\rm s} \sim 0.45$ — than the second one, and at $\alpha_{\rm s} \sim 0.56$ it is even bigger the first one.

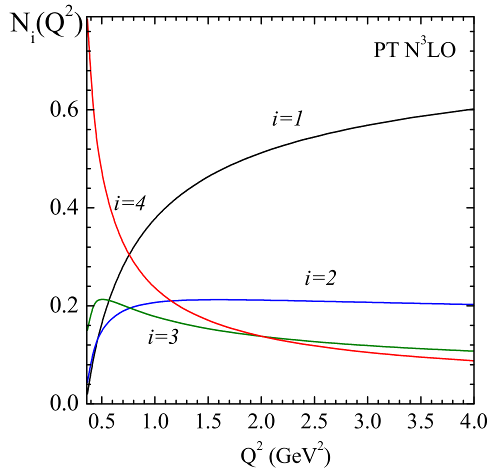

Figure 3. The Q^2-dependence of the relative contributions of the perturbative expansion terms in Eq. (3) at the four-loop level ($\mathrm{N^3LO}$) in the standard PT.

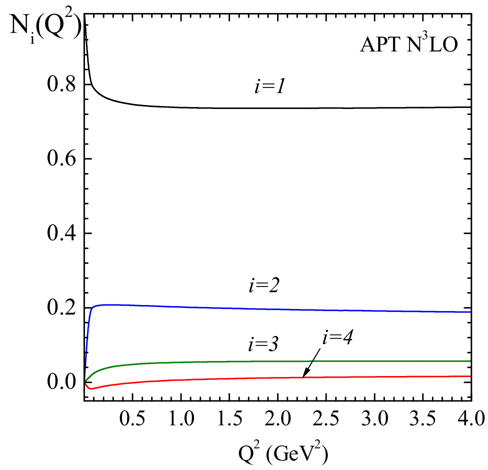

Figure 4. The Q^2-dependence of the relative contributions of the perturbative expansion terms in Eq. (3) at the four-loop level ($\mathrm{N^3LO}$) in the APT.

Similar behavior can be seen from Fig. 3, where we present the relative contributions: $\mathrm{N}_i(Q^2) = \delta_i(Q^2)/\Delta_{\mathrm{Bj}}(Q^2)$. As it is seen from this figure, in the region of small momentum transfers $Q^2 < 1\ \mathrm{GeV}^2$ the dominant contribution comes from the four-loop term. Moreover, when decreasing Q^2 its relative contribution increases. In the region $Q^2 > 2\ \mathrm{GeV}^2$ the situation changes – the major contribution comes from one- and two-loop orders there. So, the fourth order PT correction does not improve the convergence that is presumably due to an asymptotic character of the PT series.

Analogical investigation for the APT series (6) is presented at Fig. 3. This figure demonstrates that there is an essential difference between PT and APT cases. The APT expansion obeys much better convergence than the PT one. In the APT case, the higher order contributions are stable at all Q^2, and one-loop contribution gives about 70 %, two-loop – 20 %, three-loop – not exceeds 5%, and four-loop – up to 1 %.

2.3 The μ-scale dependence

As it is known, any observable obtained to all orders in pQCD expansion should be independent of the renormalisation scale μ, but in any truncated-order perturbative series the cancelation is not perfect, such that the pQCD predictions depend on the choice of the μ-scale (for a review see, e.g., Ref. [7]).

Now let us estimate the ambiguity in choosing the renormalization scale, μ for the coefficient function $C_{\mathrm{Bj}}(\alpha_{\mathrm{s}})$. The corresponding four-loop expression [14] can be rewritten as

$$\begin{aligned} C_{Bj}\left(\frac{\mu^2}{Q^2},\alpha_{\mathrm{s}}\right) &= 1 - 0.31831\,\alpha_{\mathrm{s}}(\mu^2) + \left[-0.36307 - 0.22797\ln\left(\frac{\mu^2}{Q^2}\right)\right]\alpha_{\mathrm{s}}^2(\mu^2) \\ &+ \left[-0.65197 - 0.64906\ln\left(\frac{\mu^2}{Q^2}\right) - 0.16327\ln^2\left(\frac{\mu^2}{Q^2}\right)\right]\alpha_{\mathrm{s}}^3(\mu^2) \\ &+ \left[-1.8042 - 1.7984\ln\left(\frac{\mu^2}{Q^2}\right) - 0.78968\ln^2\left(\frac{\mu^2}{Q^2}\right) - 0.11694\ln^3\left(\frac{\mu^2}{Q^2}\right)\right]\alpha_{\mathrm{s}}^4(\mu^2)\,. \end{aligned} \tag{7}$$

Next, we introduce the dimensionless parameter x_μ ($\mu^2 = x_\mu Q^2$), which is changed within the interval $x_\mu = 0.5 \div 2$ (see, for example, the analysis in Ref. [29]), and compare the μ-scale ambiguities between the two-, three- and four-loop PT and APT series at low Q^2.

We present our results in Fig. 5, where the perturbative part of the BjSR is plotted as a function of Q^2. This figure demonstrates that in the considered region of Q^2 there is an essential difference between μ-dependence of PT and APT expansions. The APT result is practically μ-scale independent even at small momentum transfers. The standard PT results are rather sensitive to μ-scale variations indicating quite significant theoretical uncertainties of the corresponding PT expansions.

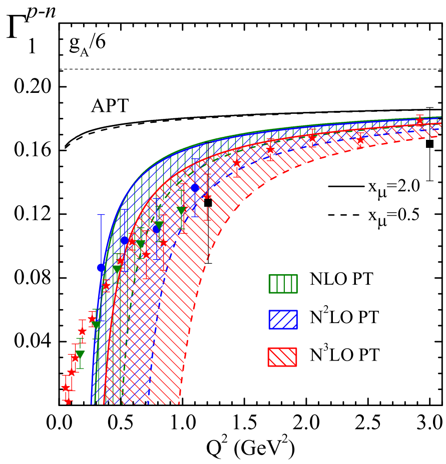

Figure 5. The μ-scale ambiguities for the perturbative part of the BjSR vs. Q^2 for two-, three- and four-loop orders of PT and APT series corresponding to x_μ in the interval $0.5 \div 2$.

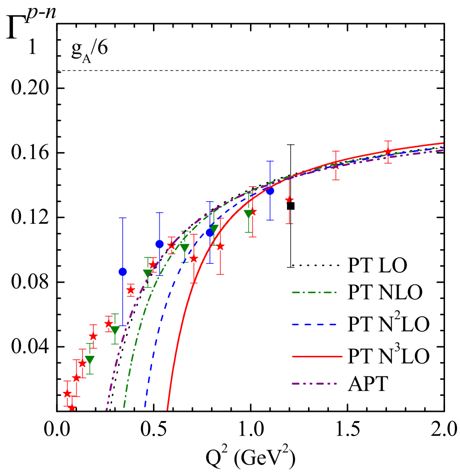

Figure 6. The μ_4-fits of the BjSR JLab data in various orders of PT and APT.

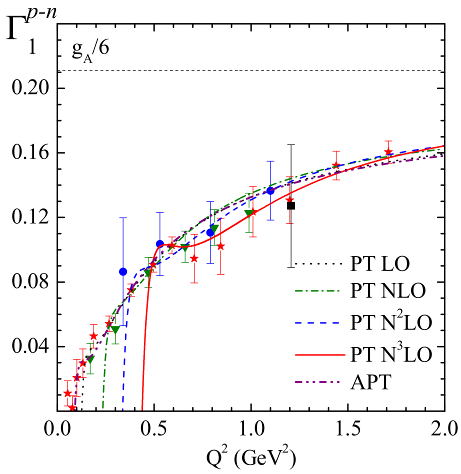

Figure 7. The $\mu_{4,6,8}$-fits of the BjSR JLab data in various orders of PT and APT.

3 Higher twists contribution

3.1 The results of the QCD fit

Previously, a detailed higher-twist analysis of the two- and three-loop expansions in powers of α_s was performed in Refs. [9]. Now, we extend the analysis up to an order $\sim \alpha_s^4$. Using the expression (2) being fitted to above mentioned experimental data [8, 16], we extract coefficients μ_{2i} of the higher twist OPE corrections. The minimal borders of fitting domains in Q^2 are settled from the *ad hoc* restriction $\chi^2 < 1$ and monotonous behavior of the resulting fitted curves.

In Figs. 6 and 7 we present the results of 1- and 3-parametric fits in various orders of PT and APT. Corresponding fit results for HT terms, extracted in different orders of PT and APT, are given in Tables 1 and 2 (all numerical results are normalized to corresponding powers of the nucleon mass M). It should be stressed that the APT result is practically coincides in various orders, therefore for it we draws one curve.

From these figures and tables we see that APT allows to move down to lower Q^2 in description of the experimental data. At the same time, in the framework of standard PT the lower border shifts up to higher Q^2 scales when increasing the order of PT expansion. This is caused by extra unphysical singularities in the higher-loop strong coupling. Note that in Figs. 6 and 7 the LO PT and APT curves are close to each other. This is explained by the fact that unphysical Landau pole in the one-loop PT α_s ($\Lambda = 285$ MeV) is located out of the fit domain, $Q_{min} \ll \Lambda$. So, the pole does not affect the results of the higher twists extraction in contrast to much stronger singularities in the higher PT orders.

Table 1. Results of higher twists extraction from the JLab data on BjSR in various orders of the PT.

PT	Q^2_{min},	μ_4/M^2	μ_6/M^4	μ_8/M^6
The best μ_4-fit results				
NLO	0.5	−0.028(5)	–	–
N^2LO	0.66	−0.014(7)	–	–
N^3LO	0.707	0.006(9)	–	–
The best $\mu_{4,6,8}$-fit results				
NLO	0.268	−0.03(1)	−0.01(1)	0.008(4)
N^2LO	0.34	0.01(2)	−0.06(4)	0.04(2)
N^3LO	0.47	0.05(4)	−0.2(1)	0.12(6)

Table 2. Results of higher twists extraction from the JLab data on BjSR in various orders of the APT.

APT	Q^2_{min}, GeV^2	μ_4/M^2	μ_6/M^4	μ_8/M^6
The best μ_4-fit results				
NLO	0.47	−0.049(3)	–	–
N^2LO	0.47	−0.049(3)	–	–
N^3LO	0.47	−0.050(3)	–	–
The best $\mu_{4,6,8}$-fit results				
NLO	0.078	−0.061(4)	0.009(1)	−0.0004(1)
N^2LO	0.078	−0.061(4)	0.009(1)	−0.0004(1)
N^3LO	0.078	−0.061(4)	0.009(1)	−0.0004(1)

3.2 Sensitivity of the higher twists to Λ_{QCD} variations

In the above analysis, we normalized the strong coupling $\alpha_s(Q^2)$ at the Z-boson mass, and then fixed the value of the Λ parameter separately in each order in α_s expansion (it was enough for understanding the role of the fourth order in the PT/APT perturbative series). However, corresponding values of the Λ parameter extracted in this way may be different from ones obtained in the direct QCD analysis of the experimental data on the moments of the structure functions (cf., e.g., Refs. [30, 31, 32]). Having this in mind, we investigate additionally the sensitivity of the extracted values of the higher twist coefficient μ_4 to the QCD scale parameter Λ in various orders of PT and APT. Note, the coefficient μ_4 corresponds of the $1/Q^2$ twist-4 term, which contains an information on quark-gluon correlations in nucleons. Different theoretical estimates for the μ_4 term can be found in our previous paper [10].

In figures 8 and 9, we show values of the coefficient μ_4 extracted from the JLab data using one-, two-, three- and four-loop PT and APT expansions vs. parameter Λ at $Q^2_{min} = 0.66$ GeV^2. From figures 8 and 9 it follows that results obtained in the framework of APT approach demonstrate a weak sensitivity of the extracted value of μ_4 to the QCD scale parameter Λ variations. This fact points out to the stability of extracted higher twist parameters. Analogous effect was also noticed above in the APT analysis of the perturbative part of the Bjorken integral.

The standard PT approach does not lead to a stable result for extracted μ_4 value with respect to Λ variations. Higher twist μ_4 contribution in PT rather strongly changes between different orders of the PT expansion and it happens both in absolute value and in sign, namely, at $\Lambda > 320$ MeV at four-loop level the higher twist coefficient becomes positive.

4 Conclusion

In this work, we performed the QCD analysis of the precise low energy JLab data on the BjSR and extracted the OPE higher twist terms using the four-loop expression for the QCD correction to the Bjorken integral $\Delta_{\rm Bj}$ published recently in Ref. [14].

Our analysis has demonstrated that in the low region $Q^2 < 2$ GeV^2, the four-loop PT order, as well as the three-loop one, contributes more than 10 % to the QCD correction $\Delta_{\rm Bj}$. It means that inclusion of the N^3LO correction to the PT series for $\Delta_{\rm Bj}$ does not improve the precision of the theoretical description of the BjSR preserving it at the level of N^2LO approximation. Additionally, in standard PT analysis the N^3LO correction induces extra unphysical singularities in the PT series, which shift the lower border of the pQCD applicability range Q_{min} to higher values compared to ones in the lower PT orders.

In the case of APT analysis, the N^3LO correction is sufficiently smaller than the N^2LO one and provides the percentage precision of the theoretical predictions. Since APT series obey the analytic properties at any

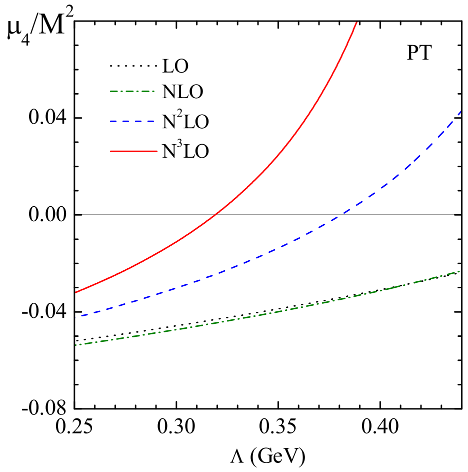

Figure 8. Value of the higher twist coefficient μ_4 extracted from the JLab data using the standard PT at different orders at $Q^2_{min} = 0.66$ GeV^2.

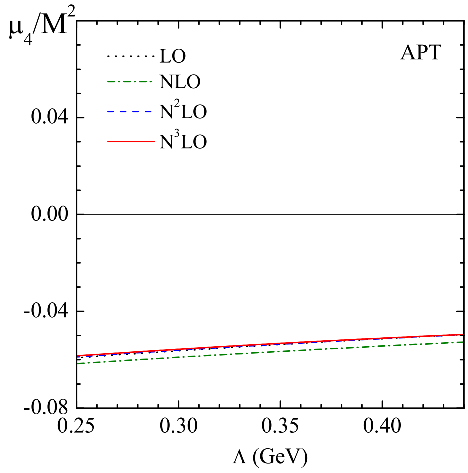

Figure 9. Value of the coefficient μ_4 extracted from the JLab data using the APT at different orders at $Q^2_{min} = 0.66$ GeV^2.

Q^2 (i.e. free of unphysical singularities), then APT allows to describe the precise low energy JLab data down to the scales $Q \sim \Lambda$ and give a possibility for reliable extraction of the higher twist coefficients.

Performing the higher-twist analysis of the JLab data on BjSR at low Q^2 in the framework of PT and APT approaches, we have shown that

- values of the higher twist coefficients extracted from the data in the standard PT approach in different PT orders are different, i.e. they are not stable with respect to higher loop corrections. The value of μ_4 coefficient at two- and three-loop levels is negative, whereas at the four-loop level it becomes positive.

- since APT exhibits the higher loop stability then the values of μ_4 extracted in the NLO, N^{2}LO and N^{3}LO APT coincide within the data fits uncertainty.

Therefore, the analysis of the standard N^{3}LO PT series for $\Delta_{\rm Bj}$ using precise JLab data on the Bjorken integral Γ^{p-n} demonstrates an asymptotic character of the standard PT expansions in the region of low $Q^2 < 2$ GeV^2, and account the four-loop corrections does not improve precision of the theoretical predictions. In addition, extra unphysical singularities emerging at the four-loop level do not allow to move deeper to the low-energy domain and to obtain stable values of the higher twist coefficients. Using the APT one can move the lower border of the pQCD JLab data description down to scales $Q \sim \Lambda \sim 300$ MeV. Extracted value of the higher twist coefficient μ_4 is universal due to its very little sensitivity to the higher loop APT corrections and $\Lambda_{\rm QCD}$ variations. So, the remarkable properties of the APT in the low-energy domain create a basis for preferring the application of this technique.

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