



Z-Z' MIXING AND ITS DETERMINATION AT THE LHC AND ILC

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New heavy neutral gauge bosons Z' are predicted by many models of physics beyond the Standard Model. The potential for measuring the $Z'W^+W^-$ coupling in W^+W^- pair production at the LHC is investigated. Also, we discuss the foreseeable sensitivity to Z' 's of W^\pm -pair production cross sections at the e^+e^- ILC. We show that the sensitivity of the ILC for probing the Z - Z' mixing is substantially enhanced when the polarization of the initial beams and the produced W^\pm bosons are considered.

1 Introduction

interactions describes nearly all experimental data available today [1], it is widely believed that it is not the ultimate theory. Grand Unified Theories (GUTs), eventually supplemented by Supersymmetry to achieve a successful unification of the three gauge coupling constants at the high scale, are prime candidates for the physics beyond the SM. Many of these GUTs, including superstring and left-right-symmetric models, predict the existence of new neutral gauge bosons, which might be light enough to be accessible at current and/or future colliders [2–4].

The search for these Z' particles is an important aspect of the experimental physics program of current and future high-energy colliders. Present limits from direct production at the LHC and Tevatron, and virtual effects at LEP, through interference or mixing with the Z boson, imply that new Z' bosons are rather heavy and mix very little with the Z boson. Depending on the considered theoretical models, Z' masses of the order of 2.4–2.9 TeV (LHC) [5, 6] and Z - Z' mixing angles at the level of a few per-mille are excluded [7]. A Z' boson, if lighter than about 5 TeV, could be discovered at the LHC with $\sqrt{s} = 14$ TeV in the Drell-Yan process

$$pp \rightarrow Z' \rightarrow \ell^+\ell^- \quad (1)$$

with $\ell = e, \mu$. Future e^+e^- International linear collider (ILC) with high c.m. energies and longitudinally polarized beams could indicate the existence of Z' bosons via its interference effects, with masses up to about $6 \times \sqrt{s}$. After the discovery of a Z' boson at the LHC, some diagnosis of its coupling and $Z - Z'$ mixing needs to be done in order to identify the correct theoretical frame.

The W^\pm boson pair production processes

$$e^+ + e^- \rightarrow W^+ + W^- \quad (2)$$

and

$$p + p \rightarrow W^+W^- + X \quad (3)$$

are a crucial ones for studying the electroweak gauge symmetry in the ILC and LHC, respectively. Properties of the weak gauge bosons are closely related to electroweak symmetry breaking and the structure of the gauge sector in general. Thus, detailed examination of (2) at the ILC and (3) at the LHC will both test this sector of the standard model with the highest accuracy and throw light on New Physics (NP) that may appear beyond the SM. We shall here focus on the phenomenological effects in reactions (1)-(3) of the so-called Z'_{SSM} and Z'_{E_6} models. Actually, in some sense, we may consider these Z' models as representative of this NP sector.

The discovery of the Z' boson would constrain models of electroweak symmetry breaking. The Drell-Yan process (1) is sensitive to the Z' mass, while decay rate $Z' \rightarrow W^+W^-$ is sensitive to the $Z - Z'$ mixing angle. In the present paper, we examine the feasibility of detecting the Z' boson via the decay rate $Z' \rightarrow W^+W^-$ at the LHC, which is not the principal discovery channels as Drell-Yan process, but can help to understand the origin of the new gauge bosons. Also, we study the indirect effects evidencing the mentioned extra Z' gauge bosons in W^\pm pair production (2) at ILC, with a center of mass energy $\sqrt{s} = 0.5 - 1$ TeV and typical time-integrated luminosities of $\mathcal{L}_{\text{int}} \sim 0.5 - 1 \text{ ab}^{-1}$. At the foreseen, really high luminosity this process should

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be quite sensitive to the indirect NP effects at a collider with $M_Z \ll \sqrt{s} \ll M_{Z'}$, the deviations of cross sections from the SM predictions being expected to increase with \sqrt{s} due to the violation of the SM gauge cancellation among the different contributions.

2 Z' models

The Z' models that will be considered in our analysis are the following [8–10]:

- (i) The four possible $U(1)$ Z' scenarios originating from the spontaneous breaking of the exceptional group E_6 . In this case, two extra, heavy neutral gauge bosons appear as consequence of the symmetry breaking and, generally, only the lightest is assumed to be within reach of the collider. It is defined, in terms of a new mixing angle β , by the linear combination

$$Z' = Z'_\chi \cos \beta + Z'_\psi \sin \beta. \quad (4)$$

Specific choices of β : $\beta = 0$; $\beta = \pi/2$; $\beta = -\arctan \sqrt{5/3}$ and $\beta = \arctan \sqrt{3/5}$, corresponding to different E_6 breaking patterns, define the popular scenarios Z'_χ , Z'_ψ , Z'_η and Z'_I , respectively.

- (ii) The Z'_{ALR} predicted by the so-called ‘alternative’ left-right scenario. For the LR model we need not introduce additional fermions to cancel anomalies. However, in the E_6 case a variant of this model (called the Alternative LR model) can be constructed by altering the embeddings of the SM and introducing exotic fermions into the ordinary 10 and 5 representations.

- (iii) The so-called sequential Z'_{SSM} , where the couplings to fermions are the same as those of the SM Z .

In the extended gauge theories predicting the existence of an extra neutral Z' gauge boson, the mass-squared matrix of the Z and Z' can have non-diagonal entries δM^2 , which are related to the vacuum expectation values of the fields of an extended Higgs sector:

$$M_{ZZ'}^2 = \begin{pmatrix} M_Z^2 & \delta M^2 \\ \delta M^2 & M_{Z'}^2 \end{pmatrix}. \quad (5)$$

Here, Z and Z' denote the weak gauge boson eigenstates of $SU(2)_L \times U(1)_Y$ and of the extra $U(1)'$, respectively. The mass eigenstates, Z_1 and Z_2 , diagonalizing the matrix (5), are then obtained by the rotation of the fields Z and Z' by a mixing angle ϕ :

$$Z_1 = Z \cos \phi + Z' \sin \phi, \quad (6)$$

$$Z_2 = -Z \sin \phi + Z' \cos \phi. \quad (7)$$

Here, the mixing angle ϕ is expressed in terms of masses as:

$$\tan^2 \phi = \frac{M_Z^2 - M_1^2}{M_2^2 - M_Z^2} \simeq \frac{2M_Z \Delta M}{M_2^2}, \quad (8)$$

where $\Delta M = M_Z - M_1 > 0$, M_Z is the mass of the Z_1 -boson in the absence of mixing, i.e., for $\phi = 0$. Once we assume the mass M_1 to be determined experimentally, the mixing depends on two free parameters, which we identify as ϕ and M_2 .

The mixing angle ϕ will play an important role in our analysis. In general, such mixing effects reflect the underlying gauge symmetry and/or the Higgs sector of the model. To a good approximation, for $M_1 \ll M_2$, in specific ‘‘minimal-Higgs models’’,

$$\phi \simeq -s_W^2 \frac{\sum_i \langle \Phi_i \rangle^2 I_{3L}^i Q_i'}{\sum_i \langle \Phi_i \rangle^2 (I_{3L}^i)^2} = \mathcal{C} \frac{M_1^2}{M_2^2}. \quad (9)$$

Here $\langle \Phi_i \rangle$ are the Higgs vacuum expectation values spontaneously breaking the symmetry, and Q_i' are their charges with respect to the additional $U(1)'$. In addition, in these models the same Higgs multiplets are responsible for both generation of mass M_1 and for the strength of the Z - Z' mixing. Thus \mathcal{C} is a model-dependent constant. For example, in the case of E_6 superstring-inspired models \mathcal{C} can be expressed as [11]

$$\mathcal{C} = 4s_W \left(A - \frac{\sigma - 1}{\sigma + 1} B \right), \quad (10)$$

where σ is the ratio of vacuum expectation values squared, and the constants A and B are determined by the mixing angle β : $A = \cos \beta / 2\sqrt{6}$, $B = \sqrt{10}/12 \sin \beta$.

An important property of the models under consideration is that the gauge eigenstate Z' does not couple to the W^+W^- pair since it is neutral under $SU(2)_L$. Therefore the process (1), and the searched-for deviations

of the cross sections from the SM, are sensitive to a Z' only in the case of a non-zero Z - Z' mixing. The mixing angle is rather highly constrained, to an upper limit of a few $\times 10^{-3}$, mainly from LEP measurements at the Z [7, 12]. The high statistics on W -pair production expected at the ILC might in principle allow to probe such small mixing angles effectively.

From (6) and (7), one obtains the vector and axial-vector couplings of the Z_1 and Z_2 bosons to fermions:

$$v_{1f} = v_f \cos \phi + v'_f \sin \phi, \quad a_{1f} = a_f \cos \phi + a'_f \sin \phi, \quad (11)$$

$$v_{2f} = -v_f \sin \phi + v'_f \cos \phi, \quad a_{2f} = -a_f \sin \phi + a'_f \cos \phi, \quad (12)$$

with $(v_f, a_f) = (g_L^f \pm g_R^f)/2$, and (v'_f, a'_f) similarly defined in terms of the Z' couplings. The fermionic Z' couplings can be found in [8, 9].

Analogously, one obtains according to the remarks above:

$$g_{WWZ_1} = \cos \phi g_{WWZ}, \quad (13)$$

$$g_{WWZ_2} = -\sin \phi g_{WWZ}, \quad (14)$$

where $g_{WWZ} = \cot \theta_W$.

3 Discovery reach on Z' in lepton pair production at LHC

The cross section of process (1) for inclusive production of a dilepton with invariant mass M can be written as ($R = Z'$) [13]

$$\frac{d\sigma(R_{ll})}{dM dy dz} = K \frac{2M}{s} \sum_{ij} f_i(\xi_1, M) f_j(\xi_2, M) \frac{d\hat{\sigma}}{dz}(i + j \rightarrow l^+ + l^-). \quad (15)$$

Here, s is the proton-proton center-of-mass energy squared; $z = \cos \theta_{\text{c.m.}}$ with $\theta_{\text{c.m.}}$ the lepton-quark angle in the dilepton center-of-mass frame; y is the dilepton rapidity; $f_{i,j}(\xi_{1,2}, M)$ are parton distribution functions in the protons P_1 and P_2 , respectively, with $\xi_{1,2} = (M/\sqrt{s}) \exp(\pm y)$ the parton fractional momenta; finally, $d\hat{\sigma}_{ij}$ are the partonic differential cross sections. The color-averaged differential cross section for the relevant, leading order, partonic subprocess $q\bar{q} \rightarrow Z' \rightarrow l^+l^-$ can be expressed as:

$$\left. \frac{d\hat{\sigma}_{q\bar{q}}^{Z'}}{dz} \right|_{z-\text{even}} = \frac{1}{N_c} \frac{\pi \alpha_{\text{em}}^2}{2M^2} [S_q^{Z'} (1 + z^2)], \quad (16)$$

with

$$S_q^{Z'} = \frac{1}{4} (g_L^q{}'^2 + g_R^q{}'^2) (g_L^l{}'^2 + g_R^l{}'^2) |\chi_{Z'}|^2, \quad \chi_{Z'} = \frac{M^2}{M^2 - M_{Z'}^2 + i M_{Z'} \Gamma_{Z'}}. \quad (17)$$

In (15), the factor K accounts for next-to-leading order QCD contributions [14]. For simplicity, and to make our procedure more transparent, we will use as an approximation a global flat value $K = 1.3$.

Since we are interested in a (narrow) peak production and subsequent decay into the DY pair, $pp \rightarrow R \rightarrow l^+l^-$, we consider the lepton differential angular distribution, integrated over an interval of M around M_R :

$$\frac{d\sigma(R_{ll})}{dz} = \int_{M_R - \Delta M/2}^{M_R + \Delta M/2} dM \int_{-Y}^Y \frac{d\sigma}{dM dy dz} dy. \quad (18)$$

The number of events under the peak, that determines the statistics, is therefore given by:

$$\sigma(R_{ll}) \equiv \sigma(pp \rightarrow R) \cdot \text{BR}(R \rightarrow l^+l^-) = \int_{-z_{\text{cut}}}^{z_{\text{cut}}} dz \int_{M_R - \Delta M/2}^{M_R + \Delta M/2} dM \int_{-Y}^Y dy \frac{d\sigma}{dM dy dz}. \quad (19)$$

For the full final phase space, $z_{\text{cut}} = 1$ and $Y = \log(\sqrt{s}/M)$. However, if the finite detector angular acceptance is accounted for, $z_{\text{cut}} < 1$ and Y in Eqs. (18) and (19) must be replaced by a maximum value $y_{\text{max}}(z, M)$. Concerning the size of the bin ΔM , it should include a number (at least one) of peak widths to enhance the probability to pick up the resonance. In the models we will consider, widths are predicted to be small, typically of the order of a percent (or less) of the mass M_R , so that the integral under the peak should practically be insensitive to the actual value of ΔM . Conversely, the SM 'background' is expected to depend on ΔM . We denote by N_B and N_S the number of 'background' and 'signal' events in the bin, the criterion $N_S = 5\sqrt{N_B}$ or 10 events, whichever is larger, as the minimum signal for the peak discovery.

To evaluate the statistics, we shall use in Eqs. (18) and (19) the CTEQ6.5 parton distributions [15], and impose cuts relevant to the LHC detectors, namely: pseudorapidity $|\eta| < 2.5$ for both leptons assumed massless (this leads to a boost-dependent cut on z ; lepton transverse momentum $p_\perp > 20$ GeV. Moreover, the reconstruction efficiency is taken to be 90% for both electrons and muons and throughout this paper a time-integrated LHC luminosity $\mathcal{L}_{\text{int}} = 100 \text{ fb}^{-1}$.

For the proton-proton initiated process $pp \rightarrow l^+l^- + X$, only the z -even parts of the partonic differential cross sections contribute to the right-side of Eq. (18), z -odd terms do not contribute after the y -integration. Also, due to $M_Z \ll M_R$ and the narrow width peak, the resonant amplitude interference with the SM is expected to give negligible contributions to the right-hand sides of (18) and (19) after the symmetric M -integration around M_R needed there. Thus, we can retain in these equations just the SM and the resonance pole contributions.

In Fig.1 we show the predicted number of resonance (signal) events N_S in the Drell-Yan process at LHC, *vs.* M_R , where $R = Z'$. The assumed integrated luminosity is $\mathcal{L}_{\text{int}} = 100 \text{ fb}^{-1}$, the cuts in phase space relevant to the foreseen detector acceptance specified above have been imposed, and the channels $l = e, \mu$ have been combined.

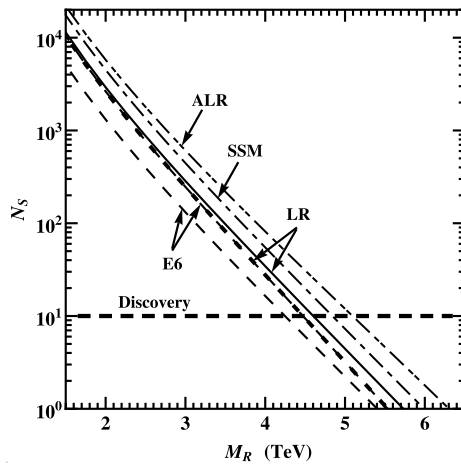


Figure 1. Number of resonance events N_S , as a function of the Z' -resonance mass M_R ($R = Z'$) at $\mathcal{L}_{\text{int}} = 100 \text{ fb}^{-1}$ in the production of Z' bosons that is followed by their decay to dileptons ($l^+l^- = e^+e^-, \mu^+, \mu^-$).

As regards the discovery of Z' we are interested in, the signature spaces in Fig. 1 reduce to the lines labeled by the different models, because the event rates are fixed, once M'_Z is given. In Fig. 1 shows that, with the assumed luminosity of 100 fb^{-1} , Z' gauge boson masses up to 4–5 TeV are in principle within the 5- σ reach of the LHC.

4 Z - Z' mixing at the ILC and LHC

The general expression for the cross section of process (2) with longitudinally polarized electron and positron beams can be expressed as [2]

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{4} \left[(1 + P_L)(1 - \bar{P}_L) \frac{d\sigma^+}{d\cos\theta} + (1 - P_L)(1 + \bar{P}_L) \frac{d\sigma^-}{d\cos\theta} \right], \quad (20)$$

where P_L and \bar{P}_L are the actual degrees of electron and positron longitudinal polarization, respectively, and σ^\pm are the cross sections for purely right-handed ($\lambda = 1/2$) and left-handed ($\lambda = -1/2$) electrons.

The polarized cross sections can generally be written as follows:

$$\frac{d\sigma^\pm}{d\cos\theta} = \frac{|\mathbf{p}|}{4\pi s\sqrt{s}} \sum_{\tau, \tau'} |F_{\lambda\tau\tau'}(s, \cos\theta)|^2. \quad (21)$$

Here, the helicities of the W^- and W^+ are denoted by $\tau, \tau' = \pm 1, 0$, the helicity amplitudes $F_{\lambda\tau\tau'}(s, \cos\theta)$, $p = |\mathbf{p}|$ the c.m. momentum of the W^- . Furthermore, s and t are the Mandelstam variables, and θ the c.m. scattering angle, with $t = M_W^2 - s(1 - \beta \cos\theta)/2$.

The sensitivity of the polarized differential cross sections to Z' is assessed numerically by dividing the angular range $|\cos\theta| \leq 0.98$ into 10 equal bins, and defining a χ^2 function in terms of the expected number of events $N(i)$ in each bin for a given combination of beam polarizations:

$$\chi^2 = \sum_{\{P_L, \bar{P}_L\}} \sum_i^{\text{bins}} \left[\frac{N_{\text{SM}+Z'}(i) - N_{\text{SM}}(i)}{\delta N_{\text{SM}}(i)} \right]^2, \quad (22)$$

where $N(i) = \mathcal{L}_{\text{int}} \sigma_i \varepsilon_W$ with \mathcal{L}_{int} the time-integrated luminosity. Furthermore,

$$\sigma_i = \sigma(z_i, z_{i+1}) = \int_{z_i}^{z_{i+1}} \left(\frac{d\sigma}{dz} \right) dz, \quad (23)$$

where $z = \cos\theta$ and polarization indices have been suppressed. Also, ε_W is the efficiency for W^+W^- reconstruction, for which we take the channel of lepton pairs ($e\nu + \mu\nu$) plus two hadronic jets, giving $\varepsilon_W \simeq 0.3$ basically from the relevant branching ratios. The procedure outlined above is followed to evaluate both $N_{\text{SM}}(i)$ and $N_{\text{SM}+Z'}(i)$.

The uncertainty on the number of events $\delta N_{\text{SM}}(i)$ combines both statistical and systematic errors where the statistical component is determined by $\delta N_{\text{SM}}^{\text{stat}}(i) = \sqrt{N_{\text{SM}}(i)}$. Concerning systematic uncertainties, an important source is represented by the uncertainty on beam polarizations, for which we assume $\delta P_L/P_L = \delta \bar{P}_L/\bar{P}_L = 0.5\%$ with the ‘‘standard’’ envisaged values $|P_L| = 0.8$ and $|\bar{P}_L| = 0.5$ [16, 17]. As for the time-integrated luminosity, for simplicity we assume it to be equally distributed between the different polarization configurations. Another source of systematic uncertainty originates from the efficiency of reconstruction of W^\pm pairs which we assume to be $\delta\varepsilon_W/\varepsilon_W = 0.5\%$. Also, in our numerical analysis to evaluate the sensitivity of the differential distribution to model parameters we include initial-state QED corrections to on-shell W^\pm pair production in the flux function approach that assures a good approximation within the expected accuracy of the data.

As a criterion to derive the constraints on the coupling constants in the case where no deviations from the SM were observed within the foreseeable uncertainties on the measurable cross sections, we impose that

$$\chi^2 \leq \chi_{\text{min}}^2 + \chi_{\text{CL}}^2, \quad (24)$$

where χ_{CL}^2 is a number that specifies the chosen confidence level, χ_{min}^2 is the minimal value of the χ^2 function. With two independent parameters the 95% CL is obtained by choosing $\chi_{\text{CL}}^2 = 5.99$.

Fig. 2 shows the discovery reach (at a 95% C.L.) in the plane spanned by the $Z - Z'$ mixing angle and the Z_2 -boson mass for the Z'_{SSM} and Z'_ψ models, respectively, from an analysis of the polarized cross sections for $P_L = \pm 0.8$ and $\bar{P}_L = \pm 0.5$. Two options of energy and time integrated luminosity have been chosen, namely, $\sqrt{s} = 0.5$ TeV and $L_{\text{int}} = 0.5 \text{ ab}^{-1}$ (thin dot-dashed lines) and $\sqrt{s} = 1.0$ TeV and $L_{\text{int}} = 1 \text{ ab}^{-1}$ (dashed lines). The ILC with energy of 0.5 TeV is able to place the limits on the $Z - Z'$ mixing angle at the level of $\text{few} \times 10^{-3}$ that comparable with the current ones obtained from the global analysis electroweak data and shown in the figures as thick dot-dashed lines with attached label of ‘‘EW data’’. It turns out that doubling energy and luminosity leads to further improvement of the limit on $Z - Z'$ mixing angle up to $|\phi| \sim 10^{-4}$.

One can perform a similar analysis of $Z - Z'$ mixing effects at the LHC in process (3) taking into account the pure leptonic decay channels of W^\pm bosons, $pp \rightarrow W^+W^+ \rightarrow l\nu l'\nu' + X$. The result of this analysis shown in Fig. 2 where corresponding limits indicated by solid lines. These figures demonstrate a high ability of the LHC to study the $Z - Z'$ mixing effects that comparable with those obtained at the ILC operating in high energy and luminosity option.

Also, the Z' mass limits obtained from the Drell-Yan process (1) in the current experiments at the LHC at $\sqrt{s} = 8$ TeV and $\mathcal{L}_{\text{int}} = 20 \text{ fb}^{-1}$ as well as at nominal energy and luminosity, $\sqrt{s} = 14$ TeV and $\mathcal{L}_{\text{int}} = 100 \text{ fb}^{-1}$, are depicted in Fig. 2 by horizontal lines. The resulting area on (ϕ, M_2) parameter space lays above of those horizontal lines and constrained by the forceable data on process (3). From those figures one can conclude that LHC and ILC would provide a complementary information on $Z - Z'$ mixing angle.

In conclusion, the possibility to observe a new Z' boson in the $Z' \rightarrow W^+W^-$ channel at the LHC was examined. We demonstrated that even if the Z' boson is first observed via purely leptonic decay mode in process (1), the measurement of the $Z'W^+W^-$ vertex would give independent information on the new physics. We also show that LHC and ILC will be able to provide a complementary information on $Z - Z'$ mixing angle and Z' mass.

Acknowledgements. This research has been partially supported by the Abdus Salam ICTP under the TRIL and Associates Scheme and the Belarusian Republican Foundation for Fundamental Research.

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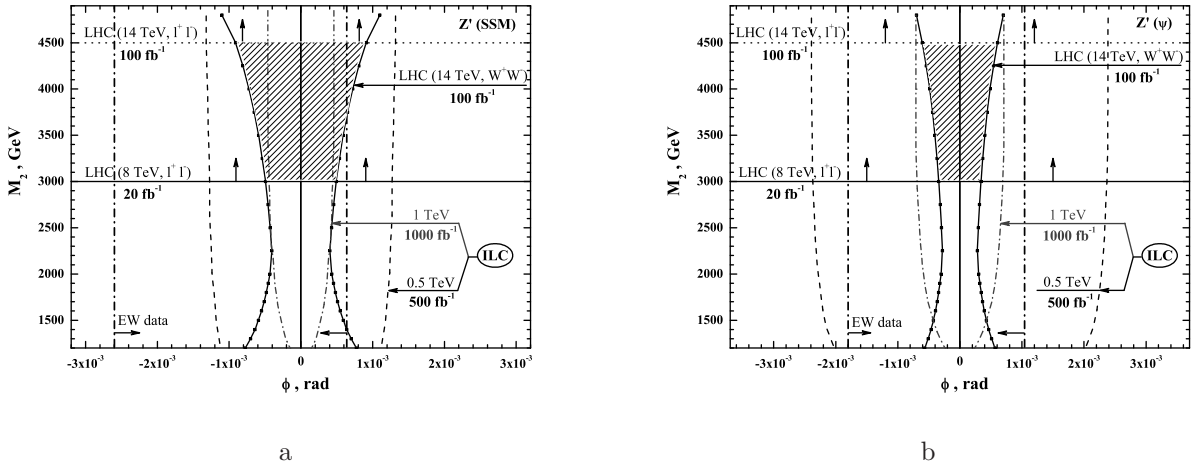


Figure 2. a) Discovery reach (at 95% C.L.) in the plane spanned by the $Z - Z'$ mixing angle and the Z_2 -boson mass for SSM model obtained from an analysis of the polarized cross sections of process $e^+e^- \rightarrow W^+W^-$ with $P_L = \pm 0.8$ and $\overline{P}_L = \pm 0.5$ at the ILC. Two options of energy and time integrated luminosity have been taken as follows: $\sqrt{s} = 0.5$ TeV and $L_{int} = 0.5 \text{ ab}^{-1}$ (thin dot-dashed lines) and $\sqrt{s} = 1.0$ TeV and $L_{int} = 1 \text{ ab}^{-1}$ (dashed lines). Current limits obtained from the global analysis of electroweak data are also shown and indicated by label “EW data” (thick dot-dashed lines). Also, limits derived at the LHC from process $pp \rightarrow W^+W^- \rightarrow \nu\nu' + X$ (thick solid lines) and from the Drell-Yan process (1) at $\sqrt{s} = 8$ TeV and $\mathcal{L}_{int} = 20 \text{ fb}^{-1}$ (thin solid line) as well as at nominal energy and luminosity, $\sqrt{s} = 14$ TeV and $\mathcal{L}_{int} = 100 \text{ fb}^{-1}$ (dotted line), are displayed; b) Same as in a), but for ψ model.

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