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TARGET MASS CORRECTIONS TO STRUCTURE FUNCTIONS IN NEUTRINO DEEP-INELASTIC SCATTERING

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With the improvement of accuracy in the inclusive deep-inelastic scattering experiments, it has become apparent that comparable improvements are needed in the accuracy of the theoretical analysis tools. As well-known, extracting structure functions in the region large values of the Bjorken variable x, it is crucial to correct data by effects associated with the non-zero mass of the target. We investigate this question and demonstrate that target mass corrections to structure functions calculated by using the new approach have a correct spectral property and noticeably differ that the standard Georgi-Politzer method gives, especially for values Q^2 of the order of $1 \div 2$ GeV^2 and large values of the Bjorken variable x.

1 Introduction

Presently the structure functions in deep-inelastic scattering is a well-proved way of probing the internal structure of nucleons and testing the physical theories governing their structure. The cross section of inelastic lepton-hadron scattering in the case of a unpolarized nucleon is determined by the hadron tensor $W_{\mu\nu}$, which is parameterized by the nucleon structure functions F_1, F_2, and F_3. (Other structure functions are proportional to the lepton mass and are therefore negligible for the kinematics of the deep-inelastic region.)

We concentrate here on the unpolarized neutrino-nucleon scattering. Neutrinos interact with matter only via the weak-interactions, which are classified as the neutral-current (NC) and the charged current (CC) interactions. CC interactions are mediated by charged $W^{\pm}$ bosons, while NC interactions are mediated by electrically neutral Z bosons. A CC reaction transforms a neutrino into its corresponding charged lepton, and vice versa. We do not consider the first two structure function F_1 and F_2 in the neutrino-nucleon scattering as these functions are simile to charged lepton deep-inelastic scattering: the electromagnetic current replaced by the corresponding weak current. Therefore, F_1 and F_2 can be easily obtained from our results which have been presented earlier in Ref. [1]. Instead we focus on the structure function F_3 associated to the totally antisymmetric tensor, which arises from the vector/axial-vector interference of the two VA currents.

To analyze the properties of structure functions the operator product expansion (OPE) method is usually used. According to the OPE, the contributions from different quark-gluon operators to hadronic tensor can be ordered according to their twist. For the structure functions this leads to the expansion in inverse powers of Q^2. The first term is the leading twist (LT) contribution which is directly related to the distributions of quarks and gluons inside the nucleon, the parton distribution functions via the factorization theorem as a convolution with coefficient functions. The coefficient functions depend on the process and the type of the structure function but are independent of the target. These functions are computable as power series in α_s. The parton distributions are independent of the process but do depend on the target. The parton distribution functions have non-perturbative origin and cannot be calculated in the perturbative QCD. The twist-4 contributions (higher twist, HT) involve interactions between quarks and gluons and lack simple probabilistic interpretation.

It must be stressed that the OPE expansion was derived in the massless limit. If a finite mass for the nucleon target is considered, the new terms arise: leading to additional power terms of kinematical origin called the target mass corrections (TMC). The TMC play a somewhat special role become larger and larger at low Q^2 and approaching to the kinematic limit as the Bjorken variable x tends to unity. The OPE was first used to study target mass effects by Georgi and Politzer in Ref. [2]. Such an approach for considering TMS became known as the Georgi and Politzer (GP) approach or ξ-scaling method because was formulated through the Nachtmann ξ variable [3]. However, the expressions for the structure functions obtained by using GP method have a difficulty arising from the violation of the spectral condition. It hence became a problem to describe the structure functions as the Bjorken variable x tends to unity. This problem has been widely discussed in the literature ever since its appearance (see, e.g., [4–6]).

As it was shown by Solovtsov [7] that this problem is similar to the problem that appears for an invariant charge in quantum chromodynamics, when the violation of the general principles of the theory, which are

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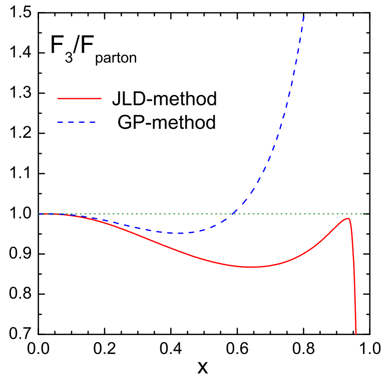

Figure 1. The behavior of the ratio vs x (without QCD corrections).

reflected in the Källén–Lehmann representation, leads to unphysical singularities. A solution of this problem was proposed in the Shirkov-Solovtsov analytic approach [8]. By using the Jost-Lehmann-Dyson (JLD) integral representation [9, 10] it was shown [7] that the natural scaling variable is a new variable s, which leads to the moments $\mathcal{M}_n(Q^2)$ that are analytic functions. In this case, the spectral property for the structure functions is satisfied automatically, and no problem arises in the limit as the Bjorken variable x tends to unity. Note the proof of the JLD representation is based on the most general principles of the theory, such as the covariance, Hermiticity, spectrality, and causality. In the present work we continue the researches begun earlier in Refs. [1] and we extend them on the structure function F_3 in charged-current, measured with high precision in neutrino-nucleon DIS [11]. As in the previous works, we concentrate on effects associated with the non-zero mass of the target including next-to-leading order (NLO) QCD corrections.

2 Threshold problem

To begin let us demonstrate a threshold problem within the GP approach. The structure function F_3 corrected by the TMC reads as follows [2]:

$$F_3(x,Q^2) = \frac{x\,F_3^{(0)}(\xi,Q^2)}{\xi(1+4\varepsilon x^2)} + \frac{2\varepsilon x^2}{\sqrt{(1+4\varepsilon x^2)^3}} \cdot \int\limits_{\xi}^{1} \frac{F_3^{(0)}(y,Q^2)}{y} \cdot dy\,, \tag{1}$$

where the Bjorken variable $x = Q^2/2\nu$, the Nachtmann variable [3] reads

$$\xi = \frac{2\,x}{1+\sqrt{1+4\varepsilon x^2}}\,, \qquad \epsilon \equiv \frac{M^2}{Q^2}\,, \qquad F_3^0(\xi,Q^2) = \lim_{M\to 0} F_3(x,Q^2)_{x=\xi}\,, \tag{2}$$

M is the nucleon mass. In Fig. 1 we show the behavior of the ratio of F_3 structure function with TMC according to the expression (1) to the parton distribution $F(x) = \sqrt{x}(1-x)^3$ (see Ref. [12]) for the JLD-method (solid line) and the ξ-approach (dashed curve) at the $\varepsilon \equiv M^2/Q^2 = 1/2$. This figure shows that there is a difference in results got by different methods (see Ref. [5] for more details) and within the method based on the JLD representation it is possible to get a correct behavior of the structure function as $x \to 1$.

Accounting the NLO QCD corrections, we use the following expressions for the proton and the neutron structure functions (see, e.g., Refs. [13, 14]):

$$\begin{aligned}
F_3^{\nu p}(x,Q^2) &= (\delta(1-x) + a_s(Q^2)\cdot C_3^1(x)) \otimes q^{\nu p}(x,Q^2)\,,\\
F_3^{\bar\nu p}(x,Q^2) &= (\delta(1-x) + a_s(Q^2)\cdot C_3^1(x)) \otimes q^{\bar\nu p}(x,Q^2)\,,\\
F_3^{\nu n}(x,Q^2) &= (\delta(1-x) + a_s(Q^2)\cdot C_3^1(x)) \otimes q^{\nu n}(x,Q^2)\,,\\
F_3^{\bar\nu n}(x,Q^2) &= (\delta(1-x) + a_s(Q^2)\cdot C_3^1(x)) \otimes q^{\bar\nu n}(x,Q^2)\,,
\end{aligned} \tag{3}$$

where $a_s(Q^2)) = \alpha_s(Q^2)/4\pi$ and the splitting function reads as

$$C_3^1(x) = \frac{4}{3}\left\{4\left[\frac{\ln(1-x)}{1-x}\right]_+ - 3\left[\frac{1}{1-x}\right]_+ - (9+\frac{2\pi^2}{3})\cdot\delta(1-x)\right\} - \frac{4}{3}\left\{2(1+x)\cdot\ln\frac{1-x}{x} + 4\cdot\frac{\ln x}{1-x} - 4 - 2x\right\}.$$

The convolution of functions means

$$a(x) \otimes f(x) = \int\limits_x^1 a(x/y) \cdot \frac{f(y)}{y} dy\,,\; a(x)_+ \otimes f(x) = \int\limits_x^1 a(x/y) \cdot \left[f(y) - \frac{x}{y} \cdot f(x) \right] \cdot \frac{dy}{y} - f(x) \cdot \int\limits_0^x C(y) dy.$$

As target mass corrections most essential at small $Q^2 < 1 \div 2$ GeV^2, we consider only the light quarks:

$$q^{\nu p}(x,Q^2) = 2 \cdot \left\{ d(x,Q^2) \cdot \cos^2\theta_c - \bar{u}(x,Q^2) + s(x,Q^2) \cdot \sin^2\theta_c \right\},$$
$$q^{\bar{\nu} p}(x,Q^2) = 2 \cdot \left\{ -\bar{d}(x,Q^2) \cdot \cos^2\theta_c + u(x,Q^2) - s(x,Q^2) \cdot \sin^2\theta_c \right\},$$
$$q^{\nu n}(x,Q^2) = 2 \cdot \left\{ u(x,Q^2) \cdot \cos^2\theta_c - \bar{d}(x,Q^2) + \bar{s}(x,Q^2) \cdot \sin^2\theta_c \right\},$$
$$q^{\bar{\nu} n}(x,Q^2) = 2 \cdot \left\{ -u(x,Q^2) \cdot \cos^2\theta_c + d(x,Q^2) - \bar{s}(x,Q^2) \cdot \sin^2\theta_c \right\},$$

where $u, d, s \,/\, \bar{u}, \bar{d}, \bar{s}$ are quark/anti-quark distributions, and θ_c is the usual Cabibbo angle. In our calculations we use quark/anti-quark distributions from Ref. [15], where was also fixed NLO value of the QCD parameter $\Lambda_{\rm QCD} = 0.248$ GeV. Note we have tested quark distributions given by other groups, e.g., the MRST/MSTW [16] and have found that no significant difference in behaviour in the region of $x > 0.2$ for which become essential the target mass corrections.

3 New approach

The method suggested by Solovtsov [7] is based on the JLD representation and gives a new scaling variable, $s = x\sqrt{(1+4\varepsilon)/(1+4\varepsilon x^2)}$, the moments on which become analytical functions. It has been shown, that for any physical structure function from the JLD representation follows

$$W(x,Q^2) = \int_0^1 d\beta\, \theta[f(\beta;x,\varepsilon)]\, H_0(\beta)\,, \tag{4}$$

where $H_0(\beta) = -dF(\beta)/d\beta$, $f(\beta;x,\varepsilon) = \dfrac{\beta}{s}\sqrt{1+4\varepsilon} - 1 - 2\varepsilon(1 - \sqrt{1-\beta^2})$. One can find the roots of the equation $f(\beta;x,\varepsilon) = 0$. If $x > \tilde{x} \equiv 1/\sqrt{1+4\varepsilon^2}$, there are the two roots

$$\beta_\pm = \frac{x\sqrt{1+4\varepsilon x^2}}{1+4\varepsilon x^2 + 4\varepsilon^2 x^2} \times \left(1 + 2\varepsilon \pm 2\varepsilon \sqrt{\frac{1-x^2}{1+4\varepsilon x^2}} \right),$$

if $x < \tilde{x}$, then there is one root β_-.

According to the method the function $W(x,Q^2)$ for any physical structure function is expressed via the corresponding parton distribution $F(x)$ as follows

$$W(x,Q^2) = \begin{cases} F(\beta_-) - F(1)\,, & 0 \le x < \tilde{x}\,, \\ F(\beta_-) - F(\beta_+)\,, & \tilde{x} \le x \le 1\,, \end{cases} \tag{5}$$

where $\tilde{x} = 1/(\sqrt{1+4\varepsilon^2})$. The spectral property of the function $W(x,Q^2)$ (that it vanishes as $x \to 1$) and its continuity for $x = \tilde{x}$ follow because $\beta_-(x=1) = \beta_+(x=1)$ and $\beta_+(\tilde{x}) = 1$.

The results of calculations for the average neutrino and anti-neutrino structure function $xF_3^p(x,Q^2) = [F_3^{\nu p}(x,Q^2) + F_3^{\bar{\nu} p}(x,Q^2)])/2$ are presented in Figs. 2 and 3 which show the behavior of the proton structure function xF_3^p as a function of x for $Q^2 = 1$ GeV^2 and $Q^2 = 3$ GeV^2. The solid line corresponds to our result obtained by using the Solovtsov's approach, the dashed curve reflects the result obtained by standard GP method [2], the dotted line corresponds to the behavior without target mass corrections [15]. It should be mentioned that the integral of over all x of the $F_3^p(x,Q^2)$ structure function gives the Gross–Llewellyn Smith sum [17]. As may be seen from Fig. 2 for this sum rule, the target mass corrections will give a noticeable contribution.

4 Conclusion

We have reported a new result of including the target mass corrections to the structure function F_3 in neutrino nucleon deep inelastic scattering. We observed that at low $Q^2 \sim 1 \div 2$ GeV^2 the target mass corrections to structure function F_3 calculated by using new method (S-method) noticeably differ from the standard Georgi-Polizer method result. It is necessary to emphasize that the inclusion of target mass corrections in the fits of deep-inelastic scattering data is important [18, 19] as reduces everywhere the magnitude of the HT terms needed to describe the data [20]. We believe that the new method including target mass effects will be useful in extracting the magnitude of the structure functions from the experimental data.

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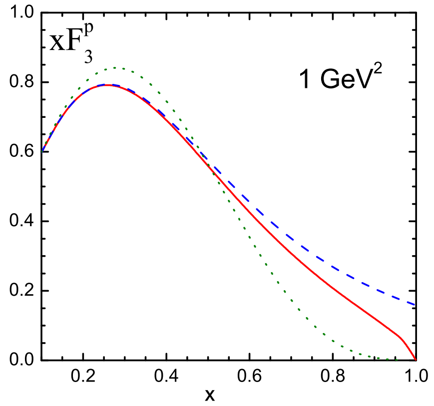

Figure 2. The function xF_3^p vs x at Q^2=1 GeV^2.

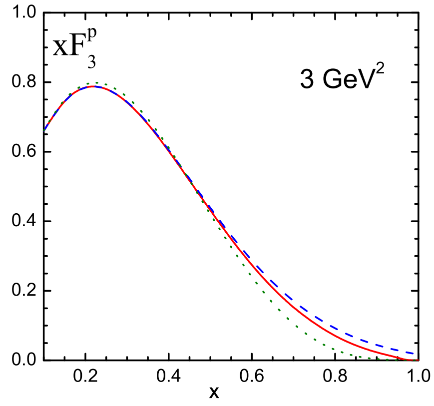

Figure 3. The function xF_3^p vs x at Q^2=3 GeV^2.

References

[1] V.I. Lashkevich and O.P. Solovtsova, Nonlin. Phenom. Complex Syst. **14**, 184 (2011); *Proc. of XVIII Annual Seminar "NPCS'2011"*, 230 (2011).

[2] H. Georgi and H.D. Politser, Phys. Rev. D **14**, 1829 (1976).

[3] O. Nachtmann, Nucl. Phys. B **63**, 237 (1973).

[4] W. Melnitchouk, R. Ent, and C. Keppel, Phys. Rep. **406**, 127 (2005); I. Schienbein *et al.*, J. Phys. G. **35**, 053101 (2008).

[5] V.I. Lashkevich and O.P. Solovtsova, Teor. Mat. Fiz. **160**, 471 (2009); Nonlin. Phenom. Complex Syst. **13**, 286 (2010).

[6] A. Accardi *et al.*, Phys. Rev. D **81**, 034016 (2010).

[7] I.L. Solovtsov, Part. Nucl. Lett. **101**, 10 (2000).

[8] D.V. Shirkov and I.L. Solovtsov, Phys. Rev. Lett. **79**, 1209 (1997).

[9] R. Jost and H. Lehmann, Nuov. Cim. **5**, 1598 (1957).

[10] F.J. Dyson, Phys. Rev. **110**, 1460 (1958).

[11] J. Beringer *et al.*, Particle Data Group, Phys. Rev. D **86**, 010001 (2012).

[12] A.L. Kataev, G. Parente, and A.V. Sidorov, Phys. Part. Nucl. **34**, 43 (2003).

[13] A. De Rújula, H. Georgi, and H.D. Politzer, Annals of Phys. **103**, 315 (1977).

[14] S. Kretzer and M.H. Reno, Phys. Rev. D **69**, 034002 (2004).

[15] M. Glück, E. Reya, and A. Vong, Z. Phys. C **7**, 433 (1995); Eur. Phys. J. C **5**, 461 (1998).

[16] A.D. Martin, R.G. Roberts, W.J. Stirling, and R.S. Thorne, Eur. Phys. J. C **16**, 117 (2000); A.D. Martin, W.J. Stirling, R.S. Thorne, and G. Watt, Eur. Phys. J. C **63**, 189 (2009).

[17] D.J Gross and C.H. Llewellyn Smith, Nucl. Phys. B **14**, 337 (1969).

[18] M. Virchaux and A. Milsztajn, Phys. Lett. B **274**, 221 (1992).

[19] S. Alekhin, J. Blümlein, S. Klein, and S. Moch, Phys. Rev. D **81**, 014032 (2009).

[20] J. Blümlein, Prog. Part. Nucl. Phys. **69**, 28 (2013).