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$\pi^0 \to \gamma\gamma$ DECAY IN POINT FORM OF POINCARE-INVARIANT QUANTUM MECHANICS

Introduction

The precision of modern experimental data has renewed interest in studying the mechanism of interaction of quarks within hadrons. Mesons of the light sector, consisting of light u- and d-quarks are of interest. Among the variety of approaches and models devoted to the description of various characteristics of bound quark-antiquark states, note the models based on the Poincare group. It is known that such models are relativistic [1], which makes their use for describing the characteristics of light sector mesons appropriate.

In the work authors demonstrate the procedure of calculation form-factor of pseudoscalar mesons to photon pair decay $P(q\overline{q}) \to \gamma\gamma$. It's shown that quark annihilation mechanism leads to the simple expression for decay form-factor integral representation. As a result, numerical studying neutral π^0-meson decay constant consisted of light quarks in point form of Poincare-invariant quantum mechanics (further PiQM) is conducted. Obtained relations used for estimation constituent quark masses and $\beta(q\overline{q})$-parameters dependence with oscillator wave function.

1. Basic futures of the model, based on point form of PiQM

The basis of the two-particle irreducible representation is defined by the quantum numbers of the total momentum of the total angular momentum J with a projection μ, effective mass of noninteracting particles [1]

$$M_0 = M(q\bar{Q}) = \omega_{m_q}(\mathrm{k}) + \omega_{m_{\bar{Q}}}(\mathrm{k}),\ \omega_m(\mathrm{k}) = \sqrt{\mathrm{k}^2 + m^2}\,, \tag{1}$$

where $\mathrm{k} = |\mathbf{k}|$, and two additional numbers that remove the degeneracy of this basis. As a result, state vector of a meson with momentum $\mathbf{Q}$, mass M is given by [2]

$$\left|\mathbf{Q}, J\mu, M(q\bar{Q})\right\rangle = \int \mathrm{d}\mathbf{k} \sqrt{\frac{\omega_{m_q}(\mathrm{p}_1)\,\omega_{m_{\bar{Q}}}(\mathrm{p}_2)}{\omega_{m_q}(\mathrm{k})\,\omega_{m_{\bar{Q}}}(\mathrm{k}) V_0}} \sum_{\lambda_1,\lambda_2} \sum_{\nu_1,\nu_2} \Omega \begin{Bmatrix} \ell & s & J \\ \nu_1 & \nu_2 & \mu \end{Bmatrix} (\theta_k, \phi_k)\, \Phi^J_{\ell s}\left(\mathrm{k}, \beta_{q\bar{Q}}\right) \times$$

$$\times D^{1/2}_{\lambda_1,\nu_1}\left(\mathbf{n}_{W_1}\right) D^{1/2}_{\lambda_2,\nu_2}\left(\mathbf{n}_{W_2}\right) \left|\mathbf{p}_1, \lambda_1, \mathbf{p}_2, \lambda_2\right\rangle. \tag{2}$$

Wave function $\Phi^J_{\ell s}(\mathrm{k})$ taking into account the number of colors of quarks N_c is normalized by the following condition:

$$\sum_{\ell,s} \int_0^\infty \mathrm{dk}\ \mathrm{k}^2 \left|\Phi^J_{\ell s}\left(\mathrm{k}, \beta_{q\bar{Q}}\right)\right|^2 = N_c. \tag{3}$$

2. $P(q\bar{q}) \to \gamma\gamma$ decay in point form of PiQM

Parametrization of pseudoscalar meson decay $P(q\bar{q}) \to \gamma\gamma$ is given by [3]

$$\langle \gamma\gamma \,|\, \hat{J} \,|\, P(q\bar{Q}) \rangle = \frac{i e^2}{(2\pi)^3} \frac{g_P(q_1^2, q_2^2)}{\sqrt{2\,\omega_{M_P}(Q)}} \mathsf{T}_{\mu\nu\rho\sigma}\, q_1^\mu\, q_2^\nu\, \varepsilon^{*\rho}(\lambda_1) \varepsilon^{*\sigma}(\lambda_2), \tag{4}$$

where $q_{1,2}$ – momentums of outer photons and $g_P(q_1^2, q_2^2)$ – pseudoscalar meson decay constant. In the rest system of meson equation (4) could be written as

$$\langle \gamma\gamma \,|\, \hat{J} \,|\, P(q\bar{q}) \rangle = \frac{i e^2}{(2\pi)^3} \frac{g_P(q_1^2, q_2^2)}{\sqrt{2 M_P}} \mathsf{T}_{\mu\nu\rho\sigma}\, q_1^\mu\, q_2^\nu\, \varepsilon^{*\rho}(\lambda_1) \varepsilon^{*\sigma}(\lambda_2). \tag{5}$$

Further we will examine the case of real photons emission by pseudo-scalar meson, so decay kinematics in the rest system can be written as

$$q_1^\mu = \{\frac{M_P}{2},0,0,\frac{M_P}{2}\}, q_2^\mu = \{\frac{M_P}{2},0,0,-\frac{M_P}{2}\}. \tag{6}$$

Since photons 4-vectors polarizations are limited by the conditions $(\varepsilon^* \cdot \varepsilon) = -1$ and $(q \cdot \varepsilon) = 0$ one can get

$$\varepsilon^\alpha(\lambda_1) = \{0,\frac{\lambda_1^2}{\sqrt{2}},i\frac{\lambda_1}{\sqrt{2}},0\},\ \varepsilon^\alpha(\lambda_2) = \{0,\frac{\lambda_2^2}{\sqrt{2}},i\frac{\lambda_2}{\sqrt{2}},0\}. \tag{7}$$

In our approach we consider following quark-antiquark annihilation mechanism decay (figure 1).

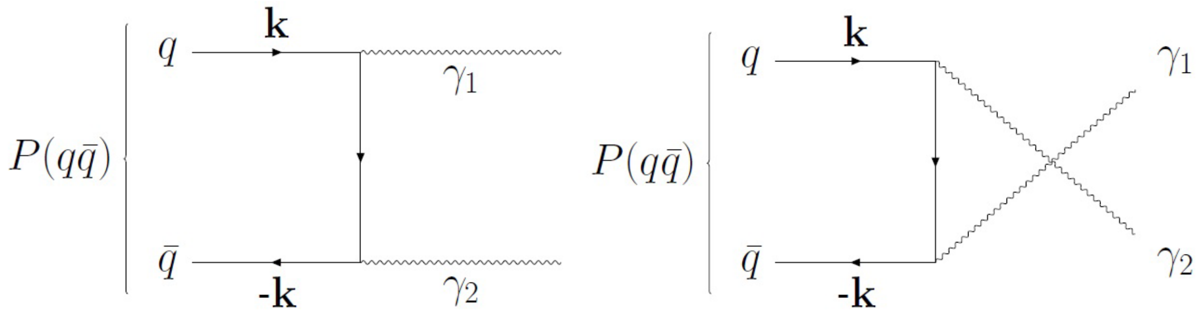

Figure 1 – quark-antiquark annihilation $P(q\bar{q}) \to \gamma\gamma$ decay mechanism

Matrix elements corresponding to diagrams

$$\frac{1}{(2\pi)^3}\frac{\bar{\upsilon}_{\nu_2}(-\mathbf{k},m_q)}{\sqrt{2\omega_{m_q}(\mathrm{k})}}\left(M_I + M_{II}\right)\frac{u_{\nu_1}(\mathbf{k},m_q)}{\sqrt{2\omega_{m_q}(\mathrm{k})}}, \tag{8}$$

where

$$M_I = e_q^2\gamma_\mu\varepsilon^{*\mu}(q_2)\frac{\hat{k}-\hat{q}_2+m_q}{(k-q_2)^2-m_q^2}\gamma_\nu\varepsilon^{*\nu}(q_1) \tag{9}$$

and

$$M_{II} = e_q^2\gamma_\mu\varepsilon^{*\mu}(q_1)\frac{\hat{k}-\hat{q}_1+m_q}{(k-q_1)^2-m_q^2}\gamma_\nu\varepsilon^{*\nu}(q_2). \tag{10}$$

After spinor part calculation and integration over solid angle of relation (8) from (5) one can get integral representation of pseudoscalar meson decay:

$$g_P(q_1^2=0,q_2^2=0)=\int \mathrm{dk}\ \ \mathrm{k}^2\ \ \Phi(\mathrm{k},\beta_{q\bar{q}})\,e_q^2\ \ \frac{2\sqrt{2\pi}\,m_q}{\omega_{m_q}^{5/2}(\mathrm{k})\mathrm{k}}\ln\left(\frac{\omega_{m_q}(\mathrm{k})+\mathrm{k}}{\omega_{m_q}(\mathrm{k})-\mathrm{k}}\right). \quad (11)$$

Obtained relation (11) correspond with results of work [4], based on relativistic quark model with quasipotential approach.

3. Numerical calculations and results

Using oscillator wave functions

$$\Phi(\mathrm{k},\beta_{q\bar{q}})=\frac{2}{\pi^{1/4}\left(\beta_{q\bar{q}}\right)^{3/2}}\exp\left[-\frac{\mathrm{k}^2}{2\left(\beta_{q\bar{q}}\right)^2}\right] \quad (12)$$

and π^0 – meson quark structure

$$\pi^0=\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d}) \quad (13)$$

from the experimental value π^0-meson decay constant [5]

$$\Gamma=\frac{\pi}{4}\alpha_{\mathrm{qed}}^2\left|g_{\pi^0\gamma\gamma}^{(\exp.)}\right|^2 M_P^3,\quad g_{\pi^0\gamma\gamma}^{\exp.}=0{,}272\pm0{,}002\,GeV^{-1} \quad (14)$$

with equality assumption of m_u and m_d constituent quark masses one can get following dependence between constituent quark $m_{u,d}$ masses and $\beta_{u\bar{u}}/\beta_{d\bar{d}}$ – parameters for $m_{u,d}\in[0{,}2;\ 0{,}33]$ GeV [6, 7] (see figure 2).

Conclusion and remarks

In the course of work authors obtained integral representation of radiative decay constant for pseudoscalar meson decay $P(q\bar{q})\to\gamma\gamma$ using quark annihilation mechanism. Obtained results correlates with calculations in other models, which confirms the reliability of the proposed model.

As a result, a numerical study of the model parameters dependence in point form of PiQM was carried out.

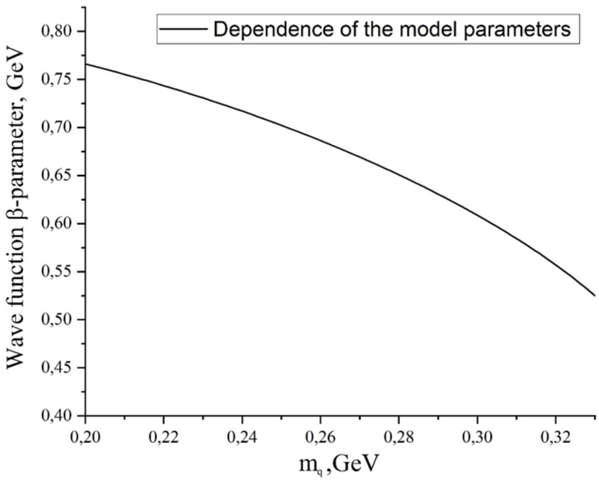

Figure 2 – Numerical calculation of dependence between constituent quark masses and $\beta_{u\bar{u}}$ / $\beta_{d\bar{d}}$ -parameters

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