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Manifestation of Quark-Hadron Duality in e^+e^- Annihilation into Hadrons

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The Adler D-function which appears in the process of the e^+e^- annihilation into hadrons is of interest from the point of view of quark-hadron duality, as this function turned out to be a smooth function with no traces of a resonance structure which is observed for the function $R(s)$, the normalized cross-section for the process of e^+e^- annihilation into hadrons. We consider various physical quantities and functions generated by $R(s)$ and obtain good agreement between our results and experimental data down to the lowest energy scale. We found that the reason of such consent is a consequence of quark-hadron duality, which connects the $R(s)$ and the D-function.

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1. Introduction

To compare theoretical results and experimental data one often uses the concept of quark-hadron duality which establishes a bridge between quarks and gluons, a language of theoreticians, and real measurements with hadrons performed by experimentalists. The idea of quark-hadron duality was formulated in the paper of Poggio, Quinn, and Weinberg [1] as follows: Inclusive hadronic cross sections, once they are appropriately averaged over an energy interval, must approximately coincide with the corresponding quantities derived from the quark-gluon picture.

To check direct consequences of the theory without using model assumptions, it is important to connect measured quantities with the "simplest" theoretical objects. Some single-argument functions, which include the Adler D-function [2], directly related to the experimentally measured quantities can play the role of these objects. Let us remind that the cross-section for the process of e^+e^- annihilation into hadrons, or the Drell ratio $R(s)$, is defined for the timelike momentum transfer, and at low energy scale has a resonance structure which is difficult to describe without model considerations.

In this report, we concentrate on the D-function, corresponding to the e^+e^- annihilation and quantities which are expressed through the Drell ratio $R(s)$ integrated with some known function: the hadronic contribution to the anomalous magnetic moment of the lepton (in the leading order in electromagnetic coupling constant)

$$a_l^{\rm had} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int\limits_0^\infty \frac{ds}{s}\, K_l(s)\, R(s), \qquad (1)$$

where $l = \mu,\ e,\ \tau$, and $K_l(s)$ is a known QED kernel function, the strong interaction contribution to the running fine structure constant at M_Z, and so on. By definition, all these quantities include the infrared region as a part of the interval of integration and, therefore, they cannot be directly calculated within perturbative QCD. We use the approach which is based on the nonperturbative expansion method called the variational perturbation theory (VPT) [3, 4]. In the case of QCD the VPT approach leads to a new small expansion parameter. Even going into the infrared region of small momenta, where

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the perturbative coupling becomes large, the nonperturbative expansion parameter remains small and the approach holds valid.

2. Interrelation of $R(s)$ and $D(Q^2)$

The Adler D-function is determined in the Euclidean region (for a spacelike momentum transfer), $Q^2 = -q^2 > 0$, and is a smooth function with no traces of a resonance structure. Figure 1 shows the D-function behavior. The solid curve corresponds the VPT result for five active quarks. The experimental curve is taken from Ref. [5].

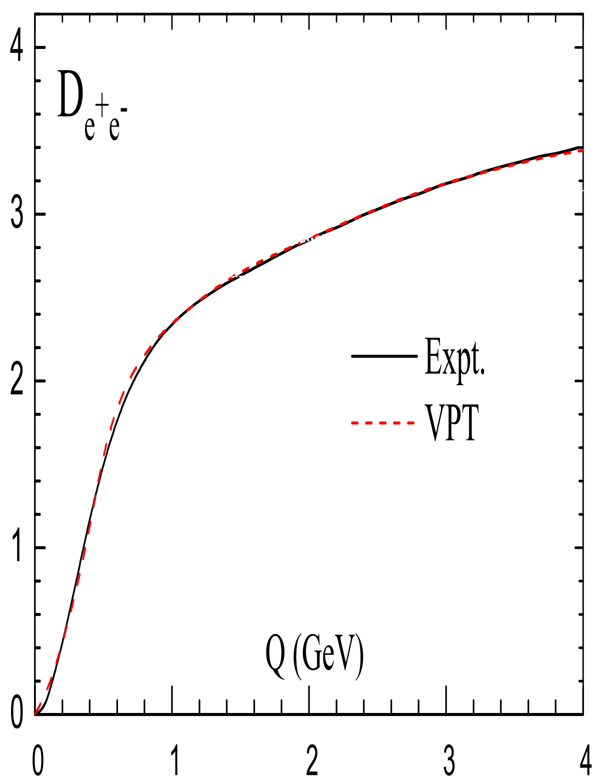

FIG. 1. The Adler function for e^+e^- annihilation into hadrons.

By using the dispersion relation it is possible to express the Adler function through the $R(s)$ and to see that the D-function has to be an analytic function in the complex Q^2-plane with a cut along the negative real axis. Analyzing various physical quantities and functions generated by

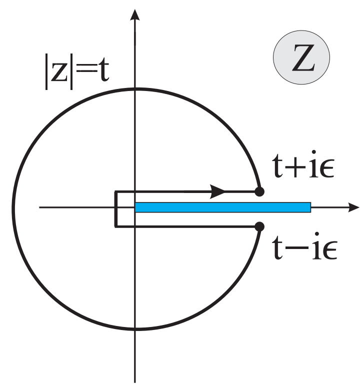

FIG. 2: The integration contour in Eq. (4).

$R(s)$ within the nonperturbative VPT-method we obtained that the method maintains necessary analytic properties and allows us to describe these quantities well down to a low energy scale [6]. It should be noted that the method based on the analytic perturbation theory in QCD [7] also preserves the correct D-function analytic properties and leads to very close to VPT results [8].

Let us investigate the question: can the expression for some quantity, say for the anomalous magnetic moment of the lepton (1), be presented equivalently through the $D(Q^2)$ and $R(s)$ functions? We reformulate this question as the criterion of the R-D self-duality. We write down some quantity, Q, in two representations:

$$Q_M = \int_0^\infty \frac{ds}{s}\, M(s) R(s) \tag{2}$$

for the Minkowskian (timelike) region,

$$Q_E = \int_0^\infty \frac{dt}{t}\, E(t) D(t) \tag{3}$$

for the Euclidean (spacelike) region,

and investigate the equality condition: $Q_M = Q_E$.

The answer to the question gives the interrelation between the kernels

$$M(s) = s \int_0^\infty dt \frac{E(t)}{(s+t)^2}\,,$$
$$E(t) = -\frac{1}{2\pi \mathrm{i}} \int_{t-\mathrm{i}\,\epsilon}^{t+\mathrm{i}\,\epsilon} \frac{d\,z}{z}\, M(-z) \tag{4}$$

where the integration contour lies in the region of analyticity of the integrand and encircles the cut of $M(-z)$ on the positive real z axis (see Fig. 2).

As an example, we apply this result to the hadronic correction to the muon (1), $l \equiv \mu$, and get R-D self-duality expressions:

$$a_\mu^{\mathrm{had}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{ds}{s}\, M(s)\, R(s)$$

for the Minkowskian (timelike) region,

$$a_\mu^{\mathrm{had}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{dt}{t}\, E(t) D(t) \tag{5}$$

for the Euclidean (spacelike) region

where $M(s) = \eta^2\left(1-\frac{\eta^2}{2}\right) + (1+\eta)^2\left(1+\frac{1}{\eta^2}\right)$

$$\times \left[\ln(1+\eta) - \eta + \frac{\eta^2}{2}\right] + \frac{1+\eta}{1-\eta}\eta^2 \ln\eta,$$

$$E(t) = \frac{1}{2}\left[\frac{\sqrt{1+4m_\mu^2/t}-1}{\sqrt{1+4m_\mu^2/t}+1}\right]^2,$$

$$\eta = \frac{1-v}{1+v},\ \ v = \sqrt{1-\frac{4m_\mu^2}{s}}\,.$$

3. Summary

Considering various physical quantities and functions generated by the function $R(s)$, we obtained a good agreement between the VPT result and experimental data. We investigated the reason of such an agreement and found that these quantities can be described in terms of the Adler function, which is well described within the VPT method.

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References

[1] E.C. Poggio, H.R. Quinn, S. Weinberg. Phys. Rev. D **13**, 1958 (1976).

[2] S.L. Adler. Phys. Rev. D **10**, 3714 (1974).

[3] I.L. Solovtsov. Phys. Lett. B **327**, 335 (1994); **340**, 245 (1994).

[4] A.N. Sissakian, I.L. Solovtsov. Phys. Part. Nucl. **30**, 461 (1999).

[5] F. Jegerlehner. Nucl. Phys. Proc. Suppl. **181**–**182**, 135 (2008).

[6] O.P. Solovtsova. Phys. Atom. Nucl. **76**, 1295 (2013).

[7] D.V. Shirkov, I.L. Solovtsov. Phys. Rev. Lett. **79**, 1209 (1997).

[8] K.A. Milton, I.L. Solovtsov, O.P. Solovtsova. Mod. Phys. Lett. A **21**, 1355 (2006).