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Pionic Decay of $\rho^{\pm}$–Meson in Relativistic Quantum Mechanics

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The integral representation of $\rho^{\pm} \to \pi^{\pm}\pi^{0}$ decay constant has been obtained in relativistic quantum mechanics based on point form of dynamics. It is shown that soft pion theorem usage in the proposed approach leads to numerical results consistent with modern experimental data. As a result a self-consistent model for light $\pi^{\pm}$–,$\rho^{\pm}$–meson observed electroweak and pionic characteristic calculation is proposed.

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1. Introduction

The research of composite quark systems is directly related to the study of hadronic properties such as meson form-factors, decay widths etc. Light sector mesons are of particular interest: to date a sufficient amount of high-precision experimental data has been accumulated [1] including for pseudoscalar $\pi^{\pm}$– and vector $\rho^{\pm}$–meson decays. ItвЂ™s known that such systems are purely relativistic, which makes it possible to test the corresponding phenomenological approaches and models for studying the properties of coupled ud–systems.

It should be noted that decay $\pi^{\pm} \to \ell^{\pm}\nu_{\ell}$ and the corresponding constant $f_{\pi^{\pm}}$ which is measured with a high accuracy degree has a special place. The specified decay constant in high energy physics has become a fundamental quantity with the help of which not only parameters of various models, but also the form-factors of other particles [2–5], including heavy mesons and nucleon-meson interaction constant [6, 7] are calculated. There is also a known technique for calculating the value of current quarks masses using value $f_{\pi^{\pm}}$ and pseudoscalar density constant [8].

Among the variety of mesonic observable quantities calculation approaches we emphasize methods based on the Poincaré group [9, 10]. Of the three forms light-front dynamic [11, 12] are the most common for such studies. Thus in works [13–15] the form-factors of light and heavy mesons of spin 1 were studied. In addition to the study of $V \to P\gamma$ transitions, the decay $\rho^{\pm} \to \pi^{\pm}\pi^{0}$ constant $g_{\rho^{\pm}\pi^{\pm}\pi^{0}}$ was calculated using the hypothesis of partial axial current conservation [5].

Note that while electroweak decays of light mesons are extensively studied in all forms of Poincaré-invariant quantum mechanics (further PiQM), the hadronic transition is analyzed separately using phenomenological QCD-motivated potentials [16, 17] or within the framework of $^{3}P_{0}$–model [18, 19], based on the creation of a quark-antiquark pair from vacuum. Generally speaking, usage relativistic QCD-motivated potentials explicitly, including calculating mesonic form-factors, is a non-trivial task [20], since such potentials depend on a significant number of parameters. Thus, in

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the work [19] it was shown that the potential relativization procedure leads to decay width $\Gamma(\rho \to \pi\pi)$ value that differs by almost a factor of two. In this regard the question of determining the basic parameters without using the explicit form of the QCD-potential is being actively studied. So in work [21] based on the light-front formalism, the root-mean-square radius of mesons was used, $f_{\pi^{\pm}}$ and f_{ρ^0} decay constants values were used in the work [8]. It follows from the above that determinations parameters methods development in models based on PiQM is an important task.

The work is devoted to the study of $\pi^{\pm}$–,$\rho^{\pm}$–meson form-factors in the point form of dynamics without explicit form QCD-motivated potential usage. Despite S–matrix equivalent for three forms of relativistic dynamics [22], studies in this form of dynamics are less extended. Below we will not discuss the features of each of the three forms of dynamics: all the advantages and disadvantages are described in [9, 10]. We only note that due to the equality of 4–velocities $\mathbb{V}^{\mu} = \{\mathbb{V}^0, \vec{\mathbb{V}}\}$ with and without interaction in this form makes it the most natural for describing bound relativistic systems.

The article has the following structure: section 2 presents the basic relations of the constituent quark model based on the point form of the PiQM. According to these expressions integral representations of pseudoscalar $P^{\pm}(q\bar{Q}) \to \ell^{\pm}\nu_{\ell}$ and vector $V^{\pm}(q\bar{Q}) \to \ell^{\pm}\nu_{\ell}$ meson decay constant, as well as the pseudoscalar density constant $g_{\pi^{\pm}}$ were calculated.

In section 3 the technique for calculating observables is generalized to the case of hadronic transition $V^{\pm} \to P^{\pm}\pi^0$. It is shown that equality of 4–velocities $\mathbb{V}^{\mu}$ with and without interaction leads to relatively simple expressions for the integral representation of the constant $g_{V^{\pm}P^{\pm}\pi^0}$. As a result of the work in section 4 numerical calculations were carried out using the expressions obtained in sec. 2, 3. It was shown that usage integral representations $f_{\pi^{\pm}}$ and pseudoscalar density constants $g_{\pi^{\pm}}$ leads to results that correlate with other approaches and models. The obtained quark constituent masses values used for decay constant $g_{\rho^{\pm}\pi^{\pm}\pi^0}$ calculation using partial conservation of the axial current: the comparison showed that the proposed model predicts results consistent with modern experimental data.

2. Basic features of the model

The main relations of the model have been defined below. State vector of pseudoscalar $J^{PC} = 0^{-+}$ and vector $J^{PC} = 1^{--}$ meson with spin $J = \ell + S$ ($|\ell - S| \leq J \leq |\ell + S|$, $S = 0$ for antiparallel quark spins and $S = 1$ for parallel quark spins, $P = (-1)^{\ell+1}$ is parity and $C = (-1)^{\ell+S}$ is charge conjugation of $q\bar{Q}$-state [1]), mass M and 4-momentum Q^{μ} ($Q^2 = M^2$, $\mathbb{V}^{\mu} = Q^{\mu}/M$) in the point form of dynamics is the basic relation [9]

$$
\begin{aligned}
|\mathbf{Q}, J\mu, M\rangle = &\sum_{\lambda_1,\lambda_2}\sum_{\nu_1,\nu_2}\int \mathrm{d}\mathbf{k}\ \Phi^J_{0S}(\mathbf{k}) \\
&\times\sqrt{\frac{\omega_{m_q}(\mathrm{p}_1)\,\omega_{m_{\bar{Q}}}(\mathrm{p}_2)}{\omega_{m_q}(\mathrm{k})\,\omega_{m_{\bar{Q}}}(\mathrm{k})\,\mathbb{V}_0}}\Omega\left\{{}^{0SJ}{}_{\nu_1,\nu_2,\mu}\right\}(\theta_{\mathrm{k}},\phi_{\mathrm{k}}) \\
&\times D^{1/2}_{\lambda_1,\nu_1}(\mathbf{n}_{W_1})\ D^{1/2}_{\lambda_2,\nu_2}(\mathbf{n}_{W_2})\ |\mathbf{p_1},\lambda_1,\mathbf{p_2},\lambda_2\rangle\ ,
\end{aligned}
\tag{1}
$$

determined through relative momentum $\mathbf{k}$ of quarks with momenta $\mathbf{p}_1$, $\mathbf{p}_2$, masses m_q, $m_{\bar{Q}}$ and with spin projections λ_1, λ_2 respectively. In expression (1) value $\omega_m(\mathrm{k}) = \sqrt{\mathrm{k}^2 + m^2}$ has been defined with $\mathrm{k} = |\mathbf{k}|$ and auxiliary function

$$
\begin{aligned}
\Omega\left\{{}^{\ell\ S\ J}_{\nu_1,\nu_2,\mu}\right\}(\theta_{\mathrm{k}},\phi_{\mathrm{k}}) = Y_{\ell m}(\theta_{\mathrm{k}},\phi_{\mathrm{k}})\mathrm{C}\begin{pmatrix} s_1 & s_2 & S \\ \nu_1 & \nu_2 & \lambda\end{pmatrix} \\
\times\,\mathrm{C}\left(\begin{smallmatrix} \ell & S & J \\ m & \lambda & \mu\end{smallmatrix}\right),
\end{aligned}
$$

with Clebsh-Gordan coefficients C of $SU(2)$ group and spherical harmonic functions $Y_{\ell m}(\theta_{\mathrm{k}},\phi_{\mathrm{k}})$ of $\mathbf{k}$-vector angles has been introduced. Wigner rotation [9] functions $D^{1/2}_{\lambda,\nu}(\mathbf{n}_W)$ in (1)

$$
D^{1/2}_{\lambda,\nu}(\mathbf{n}_W) = \frac{I - i\,(\mathbf{n}_W \cdot \boldsymbol{\sigma})}{\sqrt{1 + \mathbf{n}^2_W}}
\tag{2}
$$

are defined using Pauli matrices $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$, and rotation vector-parameter $\mathbf{n}_W$ is defined as

$$\mathbf{n}_W = \frac{\mathbf{u_k} \times \mathbf{u_Q}}{1 + (\mathbf{u_k} \cdot \mathbf{u_Q})}, \; \mathbf{u_P} = \frac{\mathbf{P}}{\omega_M(\mathrm{P}) + M}. \quad (3)$$

Meson wave function in (1) is subject of the normalization condition (4)

$$\sum_{\ell,S} \int_0^\infty \mathrm{dk}\,\mathrm{k}^2 \left|\Phi^J_{\ell S}(\mathrm{k})\right|^2 = 1. \quad (4)$$

Note, that for pseudoscalar $J^{PC} = 0^{-+}$ and vector $J^{PC} = 1^{--}$ mesons of light sector wave function will be defined as [13–15]

$$\Phi^0_{00}(\mathrm{k}) = \Phi^1_{01}(\mathrm{k}) = \Phi(\mathrm{k}) =$$
$$= \frac{2}{\pi^{1/4}\left(\beta^I_{q\bar{Q}}\right)^{3/2}} \exp\left[-\frac{\mathrm{k}^2}{2\left(\beta^I_{q\bar{Q}}\right)^2}\right], \quad (5)$$

where $\beta^I_{q\bar{Q}}$ $(I = V, P)$ is parameters of the wave functions [8, 23, 24].

Using expressions

$$\langle 0 \left| \hat{J}^\mu \right| \mathbf{Q}, M_P \rangle = i\frac{1}{(2\pi)^{3/2}} \frac{Q^\mu}{\sqrt{2\,\mathbb{V}_0\, M_P}} f_P, \quad (6)$$

for pseudoscalar $I = P$ and

$$\langle 0 \left| \hat{J}^\mu \right| \mathbf{Q}, \lambda_V, M_V \rangle = i\frac{1}{(2\pi)^{3/2}} \frac{\varepsilon^\mu(\lambda_V) M_V}{\sqrt{2\,\mathbb{V}_0\, M_V}} f_V \quad (7)$$

vector $I = V$ meson in its rest frame correspondingly with electroweak (ew.) quark current

$$\langle 0 | \hat{J}^\mu_{\text{ew.}} | \mathbf{k}, \lambda_1, -\mathbf{k}, \lambda_2 \rangle = \frac{\bar{\upsilon}_{\lambda_2}(-\mathbf{k}, m_{\bar{Q}}) \Gamma^\mu_{\text{ew.}} u_{\lambda_1}(\mathbf{k}, m_q)}{(2\pi)^3 \sqrt{2\,\omega_{m_q}(\mathrm{k})\, 2\,\omega_{m_{\bar{Q}}}(\mathrm{k})}} \quad (8)$$

after calculating the spinor part and integrating over the solid $\mathbf{k}$-vector angle one can obtain an integral representation of the decay constants:

$$f_I(m_q, m_{\bar{Q}}) = \sqrt{\frac{3}{2}} \frac{1}{\pi} \int_0^\infty \mathrm{dk}\;\mathrm{k}^2\; \Phi(\mathrm{k}) \sqrt{\frac{W^+_{m_q}(\mathrm{k})\, W^+_{m_{\bar{Q}}}(\mathrm{k})}{M_0(\mathrm{k})\, \omega_{m_q}(\mathrm{k})\, \omega_{m_{\bar{Q}}}(\mathrm{k})}} \left(1 + a_I \frac{\mathrm{k}^2}{W^+_{m_q}(\mathrm{k}) W^+_{m_{\bar{Q}}}(\mathrm{k})}\right), \quad (9)$$

where $a_P = -1$, $a_V = 1/3$. In expression (9) the following notations were used

$$W^\pm_m(\mathrm{k}) = \omega_m(\mathrm{k}) \pm m, \quad (10)$$
$$M_0(\mathrm{k}) = \omega_{m_q}(\mathrm{k}) + \omega_{m_{\bar{Q}}}(\mathrm{k}).$$

In our approach determination of constituent quark masses values will be conducted using the pseudoscalar density constant g_P, which is determined from the expression

$$\langle 0 \left| \hat{J}_5 \right| \mathbf{Q}, M_P \rangle = -i\frac{1}{(2\pi)^{3/2}} \frac{g_P}{\sqrt{2\,\mathbb{V}_0\, M_P}}. \quad (11)$$

After similar calculations (see [25]) one can obtain an integral representation of the pseudoscalar density constant in point form PiQM:

$$g_P(m_q, m_{\bar{Q}}) = \sqrt{\frac{3}{2}}\frac{1}{\pi}\int_0^{\infty}\mathrm{dk}\frac{\mathrm{k}^2\ \Phi(\mathrm{k})}{\sqrt{\omega_{m_q}(\mathrm{k})\omega_{m_{\bar{Q}}}(\mathrm{k})}}\quad\sqrt{M_0(\mathrm{k})}\left(\sqrt{W^+_{m_q}(\mathrm{k})W^+_{m_{\bar{Q}}}(\mathrm{k})}+\sqrt{W^-_{m_q}(\mathrm{k})W^-_{m_{\bar{Q}}}(\mathrm{k})}\right). \tag{12}$$

Relations (9),(10) (12) will be used below to calculate the basic parameters of the model.

3. Pionic decay $\rho^{\pm} \to \pi^{\pm}\pi^{0}$ in proposed model

Hadronic transition matrix element of vector meson V to pseudoscalar P meson in the case of π^0-meson emission can be written as [8, 14]

$$\langle \mathbf{Q}', M_P \left| \hat{J}_{\pi^0} \right| \mathbf{Q}, \lambda_V, M_V\rangle = \frac{1}{(2\pi)^3}\frac{1}{\sqrt{2\,\mathbb{V}_0\,M_V}}\frac{1}{\sqrt{2\,\mathbb{V}_0{}'M_P}}\,2\,g_{VP\pi^0}\left(\varepsilon_{\lambda_V}\cdot Q'\right), \tag{13}$$

where $g_{VP\pi^0}$ is hadronic decay constant determined from the expressions

$$\Gamma_{V\to P\pi^0} = |g_{VP\pi^0}|^2\,\frac{|\mathbf{p}_{\mathrm{out}}|^3}{6\,\pi\,M_V^2},$$
$$|\mathbf{p}_{\mathrm{out}}|^2 = \left(M_V^2 - (M_P + M_{\pi^0})^2\right) \times \frac{\left(M_V^2 - (M_P - M_{\pi^0})^2\right)}{4\ M_V^2}. \tag{14}$$

Matrix element (13) can be related to axial vector current by [14]

$$\langle \mathbf{Q}', M_P \left| \hat{A}^{\mu} \right| \mathbf{Q}, \lambda_V, M_V\rangle = \frac{1}{(2\pi)^3}\frac{1}{\sqrt{4\,\mathbb{V}_0\,\mathbb{V}_0'}}\left(\,f(q^2)\,\ell^{\mu}_f(\lambda_V) + a_+(q^2)\,\ell^{\mu}_{a_+}(\lambda_V) + a_-(q^2)\,\ell^{\mu}_{a_-}(\lambda_V)\,\right), \tag{15}$$

where $\hat{A}^{\mu} = e_q\bar{\psi}\gamma^{\mu}\gamma_5\psi$. Note, that we use point from parametrization using ℓ-vectors

$$\ell^{\mu}_f(\lambda_V) = \frac{\varepsilon^{\mu}_{\lambda_V}}{\sqrt{M_P\,M_V}}, \ell^{\mu}_{a_+}(\lambda_V) = \sqrt{M_P\,M_V}\ \left(\varepsilon_{\lambda_V}\cdot\mathbb{V}'\right)\left(\mathbb{V}^{\mu} + \frac{M_P}{M_V}\mathbb{V}'^{\mu}\right),$$
$$\ell^{\mu}_{a_-}(\lambda_V) = \sqrt{M_P\,M_V}\ \left(\varepsilon_{\lambda_V}\cdot\mathbb{V}'\right)\left(\mathbb{V}^{\mu} - \frac{M_P}{M_V}\mathbb{V}'^{\mu}\right). \tag{16}$$

Partial conservation of axial current $\partial_{\mu}\hat{A}^{\mu} = f_{\pi^{\pm}}\,M^2_{\pi^{\pm}}\pi^0$ leads to the following relations between form-factors for $\rho^{\pm}\to\pi^{\pm}\pi^0$ decay [5]

$$2\,|g_{VP\pi^0}|\,f_{\pi^{\pm}} = \left|f(0) - \left(M_V^2 - M_P^2\right)a_+(0)\right|. \tag{17}$$

The following calculations will be carried out for generalized Breit system $\vec{\mathbb{V}}+\vec{\mathbb{V}}'=0$. Below we introduce value $\varpi=(\mathbb{V}\cdot\mathbb{V}')$; in our approach $\varpi\to 1$ corresponds to soft pion case $Q^2_{\pi^0}\equiv q^2\to 0$. Taking it into account, it follows for expression (15)

$$\langle \mathbf{Q}', M_P \left|\hat{A}^{\mu}\right| \mathbf{Q}, \lambda_V, M_V\rangle = \frac{1}{(2\pi)^3}\frac{1}{\sqrt{4\,\mathbb{V}_0\,\mathbb{V}_0'}}\times \left(f(q^2\to 0)\ell^{\mu}_{f}(\lambda_V)+a_{+}(q^2\to 0)\ell^{\mu}_{a_+}(\lambda_V)\right). \quad (18)$$

For integral representations calculating of $f(q^2\to 0)$ and $a_+(q^2\to 0)$, we use 4-vectors

$$\ell^{\mu}_{\perp f} = \frac{\sqrt{M_P\,M_V}\,\sqrt{\varpi+1}}{\sqrt{2}\,\left(M_V+\varpi\,M_P\right)} \times\{(M_P-M_V)\sqrt{\frac{\varpi-1}{\varpi+1}},\,0,\,0,\,-(M_P+M_V)\}, \quad (19)$$

$$\ell^{\,\mu}_{\perp a_+} = \frac{\sqrt{M_V}}{\sqrt{2\,M_P}\,\left(M_V+\varpi\,M_P\right)} \times\{\frac{1}{\sqrt{\varpi-1}},\,0,\,0,\,\frac{1}{\sqrt{\varpi+1}}\}. \quad (20)$$

with properties $\big(\ell_{\perp f}\cdot\ell_{a_+}(0)\big)=\big(\ell_{\perp a_+}\cdot\ell_f(0)\big)=0$ and $\big(\ell_{\perp f}\cdot\ell_f(0)\big)=\big(\ell_{\perp a}\cdot\ell_a(0)\big)=1$ since the helicity final state of $V\to P\pi^0$ decay is equal to zero, i.e. $\lambda_V=0$. Multiplying (18) by (19) taking into account meson state vector (1) leads to

$$f(q^2)=\left(I_{0,\lambda_V=0}\cdot\ell_{\perp f}\right), \quad a_+(q^2)=\left(I_{0,\lambda_V=0}\cdot\ell_{\perp a_+}\right), \quad (21)$$

where we define function I_{0,λ_V}

$$I^{\mu}_{0,\lambda_V} = \sum_{\nu_1,\nu_1'}\sum_{\nu_2,\nu_2'}\int \mathrm{d}\mathbf{k}\,\Phi\left(\mathrm{k}\right)\Omega\left\{\begin{smallmatrix}0 & 1 & 1\\ \nu_1, & \nu_2, & \lambda_V\end{smallmatrix}\right\}(\theta_{\mathrm{k}},\phi_{\mathrm{k}})\,\Omega\left\{\begin{smallmatrix}0 & 0 & 0\\ \nu_1', & \nu_2', & 0\end{smallmatrix}\right\}(\theta_{\mathrm{k}},\phi_{\mathrm{k}})\left(e_q\sqrt{\frac{\omega_{m_{\bar{Q}}}(\mathrm{k}_2)}{\omega_{m_{\bar{Q}}}(\mathrm{k})}}D_{\nu_2',\nu_2}(\mathbf{n}_{W_2})\right.$$
$$\times\,\Phi\left(\mathrm{k}_2\right)\frac{\bar{u}_{\nu_1'}(\mathbf{k}_2,m_q)\Gamma^{\mu}_5(\mathbf{u}_{\mathbf{Q}'},\mathbf{u}_{\mathbf{Q}})u_{\nu_1}(\mathbf{k},m_q)}{\sqrt{4\,\omega_{m_q}(\mathrm{k}_2)\,\omega_{m_q}(\mathrm{k})}}+e_{\bar{Q}}\sqrt{\frac{\omega_{m_q}(\mathrm{k}_1)}{\omega_{m_q}(\mathrm{k})}}D_{\nu_1',\nu_1}(\mathbf{n}_{W_1})\;\Phi\left(\mathrm{k}_1\right)$$
$$\left.\times\,\frac{\bar{v}_{\nu_2}(-\mathbf{k},m_{\bar{Q}})\Gamma^{\mu}_5(\mathbf{u}_{\mathbf{Q}},\mathbf{u}_{\mathbf{Q}'})v_{\nu_2'}(-\mathbf{k}_1,m_{\bar{Q}})}{\sqrt{4\,\omega_{m_{\bar{Q}}}(\mathrm{k})\,\omega_{m_{\bar{Q}}}(\mathrm{k}_1)}}\right) \quad (22)$$

with notation

$$\mathbf{k}_{1,2}=\mathbf{k} \pm\boldsymbol{v}_Q\left((\varpi+1)\,\omega_{m_{q,\bar{Q}}}\left(\mathrm{k}\right)-\mathrm{k}\sqrt{\varpi^2-1}\cos\theta_k\right), \quad (23)$$

where $\boldsymbol{v}_{\mathbf{Q}}=\vec{\mathbb{V}}_{\mathbf{Q}}/\mathbb{V}_0$ and $\Gamma^{\mu}_5(\mathbf{u}_{\mathbf{Q}'},\mathbf{u}_{\mathbf{Q}})=B^{-1}(\mathbf{u}_{\mathbf{Q}'})\gamma^{\mu}\gamma_5 B(\mathbf{u}_{\mathbf{Q}})$. From expressions (21), (22), after calculating the spinor part and integration over the solid $\mathbf{k}$-vector angle, one can get the form-factors integral representation at $q^2\to 0$

$$f(0)=\int_0^{\infty}\mathrm{d}\mathrm{k}\,\mathrm{k}^2\,\Phi\left(\mathrm{k}\right)\Phi^{*}\left(\mathrm{k}\right) \times\left(e_q\,\eta^f\left(\mathrm{k},m_q\right)+e_{\bar{q}}\,\eta^f\left(\mathrm{k},m_{\bar{q}}\right)\right), \quad (24)$$

$$a_{+}(0) = \int_0^{\infty} \mathrm{dk}\, \mathrm{k}^2\, \Phi\,(\mathrm{k})\ \Phi^*\,(\mathrm{k}) \times \qquad (25)$$
$$\left(e_q\, \eta^{\tilde{a}_+}(\mathrm{k}, m_q) + e_{\bar{q}}\, \eta^{\tilde{a}_+}(\mathrm{k}, m_{\bar{q}})\right).$$

In (24), (25) auxiliary functions are introduced

$$\eta^f(\mathrm{k}, m) = \frac{4}{3}\,\left(2\,m + \omega_m(\mathrm{k})\right), \qquad (26)$$
$$\eta^{a_+}(\mathrm{k}, m) = -\frac{m\,\left(m + 2\,\omega_m(\mathrm{k})\right)}{6\,\omega_m^3(\mathrm{k})}.$$

Numerical calculations using the expressions obtained in sections 2 and 3 are carried out below.

4. Numerical results and discussions

Calculation constituent quarks masses and wave functions parameters values will be carried out by solving the system of equations [25]

$$\begin{cases} \frac{1}{2}\,(\hat{m}_u + \hat{m}_d) = (3.42 \pm 0.11)\ \mathrm{MeV}, \\ f_P(m_u, m_d, \beta^P_{u\bar{d}}) = f^{(\mathrm{exp.})}_{\pi^{\pm}}, \\ (\hat{m}_u + \hat{m}_d)\ \ g_P(m_u, m_d, \beta^P_{u\bar{d}}) = f^{(\mathrm{exp.})}_{\pi^{\pm}} M^2_{\pi^{\pm}}, \end{cases} \qquad (27)$$

where $f^{(\mathrm{exp.})}_{\pi^{\pm}}$, $M_{\pi^{\pm}}$ are experimental values of the decay constant $\pi^{\pm}$–meson and its mass, $\hat{m}_u$, $\hat{m}_d$ are quark current masses [1]. In what follows we assume that the constituent masses values of u– and d–quark are condition $\hat{m}_d - \hat{m}_u = m_d - m_u = 2.51 \pm 0.22$ MeV subject; in this case the system (27) solution with oscillator wave function (3) leads to the following basic parameters values of the model:

$$m_u = m_d = (218.89 \pm 4.90)\ \mathrm{MeV}, \qquad (28)$$
$$\beta^P_{u\bar{d}} = (371.81 \pm 4.90)\ \mathrm{MeV}.$$

The wave function parameter for vector $\rho^{\pm}$–meson is determined from the decay of a heavy lepton $\tau^{\pm} \to \rho^{\pm}\nu_{\tau}$: using expression (5) and experimental value $f_{\rho^{\pm}} = 210.75 \pm 0.09$ MeV [1] one can get

$$\beta^V_{u\bar{d}} = 313.10 \pm 0.08\ \mathrm{MeV}. \qquad (29)$$

Below we compare the obtained values of constituent quark masses with approaches based on different form of PiQM (see table 1)

From relations (17) and (24)–(26), quark masses value and β-parameters for decay $\rho^{\pm} \to \pi^{\pm}\pi^0$ leads to the decay constant value

$$\left|g_{\rho^{\pm}\pi^{\pm}\pi^0}\right| = \frac{1}{2\ f_{\pi^{\pm}}}\left|f(0) - \left(M_V^2 - M_P^2\right) a^{+}(0)\right|$$
$$= 5.78 \pm 0.07 \qquad (30)$$

with the corresponding decay width value

$$\Gamma^{(\mathrm{th.})}_{\rho^{\pm}\to\pi^{\pm}\pi^0} = \left|g_{VP\pi^0}\right|^2 \frac{\left|\mathbf{p}_{\mathrm{out}}\right|^3}{6\ \pi\ M_V^2} = (141 \pm 2)\ \mathrm{MeV} \qquad (31)$$

(compare to experimental data $\Gamma^{(\mathrm{exp.})}_{\rho^{\pm}\to\pi^{\pm}\pi^0} = (149.1 \pm 0.8)$ MeV). The results are comparable with other models, as well as with experimental data (see table 2).

Note that in [16] decay $\rho^{\pm} \to \pi^{\pm}\pi^0$ is used as input to the fit so we do not compare the results. Analysis of the table 2 shows that the proposed approach provides good agreement with modern experimental data.

5. Conclusion and remarks

The work is dedicated to the study of $\pi^{\pm}$-, $\rho^{\pm}$-mesons characteristics in an approach based on the composite quark model and the point form of PiQM. In the course of work the authors obtain integral representations of pseudoscalar and vector mesons f_P, f_V decay constants using electroweak quark currents. The proposed observed quantities calculation method is generalized to the case of hadronic transition $V \to P\pi^0$. The results of calculations in point form of dynamics lead to relatively simple integral representations for the $g_{V^{\pm}P^{\pm}\pi^0}$ constant. As a result of the work it has been shown that the usage of model parameters obtained from $\pi^{\pm} \to \ell^{\pm}\nu_{\ell}$ and $\tau^{\pm} \to \rho^{\pm}\nu_{\tau}$ decays leads to results for $\rho^{\pm} \to \pi^{\pm}\pi^0$ observables that are close to modern experimental data.

Table 1: Values of constituent quark masses in models, based on the different forms of PiQM

Quark flavor	Light-front calculation [8]	Light-front calculation [11, 12]	Instant form calculation[23, 24]	This work
m_u, MeV	220	220	250±5	218.89±4.90
m_d, MeV	220	220	250±5	218.89±4.90

Table 2. Comparison of decay constant $g_{\rho^{\pm}\pi^{\pm}\pi^{0}}$ value with other approaches and models as well as experimental data

Proposed approach	Value of decay constant $g_{\rho^{\pm}\pi^{\pm}\pi^{0}}$, MeV^0
Light-front model [8]	6.10
Light-front model with spurioun contribution [26]	10.00
$^3P_0$–model [18]	4.32
Quasipotential approach [19]	5.45
Bethe-Salpeter based approach [27]	5.13±0.25
Current algebra calculation [5]	5.87
This work	5.78±0.07
Experimental data [1]	5.98±0.02

The authors note that the proposed model has been successfully used for radiative decay $V(P) \to P(V)\gamma$ investigation [28], $\rho^{\pm}$-meson electromagnetic form-factor calculations [29] as well as studies of two-photon decay of a pseudoscalar π^0-,η- and $\eta\prime$-mesons [30, 31].

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