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Is it Possible to Match Higher-Twist Contributions with Chiral Perturbation Theory?

D. A. Volkova,[1] N. A. Gramotkov,[2] O. V. Teryaev,[3] and O. P. Solovtsova[4]

[1] *Joint Institute for Nuclear Research, Dubna, RUSSIA*
Dubna State University, Dubna, RUSSIA
[2] *Joint Institute for Nuclear Research, Dubna, RUSSIA*
Moscow State University, Physics Department, Moscow, RUSSIA
[3] *Joint Institute for Nuclear Research, Dubna, RUSSIA*
[4] *Gomel State Technical University, Gomel, BELARUS,*
Joint Institute for Nuclear Research, Dubna, RUSSIA
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We present a QCD-motivated approach to the analysis of polarized Bjorken sum rule in the nonperturbative infrared region $Q^2 < 1$ GeV^2. In this approach, we use the Gerasimov–Drell–Hearn sum rule as a boundary condition and move from the region of large momentum transfers to the low Q^2-region. We show that the developed approach works well and note a possible problem with the Jefferson Lab data at $Q^2 < 0.1$ GeV^2.

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1. Introduction

Deep inelastic scattering of leptons on nucleons is one of the key processes in studying the internal structure of a nucleon. The cross sections of this process are described by structure functions whose integrals form sum rules of deep inelastic scattering from various combinations. Among these sum rules, the polarized Bjorken sum rule, Γ_1^{p-n}, is central to the study of the nucleon spin structure [1, 2]. This sum rule is determined by the integral of the difference between the spin-dependent structural functions $g_1(x, Q^2)$ of the proton g_1^p and the neutron g_1^n over all possible values of the Bjorken variable x for a fixed square of the transferred momentum Q^2. Note that the spin structure function g_1 is of great physical interest because it characterizes the partial contribution of quarks to the nucleon spin, and the integral of $g^{(p/n)}(x, Q^2)$ determines the total contribution of active quarks to the proton (neutron) spin of at a fixed Q^2.

In the limit $Q^2 \to \infty$, the expression for the Bjorken sum rule obtained using the algebra of currents and isospin symmetry has the form

$$\Gamma_1^{p-n}(Q^2)|_{Q^2\to\infty} = \int_0^1 \left[g_1^p(x,Q^2) - g_1^n(x,Q^2)\right] dx = \frac{g_A}{6}, \quad (1)$$

where g_A is the axial coupling constant measured in neutron β decay.

Away from the large Q^2 limit, the Bjorken sum rule has additional terms. The theoretical description at low scales Q^2 includes both perturbative corrections and non-perturbative terms (higher-twist corrections). It is important to note that the Bjorken sum is actively measured at low and intermediate values of Q^2, allowing us to test the description of perturbative corrections and study non-perturbative ones. Thus, the $0.05 < Q^2 < 3$ GeV^2 region has been intensively studied at Jefferson Lab (JLab), see the recent work [3], references therein, and the reviews [4, 6].

For small values of $Q^2 < 1$ GeV^2, the description of perturbative contributions using the usual perturbative coupling $\alpha_s(Q^2)$ faces serious difficulties due to unphysical singularities of $\alpha_s(Q^2)$, such as the ghost pole, which contradict the fundamental principle of causality. Hence, the standard perturbative approach does not allows one to describe experimental data on Γ_1^{p-n} at low Q^2 and reliably extract the values of nonperturbative parameters. In order to avoid this difficulty, in this paper we apply an approach (combining renormalization group symmetry and Källén–Lehmann analyticity which is based upon the general principles of the local quantum field theory) called the Analytic Perturbation Theory (APT) [7] (see [8] for more detail). The analytic coupling without introducing additional parameters eliminates the non-physical features of the perturbative part and shifts the limit of applicability of the perturbative QCD to small momentum transfers. The APT has already been applied to the analysis of the Bjorken sum in a number of papers (see, e.g., [9–11]).

Since the nonperturbative part in the Bjorken sum rule has a series of powers of $1/Q^2$ (higher twists) which should move near $Q^2 \approx 0$ into a function unknown so far, we use the technique of matching the function for large Q^2 with the behavior at the lowest Q^2, using for this purpose the Gerasimov–Drell–Hearn sum rule [12], see also [13, 14].

2. Theoretical approach

Equation (1) can be generalized for finite Q^2 and according to the OPE (operator product extension) (see [15]) is represented as

$$\Gamma_1^{p-n}(Q^2) = \frac{g_A}{6}\left[1 - D_{\rm BS}(Q^2)\right] + \sum_{i=2}^{\infty} \frac{\mu_{2i}(Q^2)}{Q^{2i-2}}\,, \tag{2}$$

where $D_{\rm BS}(Q^2)$ is the perturbation part and μ_{2i}/Q^{2i-2} are the higher-twist (HT) contributions.

Considering very small values of Q^2, the representation HT consists of an infinite number of terms of the series. To avoid this, the so-called "massive" representation in the form of twist-4 [16] that includes part of the contributions of the higher twist is used. Then expression (2) takes the form

$$\Gamma_1^{p-n}(Q^2) = \frac{g_A}{6}\left[1 - D_{\rm BS}(Q^2)\right] + \frac{\hat{\mu}_4 M^2}{Q^2 + M^2}\,, \tag{3}$$

where the values of $\hat{\mu}_4$ and M^2 were fitted in [10, 11] in different analytical QCD models. Here we use the values of the power corrections as an input for our model at $Q_0 \sim 1$ GeV^2 and we get a good description of the Γ_1^{p-n} data in the low Q^2 region.

Next we follow the approach proposed in [17] and consider the integral

$$I_1(Q^2) = \frac{2M^2}{Q^2}\Gamma_1(Q^2) = \frac{2M^2}{Q^2}\int_0^1 g_1(x, Q^2)dx. \tag{4}$$

According to the GDH sum rule, the value of this integral at $Q^2 = 0$ is well known

$$I_1(0) = -\frac{\mu_A^2}{4}, \tag{5}$$

where μ_A is the anomalous magnetic moment of the nucleon in nuclear magnetons. The integral $I_1(0)$ is always negative, but its value for a large Q^2 is determined by the integral $\int_0^1 g_1(x)dx$ independent of Q^2, positive for the proton and negative for the neutron.

Dividing the contributions of g_T and g_2 (see [18]) leads to the decomposition of $I_1(Q^2)$ as the difference between $I_T(Q^2)$ and $I_2(Q^2)$:

$$I_1(Q^2) = I_T(Q^2) - I_2(Q^2),$$

$$I_T(Q^2) = \frac{2M^2}{Q^2}\int_0^1 g_T(x, Q^2)dx, \tag{6}$$

$$I_2(Q^2) = \frac{2M^2}{Q^2}\int_0^1 g_2(x, Q^2)dx\ . \tag{7}$$

It is possible to obtain a smooth interpolation between large Q^2 and $Q^2 = 0$

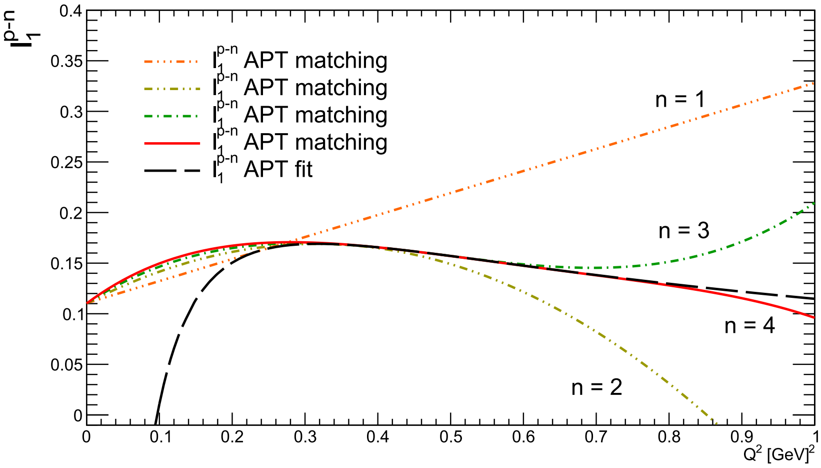

FIG. 1: (Color online) Result of the leading order APT matching for different number of terms in (11).

[18] by representing $I_T^p(Q^2)$ as

$$I_T^p(Q^2) \;=\; \theta(Q_0^2 - Q^2)\left(\frac{\mu_{A,p}}{4} - \frac{2M^2Q^2}{(Q_0^2)^2}\Gamma_1^p\right) + \theta(Q^2 - Q_0^2)\frac{2M^2}{Q^2}\Gamma_1^p, \tag{8}$$

where $\Gamma_1^p = \int_0^1 g_1^p(x)dx$. However, such interpolation neglects the perturbative and power QCD corrections and, on the other hand, assumes that at the boundary point Q_0 the contribution of g_2 is already extremely small, so

$$I_T^p(Q_0^2) = I_1^p(Q_0^2).$$

Both types of corrections are easily taken into account, but this does not allow a simple analytical parametrization.

In this paper we pay attention to I_1 without considering I_2 and I_T. Let us write for the asymptotic expression for I_1^i $(i = p, n)$ as

$$I_1^i(Q^2) \;=\; \frac{2M^2}{Q^2}\left[\int_0^1 g_1^i(x,Q^2)dx\left(1 - \frac{\alpha_s(Q^2)}{2\pi}\right) - c_i\frac{\ll O_i \gg}{Q^2}\right], \tag{9}$$

where the one-loop perturbative correction is taken into account as well as the contribution of twist-4 [19]. Here c_i is the charge factor equal to $2/9$ for the proton and $1/18$ for the neutron, and the matrix elements of combinations of the reduced twist-3 and -4 operators turn out to be equal [19] for both the proton and the neutron: $\ll O_p \gg = \ll O_n \gg = 0.09 \pm 0.06$ GeV^2.

Consider the case for the proton. In this case

$$I_{T,pert}^p(Q^2) = \theta(Q^2 - Q_0^2)\frac{2M^2}{Q^2}\left[\Gamma_1^p\left(1 - \frac{\alpha_s(Q^2)}{2\pi}\right) - c_p\frac{\ll O_p \gg}{Q^2} - I_2^p(Q^2)\right]. \tag{10}$$

Smooth interpolation to the GDH at $Q^2 = 0$ is now more difficult and cannot be performed using simple analytical formulas. Instead, we decompose (10) into a power series at the point Q_0 and define the expression at low Q^2 as:

$$I_{T,non-pert}^p(Q^2) = \theta(Q^2 > Q_0^2)I_0(Q^2) + \theta(Q^2 < Q_0^2)\sum_{n=0}^{N}\frac{1}{n!}\frac{\partial^n I_0}{\partial(Q^2)^n}|_{Q^2=Q_0^2}. \tag{11}$$

Here N is the number of continuous derivatives of these two expansions, which turns out to be a free parameter of the model along with the value of the matching point Q_0. They must be chosen in such a way that the condition is satisfied:

$$I_0(0) = I_{GDH}. \tag{12}$$

Depending on the k-order of perturbation coupling, the matching points Q_0 (they are listed in Tab.1) shift to the region of large values Q^2; however, this changes slightly depending on the approximation for the perturbative part, as shown in Fig. 1.

Table 1 shows the dependence of the matching point, Q_0, on the used order, k, for the perturbative part. As can be seen from this table, the point Q_0 varies slightly shifting from an order to a higher order to the region of large values Q^2. Figure 1 also demonstrates the same.

Table 1. Matching points Q_0^2 (in GeV^2) for different orders of APT.

	LO	NLO	N^{2}LO	N^{3}LO
k = 1	0.236	0.246	0.260	0.261
k = 2	0.331	0.343	0.361	0.361
k = 3	0.428	0.441	0.464	0.463
k = 4	0.526	0.541	0.569	0.567

3. Comparison of data

In Fig.2 we compare different levels of our approximation of I_1^{p-n} with the JLab experimental data for the $Q^2 < 1$ GeV^2 region. In Fig.3 we present different types of the calculated BSR. The solid black curve refers to the Γ_1^{p-n} obtained in [20] from the fit of a combined set of JLab experimental data, the dashed red curve refers to the Γ_1^{p-n} constructed by the decomposition method, and the orange dash-dotted curve is the phenomenological model prediction from Burkert-Ioffe [21]. In Fig.4 we present for comparison the result from [3]. One can see form Figs.3 and 4 that our curve goes closer to the data in the low Q^2 range than other approaches summarized in [3].

4. Conclusion

The applied «matching» procedure allows us to smoothly move from the region of large momentum transfers to the low-energy region $Q^2 < 1$ GeV^2 and perform a QCD analysis of the available experimental data in this area. We confirm that the experimental data contradict the Gerasimov-Drell-Hearn sum rule using the example of the function $I(Q^2)$; their is also

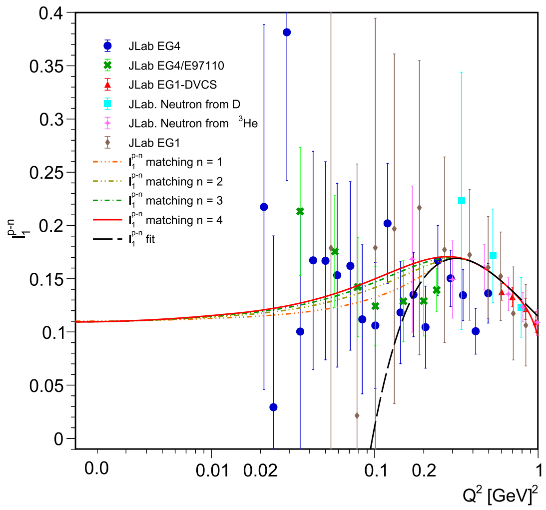

FIG. 2. (Color online) I_1^{p-n} at different matching points with respect to a combined set of JLab experimental data.

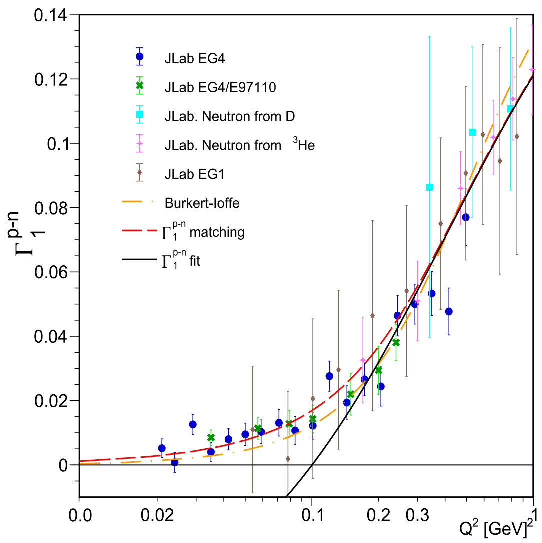

FIG. 3. (Color online) Result of comparing different types of the BSR with respect to the combined set of experimental data.

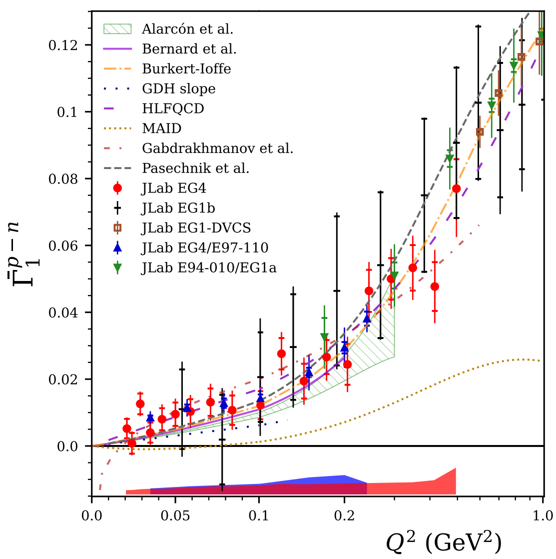

FIG. 4. (Color online) Figure similar to Fig. 3 from [3].

noticeable in the work [3]. It should be noted that a systematic error related to the method of conducting an experiment is not excluded. This approach can be applied to other problems in studying the low-energy QCD region. In addition, the approach at different loop levels of the perturbative part provides consistently good agreement with experimental data in the entire region up to the zero momentum transfers. In future works, it is planned to consider the sum rules for other channels, as well as the structure of g_2.

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