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ELECTROWEAK CHARACTERISTICS OF LIGHT π–, ρ–MESONS

Introduction. The researching of composite quark-antiquark systems is associated with the study of electroweak characteristics of hadrons, such as meson form factors, decay constants etc. To a date a sufficient volume of experimental data has been accumulated [1], including on the decays of light pseudoscalar $\pi^{\pm}$– and $\rho^{\pm}$–vector mesons. Since such systems are purely relativistic [2], it becomes possible to test phenomenological approaches and models for studying the properties of coupled $u\overline{d}$–systems. Also, the equality of the masses of constituent and quarks in the indicated mesons significantly simplifies both the calculation of the model parameters and calculations.

The paper presents a method for calculating the electromagnetic mean-square radii of pseudoscalar $\pi^{\pm}$– and vector $\rho^{\pm}$–mesons in a composite quark model based on the point form of Poincaré-invariant quantum mechanics. The authors have shown that the usage of parameters obtained from leptonic decays $\pi^{\pm} \to \ell^{\pm}\nu_{\ell}$ and $\tau^{\pm} \to \rho^{\pm}\nu_{\tau}$ as well as the pseudoscalar density constant $g_{P^{\pm}}$ [3, 4], leads to results for the mean-square radii of the pseudoscalar $\pi^{\pm}$– and vector $\rho^{\pm}$–mesons that correlate with modern experimental data and other models. As a result, a self-consistent model that describes the electroweak characteristics of pseudoscalar and vector light sector mesons is proposed.

1. Basic features of the model. Below we define the state vector of the meson with spin $J = \ell + S$ ($\ell = 0$, $S = 0,1$) and its' projection μ, 4-momentum Q^{μ} ($Q^2 = M^2$, $V^{\mu} = Q^{\mu}/M$) and mass M in the point form of dynamics

$$\left|Q,M\right\rangle_{J\mu}=\sum_{\lambda_1,\lambda_2}\sum_{\nu_1,\nu_2}\int \mathbf{dk}\ \Phi^{J}_{\ell S}\left(\mathbf{k},\beta_{q\bar{Q}}\right)\sqrt{\frac{\omega_{m_q}(\mathrm{p}_1)\omega_{m_{\bar{Q}}}(\mathrm{p}_2)}{\omega_{m_q}(\mathrm{k})\omega_{m_{\bar{Q}}}(\mathrm{k})V^0}}\times$$

$$\times\,\Omega\begin{Bmatrix}\ell & S & J\\ \nu_1 & \nu_2 & \mu\end{Bmatrix}\left(\theta_{\mathrm{k}},\phi_{\mathrm{k}}\right)D^{1/2}_{\lambda_1,\nu_1}(\mathbf{n}_{W_1})D^{1/2}_{\lambda_2,\nu_2}(\mathbf{n}_{W_2})\left|\mathbf{p}_1,\lambda_1,\mathbf{p}_2,\lambda_2\right\rangle. \tag{1}$$

In (1) the relative momentum of quarks [2] is defined $\mathbf{k}=\left\{\mathrm{k}\sin\theta_{\mathrm{k}}\cos\phi_{\mathrm{k}},\mathrm{k}\sin\theta_{\mathrm{k}}\sin\phi_{\mathrm{k}},\mathrm{k}\cos\theta_{\mathrm{k}}\right\}$ with masses m_q, $m_{\bar{Q}}$, helicities λ_1, λ_2 and momenta $\mathbf{p}_1$, $\mathbf{p}_2$ respectively. Note also that for brevity, an auxiliary function into expression (1) is introduced

$$\Omega\begin{Bmatrix}\ell & S & J\\ \nu_1 & \nu_2 & \mu\end{Bmatrix}\left(\theta_{\mathrm{k}},\phi_{\mathrm{k}}\right)=\mathrm{Y}_{\ell m}\left(\theta_{\mathrm{k}},\phi_{\mathrm{k}}\right)\mathrm{C}\begin{pmatrix}s_1 & s_2 & S\\ \nu_1 & \nu_2 & \lambda\end{pmatrix}\mathrm{C}\begin{pmatrix}\ell & S & J\\ \nu_1 & \nu_2 & \lambda\end{pmatrix} \tag{2}$$

with the Clebsch-Gordan coefficients $\mathrm{C}\begin{pmatrix}j_1 & j_2 & J\\ m_1 & m_2 & m\end{pmatrix}$ of $SU(2)$ group and the spherical functions $\mathrm{Y}_{\ell m}\left(\theta_{\mathrm{k}},\phi_{\mathrm{k}}\right)$ of $\mathbf{k}$ vector. The Wigner rotation functions in (1) are defined as

$$D^{1/2}_{\lambda,\nu}(\mathbf{n}_W)=\frac{I-i\left(\mathbf{n}_W\cdot\boldsymbol{\sigma}\right)}{\sqrt{1+\mathbf{n}_W^2}},\ \mathbf{n}_W=\frac{\mathbf{u}_{\mathbf{k}}\times\mathbf{u}_{\mathbf{Q}}}{1+\left(\mathbf{u}_{\mathbf{k}}\cdot\mathbf{u}_{\mathbf{Q}}\right)},\ \mathbf{u}_{\mathbf{P}}=\frac{\mathbf{P}}{\omega_M(\mathrm{P})+M}, \tag{3}$$

where $\omega_m(\mathrm{p})=\sqrt{\mathrm{p}^2+m}$. The wave function of the pseudoscalar $(\ell=0,S=0)$ and vector $(\ell=0,S=1)$ meson in the expression is subject to the normalization condition

$$\sum_{\ell,S}\int \mathrm{dk}\ \mathrm{k}^2\left|\Phi^{J}_{\ell S}\left(\mathrm{k},\beta_{q\bar{Q}}\right)\right|^2=1. \tag{4}$$

Using the electroweak quark current in the meson rest frame

$$\left\langle 0\left|\hat{J}^{\mu}_{\mathrm{EW.}}\right|\mathbf{k},\lambda_1,-\mathbf{k},\lambda_2\right\rangle=\frac{1}{(2\pi)^3}\frac{\bar{\upsilon}_{\lambda_2}(-\mathbf{k},m_{\bar{Q}})\Gamma^{\mu}_{\mathrm{EW.}}u_{\lambda_1}(\mathbf{k},m_q)}{\sqrt{2\omega_{m_q}(\mathrm{k})\ 2\omega_{m_{\bar{Q}}}(\mathrm{k})}} \tag{5}$$

with following calculation of the spinor part leads to an integral representation of the decay constant of pseudoscalar and vector mesons [6]:

$$f_I(m_q,m_{\bar{Q}},\beta^I_{q\bar{Q}})=\sqrt{\frac{3}{2}}\frac{1}{\pi}\int \mathrm{dk}\ \mathrm{k}^2\ \Phi\left(\mathrm{k},\beta^I_{q\bar{Q}}\right)\sqrt{\frac{W^+_{m_q}(\mathrm{k})W^+_{m_{\bar{Q}}}(\mathrm{k})}{M_0\ \omega_{m_q}(\mathrm{k})\ \omega_{m_{\bar{Q}}}(\mathrm{k})}}\times$$

$$\times\left(1+a_I\frac{\mathrm{k}^2}{W^+_{m_q}(\mathrm{k})W^+_{m_{\bar{Q}}}(\mathrm{k})}\right);\ W^{\pm}_m(\mathrm{k})=\omega_m(\mathrm{k})\pm m;\ I=P,V;\ a_P=-1,\ a_V=1/3. \tag{6}$$

Expression (6) will be used below for the model parameters calculation.

2. Electromagnetic radii of $\pi^{\pm}$ – and $\rho^{\pm}$–mesons. The integral representation of the electromagnetic root-mean-square radius of a meson is determined from the expression [5]

$$< r^2_{\pi^{\pm}} >= 6 \frac{dF(t)}{dt}\bigg|_{t\to 0}, \tag{7}$$

where the form-factor $F(t)$ is determined by the matrix

$$\left\langle Q', M \middle| \hat{J}^{\mu}_{\text{EW.}} \middle| Q, M \right\rangle = \frac{F(t)}{(2\pi)^3} \frac{Q^{\mu} + Q'^{\mu}}{\sqrt{2\ \omega_M(\mathrm{Q}')\ 2\ \omega_M(\mathrm{Q})}},\ t = -\left(Q' - Q\right)^2. \tag{8}$$

Calculation of expression (8) taking into account the meson state vector (1) and the electroweak quark current (5) in the generalized Breit system $\vec{V} + \vec{V}' = 0$ [6] with subsequent differentiation (see expression (7)) leads to

$$< r^2_{\pi^{\pm}} > = 6\int \mathrm{dk\ k}^2 \left|\Phi\left(\mathrm{k}, \beta^P_{u\bar{d}}\right)\right|^2 \left(e_u \eta^P\left(\mathrm{k}, m_u, \beta^P_{u\bar{d}}\right) + e_{\bar{d}} \eta^P\left(\mathrm{k}, m_d, \beta^P_{u\bar{d}}\right)\right). \tag{9}$$

Note that in expression (9) auxiliary functions are introduces

$$\eta^P\left(\mathrm{k}, m, \beta\right) = \frac{2\ \mathrm{k}\ \left(\omega^2_m(\mathrm{k}) - 2\beta^2\right) + \omega_m(\mathrm{k})\left(\omega^2_m(\mathrm{k}) - 3\beta^2\right)\ln\left(\dfrac{\omega_m(\mathrm{k}) - \mathrm{k}}{\omega_m(\mathrm{k}) + \mathrm{k}}\right)}{16\ \mathrm{k}\ \beta^4}. \tag{10}$$

The parametrization of form-factors for a vector $\rho^{\pm}$ – meson with a polarization vector ε_{λ} can be written as

$$\begin{gathered} {}_{\lambda'}\left\langle Q', M \middle| \hat{J}^{\mu}_{\text{EW.}} \middle| Q, M \right\rangle_{\lambda} = \frac{1}{(2\pi)^3} \frac{1}{\sqrt{2\ \omega_M(\mathrm{Q}')\ 2\ \omega_M(\mathrm{Q})}} \times \\ \times\left(-\left(\varepsilon^*_{\lambda'} \times \varepsilon_{\lambda}\right) P^{\mu} F_1(t) + \left(\left(P \cdot \varepsilon^*_{\lambda'}\right)\varepsilon^{\mu}_{\lambda} + \left(P \cdot \varepsilon_{\lambda}\right)\varepsilon^{*\mu}_{\lambda'}\right) F_2(t) + \frac{\left(P \cdot \varepsilon^*_{\lambda'}\right)\left(P \cdot \varepsilon_{\lambda}\right)}{2M^2} P^{\mu} F_3(t)\right), \end{gathered} \tag{11}$$

where $P = Q' + Q$, $t = -\left(Q' - Q\right)^2$. Assuming $t \to 0$ electromagnetic root mean square radius is defined as (see (7))

$$< r^2_{\rho^{\pm}} > = 6 \frac{dF_1(t)}{dt}\bigg|_{t\to 0}; \tag{12}$$

note that the form-factors $F_2(t)$ and $F_3(t)$ determine the magnetic and quadrupole magnetic moments of the $\rho^{\pm}$ – meson respectively, but the calculations of the latter are beyond the scope of the proposed work.

A similar calculation procedure leads to the expression

$$< r^2_{\rho^\pm} > = 6\int \mathrm{dk}\,\mathrm{k}^2 \left|\Phi\left(\mathrm{k},\beta^V_{u\bar{d}}\right)\right|^2 \left(e_u \eta^V\left(\mathrm{k}, m_u, \beta^V_{u\bar{d}}\right) + e_{\bar{d}}\eta^V\left(\mathrm{k}, m_d, \beta^V_{u\bar{d}}\right)\right), \tag{13}$$

where auxiliary functions for brevity are introduced

$$\eta^V(\mathrm{k}, m, \beta) = \frac{1}{32}\left(-\frac{8}{\beta^2} + \frac{2}{\left(m+\omega_m(\mathrm{k})\right)^2} - \frac{6\,\omega_m(\mathrm{k})\ln\left(\dfrac{\omega_m(\mathrm{k})-\mathrm{k}}{\omega_m(\mathrm{k})+\mathrm{k}}\right)}{\mathrm{k}\,\beta^2} + \right.$$
$$\left. + \frac{4\omega^2_m(\mathrm{k})}{\beta^4} + \frac{2\omega^3_m(\mathrm{k})\ln\left(\dfrac{\omega_m(\mathrm{k})-\mathrm{k}}{\omega_m(\mathrm{k})+\mathrm{k}}\right)}{\mathrm{k}\,\beta^4} + \frac{m^2\ln\left(\dfrac{\omega_m(\mathrm{k})-\mathrm{k}}{\omega_m(\mathrm{k})+\mathrm{k}}\right)}{\mathrm{k}\omega_m(\mathrm{k})\,\left(\omega_m(\mathrm{k})+m\right)} \right). \tag{14}$$

Below we will carry out numerical estimates of the observed pseudoscalar and vector mesons using the expressions from sections **1** and **2**.

3. Numerical calculations and discussion. We will determine the parameters of the model from the system of equations

$$\begin{cases} 1/2(\hat{m}_u + \hat{m}_d) = (3.45 \pm 0.42)\ \mathrm{MeV}, \\ f_P(m_u, m_d, \beta^P_{u\bar{d}}) = f^{(\mathrm{exp.})}_{\pi^\pm}, \\ (\hat{m}_u + \hat{m}_d) g_P(m_u, m_d, \beta^P_{u\bar{d}}) = f^{(\mathrm{exp.})}_{\pi^\pm}\left(M^{(\mathrm{exp.})}_{\pi^\pm}\right)^2 \\ f_V(m_u, m_d, \beta^P_{u\bar{d}}) = f^{(\mathrm{exp.})}_{\rho^\pm}, \end{cases} \tag{15}$$

where $f^{(\mathrm{exp.})}_{\pi^\pm}$, $f^{(\mathrm{exp.})}_{\rho^\pm}$ are the experimental values of the meson decay constants, $M^{(\mathrm{exp})}_{\pi^\pm}$ is the $\pi^\pm$ – meson mass, g_P is the pseudoscalar density constant, and $\hat{m}_u, \hat{m}_d$ are the values of the current masses of quarks [1]. Solution of system (15) with oscillatory wave function [6]

$$\Phi\left(\mathrm{k}, \beta^{I=P,V}_{q\bar{Q}}\right) = \frac{2}{\pi^{1/4}\left(\beta^I_{q\bar{Q}}\right)^{3/2}} \mathrm{Exp}\left[-\frac{\mathrm{k}^2}{2\left(\beta^I_{q\bar{Q}}\right)^2}\right] \tag{16}$$

leads to $m_u = m_d = (218.89 \pm 4.90)$ MeV, $\beta^P_{u\bar{d}} = (371.81 \pm 4.20)$ MeV, $\beta^V_{u\bar{d}} = (311.98 \pm 2.22)$ MeV. Substituting the obtained model parameters into expressions (9), (13) gives values

$$< r_{\pi^\pm} > = (0.534 \pm 0.006)\ \mathrm{fm},\ < r_{\rho^\pm} > = (0.615 \pm 0.006)\ \mathrm{fm}. \tag{17}$$

A comparison of the obtained results with experimental data and other models is presented in the table 1.

Table 1 – Comparison of the values of the root-mean-square radii of light mesons

R.m.s. radii	[1]	[5]	This work
$< r_{\pi^{\pm}} >$, fm	0.659 ± 0.004	–	0.534 ± 0.006
$< r_{\rho^{\pm}} >$, fm	–	0.748	0.615 ± 0.006

Analysis of table 1 shows that the proposed model gives values comparable with modern experimental data and other models. It should be noted that the contribution of the structure functions of constituent quarks was not investigated in the work: the usage of mean-square radii $\left\langle r_q^2 \right\rangle = a \big/ m_q^2$ [6] can lead to values close to those of experimental data.

Conclusion. The paper presents a calculation of the electromagnetic characteristics of mesons consisting of quarks. It is shown that the use of model parameters obtained from leptonic decays and mesons leads to results on the electromagnetic mean-square radii of mesons that correlate with experimental data and other models.

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SPIN 1/2 PARTICLE WITH ANOMALOUS MAGNETIC MOMENT AND POLARIZABILITY IN THE EXTERNAL MAGNETIC FIELD

In the paper [1], within the general method by Gel'fand–Yaglom [2], starting with the extended set of representations of the Lorentz group, it was constructed a generalized equation for a spin 1/2 particle with two additional characteristics (concerning general formalism see