Physics of Complex Systems, 2024, vol. 5, no. 4
www.physcomsys.ru

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Condensed Matter Physics. Dielectrics

UDC 621.315.61:51-74

EDN HBQYTU
https://www.doi.org/10.33910/2687-153X-2024-5-4-187-194

Field dependence of the initiation time of electrical trees in polymer insulation

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For citation: Kiselevich, V. V. (2024) Field dependence of the initiation time of electrical trees in polymer insulation. *Physics of Complex Systems*, 5 (4), 187–194. https://www.doi.org/10.33910/2687-153X-2024-5-4-187-194 EDN HBQYTU

Received 22 July 2024; reviewed 25 September 2024; accepted 25 September 2024.

Funding: The study did not receive any external funding.

Abstract. The article analyses the known expressions that describe the field dependence of the inception time of electrical trees in polymer insulation. It also highlights the need to modify these expressions to correctly determine the threshold parameters of tree initiation. We used the methods of catastrophe theory to develop the equation that describes the dependence of the tree inception time in polymer dielectrics on the value of the maximum local electric field strength. The parameters of this equation for epoxy and polyethylene insulation are determined. A good agreement has been established between the literature experimental data and the field dependencies of the inception time of trees constructed according to the proposed equation. The geometric properties of the fold catastrophe function, which characterizes the regularities of the change in the time of initiation of electrical trees in polyethylene insulation, are considered.

Keywords: polymer insulation, electrical trees, inception time of trees, maximum local electric field strength, threshold parameters of trees initiation, catastrophe theory, fold catastrophe

Introduction

The widespread use of polymers as insulating materials for various electrical devices causes considerable interest in the study of electrical aging and breakdown processes in polymer dielectrics (Tumusova et al. 2020; Zakrevskii et al. 2020). Knowledge of breakdown mechanisms allows the correct selection of permissible levels of operating and test voltages of electrical insulating structures (Borisova, Marchenko 1998; Dissado, Fothergill 1992). Studies aimed at identifying the patterns of electrical aging of polymer dielectrics, in turn, contribute to the improvement of methods for calculating the life characteristics of electrical insulation (Blythe, Bloor 2005; Borisova, Kojkov 1979).

The process of electrical aging of polymer dielectrics is closely related to the initiation of trees — hollow branched channels, the development of which is accompanied by partial discharges and, ultimately, leads to a breakdown of insulation. At moderate electric field strengths, the root cause of the occurrence and development of trees in polymer insulation, as a rule, is the presence of gas inclusions, foreign impurities in its volume, as well as the presence of mechanically strained molecular bonds (Blythe, Bloor 2005; Kuchinskij 1979). At high values of local strength, the prevailing mechanism for the initiation of trees in polymers is the injection of electrons from metal electrodes (Blythe, Bloor 2005; Dissado, Fothergill 1992).

The breakdown of polymer insulation usually occurs at the stage of development of tree-like channels. Therefore, the main attention of the authors of scientific papers is aimed at studying the mechanisms of development and distribution of trees (Sitole, Nyamupangedengu 2020). Mathematical and physical models reflecting the patterns of development of electrical trees in polymers are described in detail in (Cai et al. 2019; Dissado et al. 1997; Noguchi et al. 2020; Wang et al. 2023). Analytical expressions that form the basis of such models are often very complex and cumbersome, which seriously constrains their real practical application. Moreover, in order to be able to reliably predict the service life of insulating structures, it is extremely necessary to know the quantitative characteristics of not only the development process but also the process of tree inception. This circumstance is of particular importance in the case of high-voltage electrical devices (insulators of overhead power lines, high- and ultrahigh-voltage cables, thin-film capacitors, etc.). This is due to the fact that the insulation of such devices is often subjected to considerable short-term overvoltages, as a result of which its breakdown can occur already at the stage of the inception of trees (Dissado, Hill 1990).

To quantitatively characterize the process of occurrence of trees, it is customary (Dissado, Fothergill 1992) to use such a parameter as the inception time, determined by the maximum electric field strength affecting the dielectric. The regularities of the change in the inception time of trees in polymer dielectrics when varying the value of the applied voltage were studied in (Dissado, Hill 1990; Hu et al. 2012; Sitole, Nyamupangedengu 2020; Tanaka, Greenwood 1978), where the exponential equation (Tanaka, Greenwood 1978) was used to process the experimental data. At the same time, as is known (Dissado, Fothergill 1992), at small levels of applied voltage, the exponential law can give greatly overestimated values of the tree inception time.

The purpose of this paper is to derive an equation to describe the dependence of the tree inception time on the value of the maximum electric field strength and to determine the parameters of this equation for epoxy and polyethylene insulation.

Formulation of the problem

It is known (Kuchinskij 1979) that the time t_{tr} of the tree inception in polymer insulation depends on the maximum strength E_m of the electric field at the place of occurrence of inhomogeneity. At a given voltage level, the maximum local strength E_m depends on two main factors: the shape of the inhomogeneity and the geometry of the electrodes. The known formulas for determining E_m, taking into account these factors, are presented in publications (Blythe, Bloor 2005; Dissado, Hill 1990; Kuchinskij 1979).

In practice, the time interval from the beginning of voltage supply to the insulation until the appearance of hollow channels with partial discharges at the level of tenths of pC is taken as t_{tr}. The minimum value of E_m, at which the appearance of trees is possible in electrical insulation, is called the threshold strength E_{th} of tree initiation, and the time t_{tr} corresponding to this strength is the threshold time t_{th} of tree inception. Under the given conditions of the experiment (the geometry of the electrodes, the thickness of the samples, and the environmental parameters), the value of E_{th} depends solely on the properties and state of the dielectric material.

The experimental curves reflecting the field dependence $t_{tr} = f(E_m)$ of the tree inception time in the polymer dielectric volume are satisfactorily described by the inverse power law (Kuchinskij 1979)

$$t_{tr}(E_m) = a / E_m^b \tag{1}$$

and exponential law (Sitole, Nyamupangedengu 2020; Tanaka, Greenwood 1978)

$$t_{tr}(E_m) = c\left[\exp\left(-kW_e^{3/2} / E_m\right) - \exp\left(-kW_e^{3/2} / E_{th}\right)\right]^{-1}. \tag{2}$$

Here a, b and c, k are the parameters determined by the properties of the dielectric material; W_e is the effective work function of electrons from the metal of the electrode.

The parameters of equations (1) and (2) are most often (Sitole, Nyamupangedengu 2020) selected based on the condition of the best smoothing of the experimental curves $t_{tr} = f(E_m)$. Due to the fact that the E_{th} value is not included in expression (1), an experiment is required to assess the threshold time t_{th} of tree initiation. However, the direct experimental determination of t_{th} is fraught with great difficulties, since when E_m decreases, the statistical spread between the obtained t_{tr} values increases, and as E_m approaches the preliminary predicted limit of E_{th}, the spread can reach two or three orders of magnitude (Dissado, Hill 1990; Sitole, Nyamupangedengu 2020). This is due not so much to the conditions of the experiment and the errors inevitable when approximating the experimental curves with theoretical expressions, as to the increase in the influence of the degree of inhomogeneity of the dielectric material

 https://www.doi.org/10.33910/2687-153X-2024-5-4-187-194

on the process of initiation and development of electrical trees when the maximum local strength tends to its threshold value.

In the exponential law (2), the limit value E_{th} corresponds to time $t_{th} \to \infty$. Meanwhile, in reality, even when the insulation is under voltage, which creates local areas of strength $E_m \approx E_{th}$ in it, the time t_{th} will be finite. In addition, the hypothesis about the correlation between the effective work function W_e and the strength E_{th} of the onset of tree initiation, used in deriving the exponential dependence (2), did not find proper experimental confirmation (Tanaka, Greenwood 1978).

Based on the foregoing, it can be concluded that obtaining an expression of field dependence $t_{tr} = f(E_m)$, free from these shortcomings of the known relations (1) and (2), is a rather relevant task. To derive the equation $t_{tr} = f(E_m)$, we will use the methods of catastrophe theory (Gilmore 1993).

Theoretical analysis

To be able to analyze the dependence $t_{tr} = f(E_m)$ in line with the mathematical catastrophe theory, first, it is necessary to identify the characteristic features — the flags of catastrophes (Gilmore 1993), which can be used to judge the presence of a catastrophe in a polymer dielectric exposed to an electric field. The fact of the existence of a threshold (critical) strength E_{th}, upon reaching which the process of tree inception begins in the dielectric, is the first sign of the probable presence of a catastrophe. The second sign of a catastrophe can be the above anomalous dispersion of the experimental values of the tree inception time t_{tr} in the case $E_m \approx E_{th}$.

The description of physical systems in catastrophe theory is performed using potential functions Φ (catastrophe functions), the form of which is determined by the complexity of the system under consideration. Suppose that the time t_{tr} of the inception of electric trees mainly depends on one significant parameter — the maximum local strength E_m of the electric field. Then, to establish the analytical relationship between t_{tr} and E_m in the first approximation, we can limit ourselves to considering the one-parameter fold catastrophe.

The known expression for the function Φ of the fold catastrophe has the form (Gilmore 1993):

$$\Phi(\chi;\, \mathcal{A}) = \chi^3/3 + \mathcal{A}\chi\,, \tag{3}$$

where χ is a variable that determines the state of the system under study; $\mathcal{A}$ is a control parameter, the smooth variation of which leads to a continuous or abrupt ('catastrophic') change in the variable χ.

The mathematical parameters χ and $\mathcal{A}$ included in equation (3) are dimensionless. To establish the relationship between the dimensionless parameters χ, $\mathcal{A}$ and their corresponding dimensional analogs x, A, normalization relations are used (Gilmore 1993):

$$\chi = x/x_{\mathcal{D}} - 1; \quad \mathcal{A} = A/A_{\mathcal{D}} - 1, \tag{4}$$

where x is the physical state variable; A is the physical control parameter; $x_{\mathcal{D}}$ and $A_{\mathcal{D}}$ are the critical parameters of the catastrophe function, which are understood as the values of the quantities x and A at the physical critical point identified with the mathematical point $\mathcal{D}$. In the case of a fold catastrophe, point $\mathcal{D}$ is a doubly degenerate critical point at which $\partial\Phi/\partial\chi = \partial^2\Phi/\partial\chi^2 = 0$ (Gilmore 1993).

It follows from equation (3) that the mathematical control parameter $\mathcal{A}$ is proportional to the square of the state variable χ: $\mathcal{A}\sim\text{-}\chi^2$. Therefore, given the proportionality $1/t_{tr} \sim E_m^b$ (where $b \geq 2$) characteristic of the physical quantities t_{tr} and E_m, it is convenient to take the strength E_m and not the time t_{tr} as a state variable; the value $v_{tr} = 1/t_{tr}$ will be considered a physical control parameter.

Let us match the physical critical point for which $E_m = E_{th}$ and $t_{tr} = t_{th}$ (or $v_{tr} = v_{th}$) to the mathematical point $\mathcal{D}$. Then the threshold physical parameters t_{th}, v_{th}, E_{th} will become equivalent to the critical parameters of the catastrophe function: $t_{th} \equiv t_{\mathcal{D}}$, $v_{th} \equiv v_{\mathcal{D}}$, $E_{th} \equiv E_{\mathcal{D}}$.

Next, using the normalization relations (4), we reduce the dimensional physical quantities E_m and v_{tr} to a dimensionless form

$$\mathcal{E}_m = E_m/E_{\mathcal{D}} - 1;\ \mathcal{V}_{tr} = v_{tr}/v_{\mathcal{D}} - 1 = t_{\mathcal{D}}/t_{tr} - 1\,, \tag{5}$$

where $\mathcal{E}_m$ is the dimensionless strength (mathematical state variable); $\mathcal{V}_{tr}$ is the dimensionless quantity, inverse of the tree inception time (mathematical control parameter).

Taking $\mathcal{X} = \mathcal{E}_m$ and $\mathcal{A} = -a_t \mathcal{V}_{tr}$, we obtain the equation of fold catastrophe from formula (3)

$$\Phi(\mathcal{E}_m;\ \mathcal{V}_{tr}) = \mathcal{E}_m^3/3 - a_t \mathcal{V}_{tr} \mathcal{E}_m, \tag{6}$$

characterizing the relationship between the maximum local electric field strength and the tree inception time in polymer insulation. Through a_t in the equation (6), we denote a positive scaling factor introduced to better visualize the geometric characteristics of the fold catastrophe function Φ. The minus sign at a_t is taken for the convenience of further analysis of the function Φ.

The analysis of the functions of catastrophes is based on the study of the features of their behavior in the vicinity of critical points when the control parameters change (Gilmore 1993). The profiles $\Phi = f(\mathcal{E}_m)$ of the fold catastrophe function, corresponding to the three characteristic values of the control parameter $\mathcal{V}_{tr}$, are shown in Fig. 1.

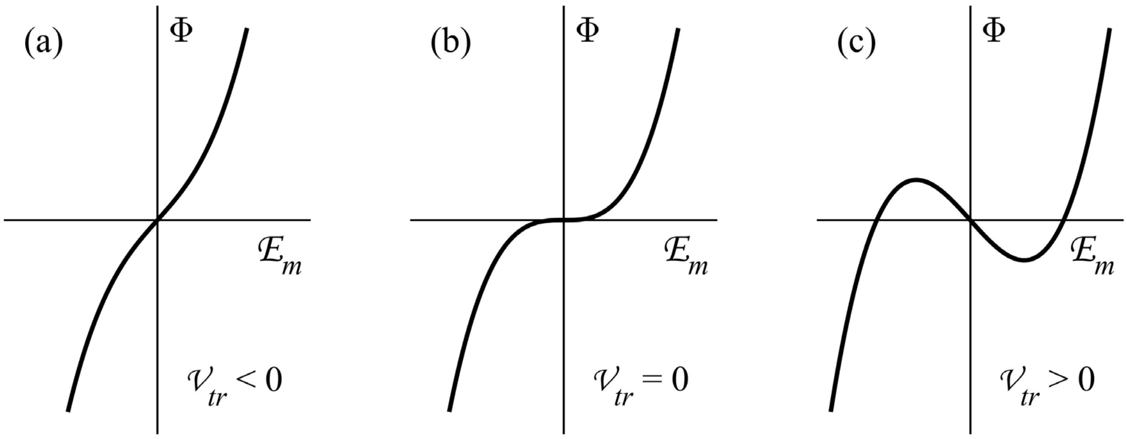

Fig. 1. Fold catastrophe function profiles

The position of the critical points of the function (6) is given by the equations:

$$\partial\Phi/\partial\mathcal{E}_m = \mathcal{E}_m^2 - a_t \mathcal{V}_{tr} = 0; \tag{7}$$

$$\partial^2\Phi/\partial\mathcal{E}_m^2 = 2\mathcal{E}_m = 0. \tag{8}$$

Since $a_t > 0$, when $\mathcal{V}_{tr} < 0$ the equation (7) has no solutions, and therefore, the catastrophe function $\Phi(\mathcal{E}_m;\ \mathcal{V}_{tr})$ has no critical points (Fig. 1a). With $\mathcal{V}_{tr} = 0$, the equation (7) has one root $\mathcal{E}_m = 0$, and the function $\Phi(\mathcal{E}_m;\ \mathcal{V}_{tr})$ has one critical point — the inflection point located at the origin (Fig. 1b). Moreover, due to the fact that equations (7) and (8) are simultaneously fulfilled in the case $\mathcal{V}_{tr} = 0$, this critical point is doubly degenerate. With $\mathcal{V}_{tr} > 0$, the equation (7) has two opposite in sign roots $\mathcal{E}_m = \pm\sqrt{a_t \mathcal{V}_{tr}}$, and the function $\Phi(\mathcal{E}_m;\ \mathcal{V}_{tr})$ has two critical points (Fig. 1c): the minimum point at $\mathcal{E}_m > 0$ and the maximum point at $\mathcal{E}_m < 0$. The minimum point characterizes the stable equilibrium state of the system, realized in practice, and the maximum point is an unstable state that does not take place in reality.

Using the conversion expressions (5), we write the relations that establish a correspondence between the characteristic values of dimensionless mathematical parameters ($\mathcal{E}_m$, $\mathcal{V}_{tr}$) and dimensional physical quantities (E_m, t_{tr}):

$$\begin{aligned} &\mathcal{E}_m < 0 \Leftrightarrow E_m < E_{\mathcal{D}}; \quad &&\mathcal{E}_m = 0 \Leftrightarrow E_m = E_{\mathcal{D}}; \quad &&\mathcal{E}_m > 0 \Leftrightarrow E_m > E_{\mathcal{D}}; \\ &\mathcal{V}_{tr} < 0 \Leftrightarrow t_{tr} > t_{\mathcal{D}}; &&\mathcal{V}_{tr} = 0 \Leftrightarrow t_{tr} = t_{\mathcal{D}}; &&\mathcal{V}_{tr} > 0 \Leftrightarrow t_{tr} < t_{\mathcal{D}}. \end{aligned} \tag{9}$$

Then, based on the relations (9) and the results of the analysis of equations (6)–(8), we can draw the following conclusions.

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In the case when an electric field is applied to the dielectric, at which the maximum strength is $E_m < E_{\mathcal{D}}$, trees in insulation are not observed, since the time of their initiation exceeds the threshold time, i. e., $t_{tr} > t_{\mathcal{D}}$. With an increase in the voltage applied to the dielectric, the local strength E_m becomes greater than the threshold strength $E_{\mathcal{D}}$, which leads to the initiation of trees in this dielectric over time $t_{tr} < t_{\mathcal{D}}$. The equality $t_{tr} = t_{\mathcal{D}}$ that occurs in the case $E_m = E_{\mathcal{D}}$ corresponds to the condition of the onset of the catastrophe — the activation of the process of the inception of trees in electrical insulation and the appearance of dependence $t_{tr} = f(E_m)$.

Substituting (5) into (7), we obtain the desired expression of the field dependence of the tree inception time in polymer insulation:

$$t_{tr}(E_m) = t_{\mathcal{D}} \big/ [1 + a_t^{-1}(E_m \big/ E_{\mathcal{D}} - 1)^2]. \tag{10}$$

Equation (10) is valid at $E_m \geq E_{\mathcal{D}}$, i. e., when conditions for the initiation of trees are created in the dielectric. The structure of this equation provides the finiteness of the value of the threshold time $t_{\mathcal{D}}$, when the minimum electric field strength $E_{\mathcal{D}}$ is induced in the dielectric. This fact is a strong point of the obtained expression (10).

Discussion of results

The processing of experimental data for polyethylene (Dissado, Hill 1990), cross-linked polyethylene (XLPE) (Hu et al. 2012), and epoxy resin (Sitole, Nyamupangedengu 2020) made it possible to find the parameters a_t, $t_{\mathcal{D}}$ and $E_{\mathcal{D}}$ of equation (10). The numerical values of these parameters are summarized in the Table 1, which additionally indicates the values of the threshold strength E_{th}, determined by the authors (Dissado, Hill 1990; Hu et al. 2012; Sitole, Nyamupangedengu 2020) using the equations of type (2).

Table 1. Parameters of the field dependence equation of the inception time of electrical trees in polymer insulation

Insulation	Parameters of the equation (10)			E_{th}, MV/m
	a_t	$t_{\mathcal{D}}$, s	$E_{\mathcal{D}}$, MV/m	
Polyethylene	1.289×10^{-4}	5.904×10^{5}	400.7	400
Cross-linked polyethylene (XLPE)	4.957×10^{-4}	3.522×10^{5}	301.4	441
Epoxy resin	1.518×10^{-3}	7.885×10^{4}	149.1	150

The Table 1 shows that for epoxy resin and polyethylene, the values of $E_{\mathcal{D}}$ and E_{th} agree well, while for cross-linked polyethylene, on the contrary, they diverge greatly. The main reason for the large scatter between $E_{\mathcal{D}}$ and E_{th} for XLPE seems to be the short duration of the experiment (60 min). A direct argument in favor of such an explanation is the authors' own indication (Hu et al. 2012) of the need to increase the duration of the experiment in order to obtain a more acceptable value of the threshold strength. An indirect argument is the fact that when estimating the $E_{\mathcal{D}}$ value for polyethylene, which was comparable to E_{th}, an experimental dependence $t_{tr} = f(E_m)$ was used, which was measured (Dissado, Hill 1990) over a substantially longer period of time (~110 h) than a similar dependence for cross-linked polyethylene.

As an example, Fig. 2 illustrates the field dependencies of the inception time of trees in the polymer dielectrics under study. Fig. 2 shows a high degree of correlation between the experimental data (Dissado, Hill 1990; Hu et al. 2012; Sitole, Nyamupangedengu 2020), indicated by symbols, and continuous curves constructed according to equation (10). This confirms the possibility of applying equation (10) to describe the field dependencies $t_{tr} = f(E_m)$ observed in polymer dielectrics.

In conclusion, we consider the main geometric images of the fold catastrophe function, given by equation (6). To do this, using the data (Dissado, Hill 1990), we will build a surface $\Phi = f(\mathcal{E}_m, \mathcal{V}_{tr})$ for polyethylene (Fig. 3).

The surface $\Phi = f(\mathcal{E}_m, \mathcal{V}_{tr})$, shown in Fig. 3, is the result of combining individual curves $\Phi = f(\mathcal{E}_m)$ corresponding to different values of the scaled control parameter $a_t\mathcal{V}_{tr}$. Therefore, it summarizes all the regularities of the change in the fold catastrophe function, resulting from the above analysis of the profiles $\Phi = f(\mathcal{E}_m)$ of this function. So, from Fig. 3, it can be seen that the curves $\Phi = f(\mathcal{E}_m)$ corresponding

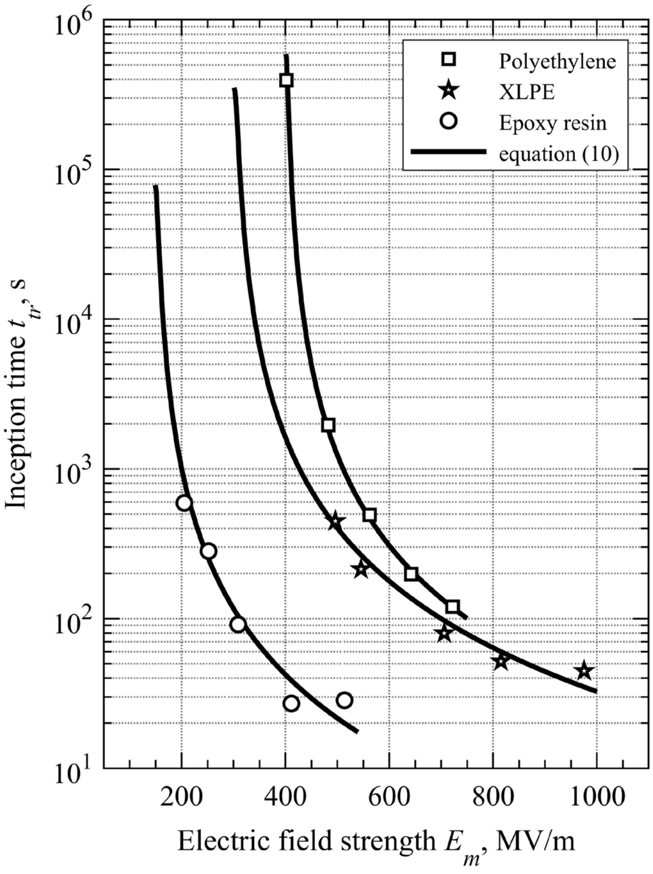

Fig. 2. Field dependencies of the inception time of trees in the polymer dielectrics

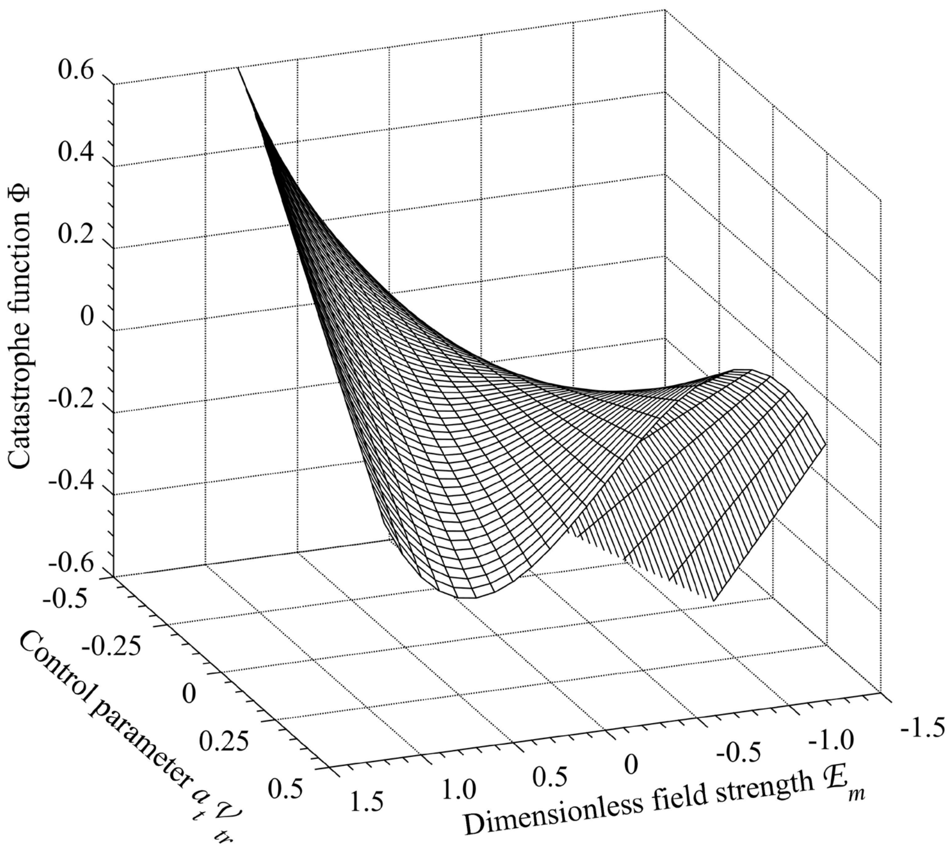

Fig. 3. Fold catastrophe function surface for polyethylene

https://www.doi.org/10.33910/2687-153X-2024-5-4-187-194

to $\mathcal{V}_{tr} < 0$ and forming the back side of the surface do not have a single critical point. On each of the curves $\Phi = f(\mathcal{E}_m)$ corresponding to $\mathcal{V}_{tr} > 0$ and forming the front side of the surface, there are two critical points: points of minimum (at $\mathcal{E}_m > 0$) and maximum (at $\mathcal{E}_m < 0$). The cross-section of the surface $\Phi = f(\mathcal{E}_m, \mathcal{V}_{tr})$ by the plane $\mathcal{V}_{tr} = 0$, of course, gives a curve $\Phi = f(\mathcal{E}_m)$ with a doubly degenerate critical point at the origin.

The projection of the surface $\Phi = f(\mathcal{E}_m, \mathcal{V}_{tr})$ onto the plane $\Phi\mathcal{E}_m$ is shown in Fig. 4. This projection is a set of curves $\Phi = f(\mathcal{E}_m)$ plotted for dimensionless strength $\mathcal{E}_m \in [0;\ 1.2]$ when the parameter $a_t\mathcal{V}_{tr}$ changes in the range from 0 to 0.5 in increments of 0.025. The upper curve $\Phi = f(\mathcal{E}_m)$ in Fig. 4 corresponds to the case $\mathcal{V}_{tr} = 0$, and the bottom one corresponds to the condition $a_t\mathcal{V}_{tr} = 0.5$. Fig. 4 shows that as the control parameter $\mathcal{V}_{tr}$ increases, the minimum of the function Φ becomes deeper and shifts towards larger values of dimensionless strength $\mathcal{E}_m$. This behavior of the function Φ is consistent with the known nature of the dependence $t_{tr} = f(E_m)$ change, namely, with a decrease in the tree inception time t_{tr} due to an increase in the maximum local strength E_m of the electric field acting on the polymer dielectric.

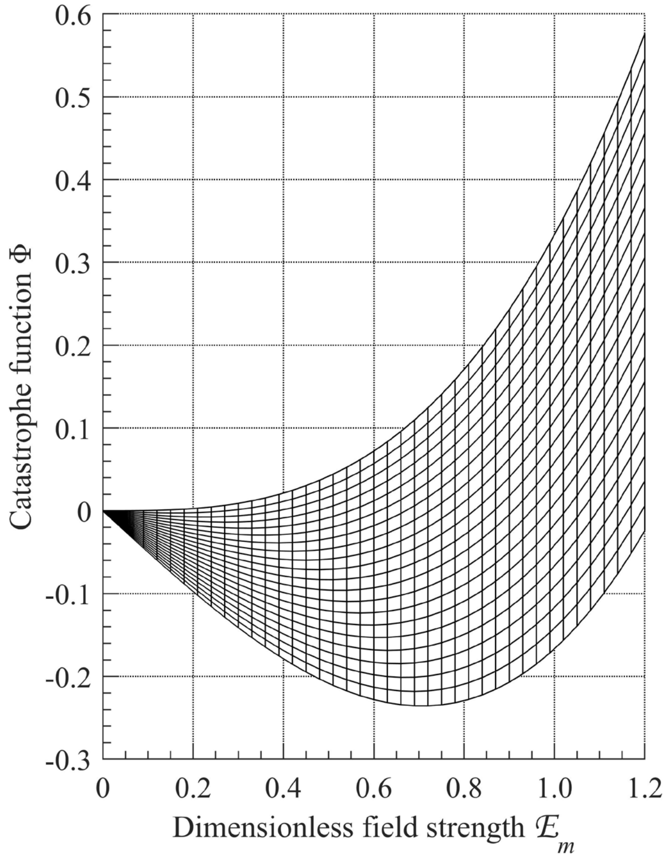

Fig. 4. Fold catastrophe function surface projection for polyethylene

Conclusions

Within the framework of catastrophe theory, the derivation of equation (10) is presented, which makes it possible, at a known maximum local electric field strength, to estimate the tree inception time in polymer insulation. The representation of equation (10) in the form of a time dependence $E_m = f(t_{tr})$ makes it possible to solve the inverse problem quite often encountered in practice — to determine the level of strength that excludes the appearance of trees in the insulation during a given time of its operation.

Field dependencies of the tree inception time are constructed and the threshold parameters of tree initiation in epoxy and polyethylene insulation are estimated. A high degree of correlation between the literature experimental data and field dependencies corresponding to equation (10) is shown. The geometric images of the fold catastrophe function, which reflect the general nature of the change in the inception time of electrical trees in polyethylene, are analyzed.

The results obtained in the article can be used to explain the patterns of tree inception arising from the data of operation and technical tests of polymer dielectrics.

Conflict of Interest

The author declares that there is no conflict of interest, either existing or potential.

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