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# Constituent quark masses in Poincaré-invariant quantum mechanics

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**Abstract.** The masses of the quarks in the Poincaré-invariant quantum mechanics are the constituent masses. Even in this framework it is possible to obtain an estimate of the constituent quark masses from the Ward identity for the axial current ant the current quark masses.

## 1. Introduction

It's well-known that quark masses are free parameters of composite quark models and quantum field theories, based on effective QCD-motivated lagrangians ($\chi$-PT model, models, based on sum rules etc.). There are two types of quark masses that do not coincide in magnitude: the mass of the current quark, estimated in processes with a significant transfer of the 4-momentum square, and constituent mass, which includes the mass of the gluon field around the quark and is estimated from the mass of hadrons and their quark composition (see, for example, [1]). The search for a connection between the current and the constituent masses of quarks in the framework of QCD is an actual and complex task.

In this paper, using the idea of the identity for the pseudoscalar density [2], we propose a procedure for fixing the values of the constituent masses of quarks with the use of current masses and calculating the parameters of the model, based on the point form of Poincaré-invariant quantum mechanics [3] (PIQM), using experimental data on electroweak decays of pseudoscalar and vector mesons.

## 2. Leptonic decay constants in PIQM

The scheme for solving the problem in PIQM is as follows: at the first stage a basis of direct product of two particles without interaction is constructed. In the case of a system of two particles with the masses $m_q$ and $m_Q$ and, respectively, with 4-momentums $p_1 = \big(\omega_{m_q}(\mathrm{p}_1), \mathbf{p}_1\big)$ and $p_2 = \big(\omega_{m_Q}(\mathrm{p}_2), \mathbf{p}_2\big)$ basis

$$|\mathbf{p_1}, \lambda_1\rangle \, |\mathbf{p_2}, \lambda_2\rangle \equiv |\mathbf{p_1}, \lambda_1, \mathbf{p_2}, \lambda_2\rangle \tag{1}$$

defines a reducible representation of the Poincaré group.

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In the second stage, using the Clebsch-Gordan decomposition for the Poincaré group (see [4],[5]) a basis of an irreducible representation, that characterizes the entire system, is constructed. To do this, we introduce a full momentum

$$\mathbf{P} = \mathbf{p_1} + \mathbf{p_2} \tag{2}$$

and the relative momentum $\mathbf{k}$ of particles [6]. The basis of the two-particle irreducible representation is defined by the quantum numbers of the total momentum (2), total angular momentum $J$ with a projection $\mu$ and effective mass of noninteracting particles

$$M_0 = \omega_{m_Q}(\mathrm{k}) + \omega_{m_q}(\mathrm{k}) \ , \ \ \omega_m(\mathrm{k}) = \sqrt{\mathrm{k}^2 + m^2}, \tag{3}$$

where $\mathrm{k} = |\mathbf{k}|$, and two additional numbers that remove the degeneracy of this basis.

As a result, the meson with momentum $\mathbf{Q}$, mass $M$, spin $J$ and it's projection $\mu$ is defined as the direct product of the state vectors of free particles (quarks) with wave function $\Phi^J_{\ell s}(\mathrm{k})$:

$$\begin{aligned} |\mathbf{Q}, J\mu, M\rangle = \int \mathrm{d}\mathbf{k} \sqrt{\frac{\omega_{m_q}(\mathrm{p}_1)\,\omega_{m_Q}(\mathrm{p}_2)\,M_0}{\omega_{m_q}(\mathrm{k})\,\omega_{m_Q}(\mathrm{k})\,\omega_{M_0}(\mathrm{P})}} \times \\ \times \sum_{\lambda_1\lambda_2}\sum_{\nu_1\nu_2} \Omega\left\{ {\ell \atop \nu_1,} {s \atop \nu_2,} {J \atop \mu} \right\}(\theta_k,\phi_k)\,\Phi^J_{\ell s}(\mathrm{k})\,D^{1/2}_{\lambda_1,\nu_1}(\mathbf{n}_{W_1})\,D^{1/2}_{\lambda_2,\nu_2}(\mathbf{n}_{W_2})\,|\mathbf{p}_1,\lambda_1,\mathbf{p}_2,\lambda_2\rangle\,, \end{aligned} \tag{4}$$

where

$$\begin{aligned} \Omega\left\{ {\ell \atop \nu_1,} {s \atop \nu_2,} {J \atop \mu} \right\}(\theta_k,\phi_k) = \mathbf{C}\left\{ {s_1 \atop \nu_1,} {s_2 \atop \nu_2,} {s \atop \lambda} \right\}\mathbf{C}\left\{ {\ell \atop m,} {s \atop \lambda,} {J \atop \mu} \right\} Y_{\ell\, m}(\theta_k,\phi_k) = \\ = \mathbf{C}\left\{ {s_1 \atop \nu_1,} {s_2 \atop \nu_2,} {s \atop \nu_1+\nu_2} \right\}\mathbf{C}\left\{ {\ell \atop \mu-(\nu_1+\nu_2),} {s \atop \nu_1+\nu_2,} {J \atop \mu} \right\} Y_{\ell\,\mu-(\nu_1+\nu_2)}(\theta_k,\phi_k)\ . \end{aligned} \tag{5}$$

In Eq.(5) the functions $\mathbf{C}\left\{ {s_1 \atop \nu_1,} {s_2 \atop \nu_2,} {s \atop \lambda} \right\}$, $\mathbf{C}\left\{ {\ell \atop m,} {s \atop \lambda,} {j \atop \mu} \right\}$ are Clebsch-Gordan coefficients of $SU(2)$ group, $Y_{lm}(\theta_k,\phi_k)$ are the spherical functions and $D(\mathbf{n}_W)$ is Wigner $D$-function.

Wave function (WF) $\Phi^J_{\ell s}(\mathrm{k})$ taking into account the number of quark colors $N_c$ is normalized by the condition

$$\sum_{\ell,s}\int_0^\infty \mathrm{d}\mathrm{k}\mathrm{k}^2 \left|\Phi^J_{\ell s}(\mathrm{k})\right|^2 = N_c. \tag{6}$$

The decay constants $P(q\bar{Q}) \to \ell + \nu_\ell$ and $\ell \to V(q\bar{Q}) + \bar{\nu}_\ell$ pseudoscalar and vector mesons with masses $M_P$ and $M_V$, after removing the element of the Kobayashi-Maskawa matrices, are determined by the expressions:

$$\left\langle 0 \left| \hat{J}^\mu_P(0) \right| \mathbf{P}, M_P \right\rangle = \frac{\mathrm{i}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_{M_P}(\mathrm{P})}} P^\mu f_P\ , \tag{7}$$

$$\left\langle 0 \left| \hat{J}^\mu_V(0) \right| \mathbf{P}, 1\lambda_V, M_V \right\rangle = \frac{\mathrm{i}}{(2\pi)^{3/2}} \frac{\varepsilon^\mu(\lambda_V)}{\sqrt{2\omega_{M_V}(\mathrm{P})}} M_V f_V\ . \tag{8}$$

Using relations (4), (7) and (8) matrix element of the quark currents one can obtain the integral representations of leptonic decays constants of pseudoscalar and vector mesons [7, 8]:

$$\begin{aligned} f_I(m_q, m_Q, \beta^I_{qQ}) = \sqrt{\frac{3}{2}}\frac{1}{\pi}\int\limits_0^\infty \mathrm{d}\mathrm{k}\mathrm{k}^2 \Phi(\mathrm{k},\beta^I_{qQ}) \sqrt{\frac{W^+_{m_q}(\mathrm{k})\,W^+_{m_Q}(\mathrm{k})}{M_0\,\omega_{m_q}(\mathrm{k})\,\omega_{m_Q}(\mathrm{k})}} \times \\ \times \left(1 + a_I \frac{\mathrm{k}^2}{W^+_{m_q}(\mathrm{k})\,W^+_{m_Q}(\mathrm{k})}\right)\ ,\ \ I = P,V;\ \ a_P = -1, a_V = 1/3\ , \end{aligned} \tag{9}$$

where

$$W_m^{\pm}(\mathrm{k}) = \omega_m(\mathrm{k}) \pm m\,. \tag{10}$$

Note, that the integral representation (9) match with the expressions for the decay constants of pseudoscalar and vector mesons in the instant and front-forms of PIQM [9, 10].

For fixing the parameters we use the constant of pseudoscalar density, which determined by relation (see e.g. [2, 11]):

$$\langle 0\,|\,\bar{Q}\gamma_5 q\,|\,\mathbf{P}, M_P\rangle = -\frac{\mathrm{i}}{(2\pi)^{3/2}}\frac{g_P}{\sqrt{2\,\omega_{M_P}(\mathrm{P})}}\,. \tag{11}$$

where axial current $\hat{J}_P^{\alpha}(x) = \bar{Q}(x)\gamma^{\alpha}\gamma_5 q(x)$ and pseudoscalar density $j^5(x) = \mathrm{i}\bar{Q}(x)\gamma_5 q(x)$ are related by [2]

$$\partial_\alpha \hat{J}_P^{\alpha}(x) = (\hat{m}_Q + \hat{m}_q) j^5(x). \tag{12}$$

The equations (7), (11) and (12) leads to the fact, that the constants $f_P$ and $g_P$ are related by [2, 11, 12]:

$$(\hat{m}_q + \hat{m}_Q)\, g_P = M_P^2 f_P\,. \tag{13}$$

where $\hat{m}_{q,Q}$ – current masses of quarks. The Eq. (13) is used, as usual, for $u$ and $d$-quarks (sometimes and for $s$-quark [2]). After the calculations we obtain the integral representation constants $g_P$ of pseudoscalar meson

$$g_P\left(m_q, m_Q, \beta_{qQ}^P\right) = \sqrt{\frac{3}{2}}\frac{1}{\pi}\int\limits_0^\infty \mathrm{dk}\,\mathrm{k}^2 \Phi\left(\mathrm{k}, \beta_{qQ}^P\right)\sqrt{\frac{M_0}{\omega_{m_q}(\mathrm{k})\,\omega_{m_Q}(\mathrm{k})}}\times$$
$$\times\left(\sqrt{W_{m_q}^{+}(\mathrm{k})\,W_{m_Q}^{+}(\mathrm{k})} + \sqrt{W_{m_q}^{-}(\mathrm{k})\,W_{m_Q}^{-}(\mathrm{k})}\right). \tag{14}$$

## 3. Numerical results and discussions

The values of the constituent masses of light quarks ($u, d$ and $s$) and the parameters of the wave function $\beta_{qQ}^I$ can be fixed by means of the experimental values of the decay constants [13]

$$f_{\pi^+}^{(\exp)} = (130.50 \pm 0.01 \pm 0.03 \pm 0.13)\ \text{MeV},\ f_{K^+}^{(\exp)} = (155.72 \pm 0.17 \pm 0.45 \pm 0.16)\ \text{MeV} \tag{15}$$

and the lattice calculations of the quark current masses [13, 14, 15]:

$$\hat{m}_u = \left(2.2^{+0.6}_{-0.4}\right)\ \text{MeV},\quad \hat{m}_d = \left(4.7^{+0.5}_{-0.4}\right)\ \text{MeV},$$
$$\frac{(\hat{m}_u + \hat{m}_d)}{2} = \left(3.7^{+0.7}_{-0.3}\right)\ \text{MeV},\quad \hat{m}_s = \left(96^{+8}_{-4}\right)\ \text{MeV}. \tag{16}$$

Wave function $\Phi(\mathrm{k}, \beta_{qQ}^I)$ is treated as a trial wave function with a parameter $\beta_{qQ}^I$, which can be chosen for the ground state in one of the species:

$$\Phi^{\mathrm{os}}(\mathrm{k}, \beta) = N_{os}\exp\left[-\frac{\mathrm{k}^2}{2\beta^2}\right],\quad N_{os} = \frac{2}{\pi^{1/4}\beta^{3/2}}\,, \tag{17}$$

$$\Phi^{\mathrm{coul}}(\mathrm{k}, \beta) = \frac{N_{coul}}{\left(1 + \mathrm{k}^2/\beta^2\right)^2},\quad N_{coul} = 4\sqrt{\frac{2}{\pi\beta^3}}\,, \tag{18}$$

$$\Phi^{\mathrm{pl}}(\mathrm{k}, \beta) = \frac{N_{pl}}{\left(1 + \mathrm{k}^2/\beta^2\right)^3},\quad N_{pl} = 16\sqrt{\frac{2}{7\pi\beta^3}}\,. \tag{19}$$

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**Table 1.** The values of the constituent masses of $u$,$d$ and $s$-quarks and parameters of the trial WF $\beta^P_{qQ}$

| WF | $m_u$, MeV | $\bar{m}_{ud}$, MeV | $\bar{m}_s$, MeV | $\beta^P_{ud}$, MeV | $\beta^P_{us}$, MeV |
|---|---|---|---|---|---|
| $\Phi^{\rm os}$ – (17) | $218.3 \pm 4.2$ | $219.6 \pm 4.2$ | $426.2 \pm 59.7$ | $370.8 \pm 9.3$ | $373.2 \pm 20.9$ |
| $\Phi^{\rm coul}$ – (18) | — | — | — | — | — |
| $\Phi^{\rm pl}$ – (19) | $235.2 \pm 14.7$ | $233.9 \pm 14.7$ | — | $562.1 \pm 28.2$ | — |

Assuming that the scale of the violation of isotopic symmetry for the constituent $u$ and $d$-quarks is the same as for current, i.e. $m_d - m_u = \hat{m}_d - \hat{m}_u = 2.5$ MeV from the system of equations

$$\begin{cases} f_P(m_q, m_Q, \beta^P_{qQ}) = f_P^{(\rm exp)}\ , \\ (\hat{m}_q + \hat{m}_Q)\ g_P(m_q, m_Q, \beta^P_{qQ}) = f_P^{(\rm exp)} M_P^2, \end{cases} \tag{20}$$

for pseudoscalar $\pi^\pm$ and $K^\pm$-mesons one can find the values of the constituent masses of the quarks and the parameters of the wave functions for $u, d$ and $s$-quarks (see table 1). Mark "–" means no solution to the system (20) within the error.

Values for the test oscillator (Gaussian) WF correlate with the model parameters [16] with linear confinement potential: $m_u = 220$ MeV, $m_s = 450$ MeV, $\beta^P_{ud} = 365.9$ MeV and $\beta^P_{us} = 388.6$ MeV. Also, constituent masses $m_u = 220$ MeV и $m_s = 419$ MeV are used in the Mock-model [17].

We note that this technique is sensitive to the choice of the trial wave function. So for the Coulomb WF (18) in general one can not find a solution of the system of equations (20). Wave function (19) is only suitable for $u, d$-quarks (using (16) for $s$-quark is problematic because of the strong violation of the chiral symmetry).

Using the values $G_F$, $|V_{ud}|$, $|V_{us}|$, $m_\tau$, $\tau_\tau$, $M_\rho$ и $M_{K^*}$ from [13] find, that experimental values $f_V^{(\rm exp)}$ are:

$$f_{\rho^+}^{(\rm exp)} = (209.3 \pm 1.5)\ \text{MeV}, \quad f_{K^{*+}}^{(\rm exp)} = (205.3 \pm 6.0)\ \text{MeV}\ . \tag{21}$$

From the equation $f_V(m_q, m_Q, \beta^V_{qQ}) = f_V^{(\rm exp)}$ we find the values of the parameters $\beta^V_{ud}$ and $\beta^V_{us}$ for oscillator WF (17):

$$\beta^V_{ud} = (310.9 \pm 2.2)\ \text{MeV}, \ \ \beta^V_{us} = (314.3 \pm 83.6)\ \text{MeV}\ . \tag{22}$$

## 4. Conclusion and remarks

The paper is dedicated to calculation the parameters of a model, based on the point form of Poincaré-invariant quantum mechanics. The obtained values of the constituent masses of quarks and parameters of the wave functions correlate with the values, obtained in models based on the light-front and instant form of dynamics. Authors note, that analysis of model parameters, depending on the type of trial wave function, also have been done.

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