The Role of Intermediate Scalar States in the Description of Pseudoscalar Meson Scattering Processes

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I. INTRODUCTION

The world of light quarks and hadrons is very reach with interesting phenomena. At short distances there are only free quarks and gluons. They are governed by Quantum Chromodynamics.At large distances there are only hadrons. This point-like particles are described by the standard quantum field equations.At intermediate distances color confinement and hadronization take place.As a result, the well-known pseudoGolstone boson octet emerges, of which pions and kaons are the two lightest particles. The determination of their low-energy interactions is crucial to testing our knowledge of QCD in the non-perturbative regime. Furthermore, light mesons appear in the final states of most hadronic interactions in experiments, which makes the understanding of their scattering processes necessary. On top of that, there exists a myriad of mesons that appear as resonances in these low-energy meson-meson scattering channels.For example, this is the case of the $\sigma/f_0(500)$ and $\kappa/K_0^*(700)$ resonances.The problem of describing scalar mesons is one of the most relevant in modern physics of elementary particles "below charm".First of all, this refers to the lightest of scalar particles $f_0(500)$, which plays a key role in describing nucleon -nucleon interactions, *pp*- scattering, and non-lepton interactions of kaons.

In contrast to vector and tensor resonances, the identification of scalar states remains a challenge throughout their study. The main experimental data on scalar mesons obtained , for example, in πN scattering on polarized /unpolarized targets, $p\overline{p}$ annihilation, central hadronic production, J/ψ, B-, D- and K-meson decays, $\gamma\gamma$ formation, and ϕ radiative decays [1]. There are several points of view on the structure of scalar mesons. There are many models [2]-[4] where this particles are treated as two quark systems.The $q\overline{q}$ quark model classification fits very well for pseudoscalar and vector mesons. Resonances without strangeness are nearly degenerate in mass, however, due to the fact that the strange quark is much heavier than the up and down quarks, resonances with strange quarks have a mass increment of about 150-300 MeV for each additional strange valence quark or anti-quark.The naive quark model predicts the lightest scalar $q\overline{q}$ nonet of mesons in a 1^3P_0 configuration.It is expected that P-wave states will be heavier than those in S-wave. In particular, models based in the naive quark model predict a mass above 1 GeV for those states .However, [5], one can see that there are many of them with a mass below this energy. Furthermore, as we can see , mesons with strangeness are lighter that some without. In some approaches scalar particles are treated as four-quark systems [6],[7]. The development of this approach have led to the models where the ground state of $qq\bar{q}\bar{q}$ system is considered as the $K\bar{K}$ molecule [8].

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In some approaches scalar mesons are associated with scalar glueballs, predicted by QCD. The lattice calculations [9] give the opportunity to estimate the scalar mesons masses. In [10]the descriptions of scalar particles in the framework of instanton liquid model of the QCD vacuum is proposed. There are a number of articles where scalar mesons are treated as a mixture of quark and gluonic states.

Experimental study of scattering processes began at the end of the 1970s.To obtain information about $\pi\pi$ scattering, one use electroweak K_{l_4}decays[11]-[13].The study of $K\pi$ scattering was carried out using $KN \to K\pi N'$ [14],[15].At present,D- meson decays are used to study these processes [16],[17].

Intensive theoretical studies of the scattering of light mesons are carried out in the Chiral Perturbation Theory (ChPT) [18], by means of a dispersion relation [1] and within framework of Lattice QCD [19]. In this paper, we study the PP-scattering in the QCM [20].

II. SCATTERING AMPLITUDE

Meson - meson scattering amplitude

$$M(s,t,u) = \langle P^c(q_1)P^d(q_2)|P^a(p_1)P^b(p_2)\rangle \tag{1}$$

is determined by the graph shown in Figure 1.

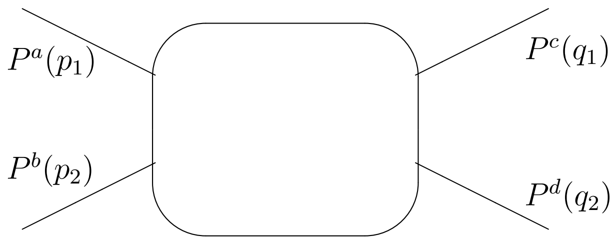

Fig.1

In general, it can be represented as

$$\begin{aligned} M(s,t,u) = &[Tr\{\lambda^a\lambda^c\lambda^d\lambda^b\} + Tr\{\lambda^a\lambda^d\lambda^b\lambda^c\}]B(s,t,u) + \\ &+[Tr\{\lambda^a\lambda^b\lambda^d\lambda^c\} + Tr\{\lambda^a\lambda^c\lambda^d\lambda^b\}]B(t,u,s) + \\ &+[Tr\{\lambda^a\lambda^d\lambda^c\lambda^b\} + Tr\{\lambda^a\lambda^b\lambda^c\lambda^d\}]B(u,s,t) \\ &+\delta^{ac}\delta^{bd}C(s,t,u) + \delta^{ab}\delta^{cd}C(t,u,s) + \delta^{ad}\delta^{bc}C(u,s,t) \end{aligned} \tag{2}$$

where

$$s = (p_1+p_2)^2, t = (p_1-q_1)^2, u = (p_1-q_2)^2 \tag{3}$$

λ^a-combination of Gell-Mann matrices that determine the kind of scattered mesons ,$B(s,t,u)$, $C(s,t,u)$ - invariant amplitudes. Generally invariant amplitude $A(s,t,u)(A = B, C)$ can be represented as

$$A(s,t,u) = Box(s,t,u) + V(s,t,u) + S(s,t,u) \tag{4}$$

where $Box(s,t,u)$,$V(s,t,u)$, $S(s,t,u)$ - denotes contributions from respectively box and graphs with intermediate vector and scalar mesons respectively.shown in Figure 2.

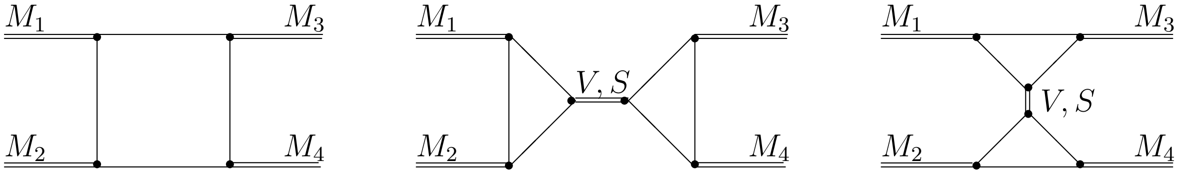

Fig.2

The hadronic interactions will be described in the QCM [20].

The hadron fields are assumed to arise after integration over gluon and quark variables in the QCM generating function. The transition of hadrons to quarks and vice versa is given by the interaction Lagrangian. In particular necessary interaction Lagrangians look like:

$$\mathcal{L}_M = \frac{g_M}{\sqrt{2}} M\bar{q}^a \Gamma \lambda^m q^a \tag{5}$$

where Γ- Dirak matrix,λ^m - is a corresponding SU(3)-matrix,q- quark vector

$$q_j^a = \begin{pmatrix} u^a \\ d^a \\ s^a \end{pmatrix}$$

The coupling constants g_M for meson-quark interaction are defined from so-called compositeness condition. It us convenient to use interaction constant in a form:

$$h_M = \frac{3g_M^2}{4\pi^2} = -\frac{1}{\tilde{\prod}'_M(m_M)} \tag{6}$$

instead of g_M in the further calculations. The QCM assumption is that the quark confinement is provided by nontrivial gluon vacuum background. The averaging of quark diagrams generated by S-matrix over vacuum gluon fields $\hat{B}_{VAC}$ is suggested to provide quark confinement and to make the ultraviolet finite theory. The confinement ansatz in the case of one-loop quark diagrams consists in following replacement:

$$\int d\sigma_{VAC} Tr|M(x_1)S(x_1,x_2|B_{VAC})...M(x_n)S(x_n,x_1|B_{VAC})| \longrightarrow$$

$$\int d\sigma_v Tr|M(x_1)S_v(x_1-x_2)...M(x_n)S_v(x_n-x_1)|, \tag{7}$$

where

$$S_v(x_1-x_2) = \int \frac{d^4p}{i(2\pi)^4} e^{-ip(x_1-x_2)} \frac{1}{v\Lambda_q - \hat{p}} \tag{8}$$

The parameter Λ_q characterizes the confinement rang of quark with flavor number $q = u,d,s$. We put $\Lambda_u = \Lambda_d = \Lambda_n$ in the most of decays. Parametre Λ_q has been fixed by fitting the decay constant of light mesons.

$$\Lambda_n = 460\ MeV \Lambda_s = 506\ MeV \tag{9}$$

The measure $d\sigma_v$ is defined as:

$$\int \frac{d\sigma_v}{v-\hat{z}} = G(z) = a(-z^2) + \hat{z}b(-z^2) \tag{10}$$

The function $G(z)$ is called the confinement function. $G(z)$ is independent on flavor or color of quark. $G(z)$ is an entire analytical function on the z-plane.$G(z)$ decreases faster then any degree of z in Euclidean region.The choice of $G(z)$,or as the same of $a(-z^2)$ andq $b(-z^2)$, is one of model assumptions.In the note [20] $a(-z^2)$ and $b(-z^2)$ are chosen as:

$$\begin{aligned} a(u) &= 2e^{-u^2-0.5u} \\ b(u) &= 2e^{-u^2-0.2u} \end{aligned} \tag{11}$$

III. BOX CONTRIBUTION

Box- diagram describing the scattering of pseudoscalar mesons is shown in Figure 2. and can be written as

$$\begin{aligned} Box = \frac{3(g_1g_2g_3g_4)}{4\pi^2}\int d\sigma \int \frac{d^4k}{4\pi^2 i} Tr\{i\gamma^5 S_{\Lambda_1\sigma}(\hat{k}+\hat{q_1})i\gamma^5 S_{\Lambda_2\sigma}(\hat{k}+\hat{q_1}+\hat{q_2})\cdot \\ \cdot i\gamma^5 S_{\Lambda_3\sigma}(\hat{k}+\hat{p_1})i\gamma^5 S_{\Lambda_4\sigma}(\hat{k})\} \end{aligned} \tag{12}$$

The following analytical expression have been obtained in the framework of the QCM:

$$\begin{aligned} Box(s,t,u,p_1^2,p_2^2,q_1^2,q_2^2,\Lambda_1,\Lambda_2,\Lambda_3,\Lambda_4) = \\ = \frac{4\pi^2\sqrt{h_1h_2h_3h_4}}{3} I_{box}(s,t,u,p_1^2,p_2^2,q_1^2,q_2^2,\Lambda_1,\Lambda_2,\Lambda_3,\Lambda_4) \end{aligned} \tag{13}$$

where $I_{box}(\ldots)$-structural integral obtained according to the QCM rules

IV. INTERMEDIATE VECTOR MESON CONTRIBUTION

The contribution of the intermediate vector mesons is determined by the diagrams shown in Figure 2 and can be written as:

$$V = T^\mu(P_+^2,p_1^2,p_2^2)h_V G^{\mu\nu}(P_+^2)T^\nu(Q_+^2,q_1^2,q_2^2) \tag{14}$$

where

$$P_+ = p_1+p_2; P_- = p_1-p_2; Q_+ = q_1+q_2; Q_- = q_1-q_2$$

Form factor $T^\mu(P_+^2,p_1^2,p_2^2)$, that is defined by triangular diagram ,was obtained as

$$\begin{aligned} T^\mu(P_+^2,p_1^2,p_2^2) = \int d\sigma \int \frac{d^4k}{4\pi^2} Tr\{\gamma^\mu S_\sigma(\hat{k})i\gamma^5 S_\sigma(\hat{k}+\hat{p_1})i\gamma^5 S_\sigma(\hat{k}+\hat{p_+})\} = \\ = F^+(P_+^2,p_1^2,p_2^2)P_+^\mu + F^-(P_+^2,p_1^2,p_2^2)P_-^\mu \end{aligned} \tag{15}$$

The product of the intermediate vector meson propagator by a constant of its interaction with the quarks in chain approximation has the form:

$$h_V G^{\mu\nu}(p^2) = \frac{1}{\Pi_1(p^2)-\Pi_1(m_V^2)}\{-g^{\mu\nu} + \frac{p^\mu p^\nu \Pi_2(p^2)}{\Pi_1(p^2)-\Pi_1(m_V^2)+p^2\Pi_2(p^2)}\} \tag{16}$$

Where $\Pi_1(p^2)$,$\Pi_2(p^2)$ - transverse and longitudinal parts of the polarization operator.

In general, the contribution of the intermediate vector meson is obtained in the form of:

$$\begin{aligned} V = \frac{1}{\Pi_1(m_V^2)}[F_p^+F_q^+(P_+Q_+) + F_p^+F_q^-(P_+Q_-) + \\ F_p^-F_q^+(P_-Q_+) + F_p^-F_q^-(P_-Q_-)] + \\ +\frac{1}{\Pi_1(P_+^2)-\Pi_1(m_V^2)}\frac{\Pi_2(P_+^2)}{\Pi_1(m_V^2)}[P_+^2(F_p^-F_q^-(P_-Q_-) + \\ F_p^-F_q^+(P_-Q_+)) + (P_+P_-)(F_p^-F_q^+(P_+Q_+) + F_p^-F_q^-(P_-Q_+))] \end{aligned} \tag{17}$$

V. INTERMEDIATE SCALAR CONTRIBUTION

In the QCM scalar meson interaction with quarks is determined by the Lagrangian:

$$L_S = \frac{g_S}{\sqrt{2}} S(x)\overline{q}(x)(I - i\frac{H}{\Lambda}(\overleftarrow{\partial} - \overrightarrow{\partial}))\lambda_s q(x) \tag{18}$$

where

$$\lambda_S = \begin{cases} \lambda^0 = \lambda^3 \Rightarrow a_0(980) \\ \lambda^0 \cos\delta_s + \lambda^8 \sin\delta_s \Rightarrow \sigma/f_0(500) \\ -\lambda^0 \sin\delta_S + \lambda^8 \cos\delta_S \Rightarrow f_0(980) \\ \frac{1}{\sqrt{2}}(\lambda^6 - i\lambda^7) \Rightarrow \kappa/K_0^*(700) \end{cases}$$

Form factor of $S \to PP$ is written in a form:

$$T_{SPP}(p^2, p_1^2, p_2^2, \Lambda_1, \Lambda_2, \Lambda_3) = \frac{3g_S g_{M_1} g_{M_2}}{4\pi^2} \times$$

$$\int d\sigma \int \frac{d^4k}{4\pi^2} Tr\{(I - i\frac{H}{\Lambda}(\overleftarrow{\partial} - \overrightarrow{\partial}))S_\sigma(\hat{k}, \Lambda_1)i\gamma^5 S_\sigma(\hat{k} + \hat{p_1}, \Lambda_2)i\gamma^5 S_\sigma(\hat{k} + \hat{p_+}, \Lambda_3)\} =$$

$$= \sqrt{h_S} F_{SPP}(p^2, p_1^2, p_2^2, \Lambda_1, \Lambda_2, \Lambda_3) \tag{19}$$

One have to mention that in the case of simple Lagrangian with $\Gamma_S = I$ the form factor $F_{SPP}(s)$ calculated for zero masses of final states and normalized to unity at $m_S = 0$ decreases with m_S and becomes equal zero at $m_S \approx 1000 \div 1100 MeV$. This leads to a theoretical value of $f_0(975) \to \pi\pi$ decay width to the underestimated $(\Gamma \sim 1 MeV)$ in comparison with the experimental one $\Gamma_{exp} = (26 \pm 5) MeV$. We propose the solution of this problem by introducing additional interaction with first derivative. So instead of simple "minimal" scalar vertex $\Gamma_S = 1$ one have to consider the following one:

$$\Gamma_S = I - i\frac{H}{\Lambda}\hat{\partial}, \tag{20}$$

where $\hat{\partial} \equiv \overrightarrow{\hat{\partial}} - \overleftarrow{\hat{\partial}}$, The parameter H and the mixing angle δ_S in (18),have been determined from the condition of consistency and have values:

$$H = 0,54; \delta_s = 17^\circ \tag{21}$$

The product of the coupling constants and scalar meson propagator has the form:

$$h_S G_S(p^2) = \frac{1}{\Pi_s(p^2, \Lambda_1, \Lambda_2) - \Pi_s(m_s^2, \Lambda_1, \Lambda_2)} \tag{22}$$

The contribution of scalar meson can be written as

$$S = \frac{1}{\Pi_s(p^2, \Lambda_1, \Lambda_3) - \Pi_s(m_s^2 \Lambda_1, \Lambda_3)} F_{SPP}(p^2, p_1^2, p_2^2, \Lambda_1, \Lambda_3, \Lambda_2) F_{SPP}(p^2, q_1^2, q_2^2, \Lambda_1, \Lambda_3, \Lambda_4) \tag{23}$$

VI. SCATTERING LENGTHES

The scattering length is a key quantity for understanding the basic properties of the hadron interaction at low energy. The scalar scattering lengthes a^I are calculated by the formula

$$a^I = \frac{1}{32\pi} T^I(m_+^2, 0, 0) \tag{24}$$

where $T^I(m_+^2, 0, 0)$-scattering amhlitude for given isospin I; $m_+ = m_{M_1} + m_{M_2}$

A. $\pi\pi$-scatterig

Scattering of π meson to π meson is possible via three channels $I = 0, 1, 2$. The scattering amplitudes for different channels T^I can be expressed by $A(s,t,u), A(t,s,u), A(u,t,s)$ as follows:

$$T^0(s,t,u) = 3A(s,t,u) + A(t,s,u) + A(u,t,s)$$
$$T^1(s,t,u) = A(t,s,u) - A(u,s,t)$$
$$T^2(s,t,u) = A(t,s,u) + A(u,s,t) \quad (25)$$

Because of the symmetry between the final mesons we have the equality $A(s,t,u) = A(s,u,t)$, therefore, are different from zero only $T^0(s,t,u)$ and $T^2(s,t,u)$. where $A(s,t,u)$ is defined by (4).

The amplitude $A(s,t,u)$ lis defined by (4) where $I^{\pi\pi}_{box}(s,t,u), S^{\pi\pi}(s,t,u), V^{\pi\pi}(s,t,u)$ are defined by (13),(17),(19) with $p_i^2 = m_\pi^2, \Lambda_i = \Lambda_n$. The contribution of intermediate vector meson turned out to be nigligeble.Thus, the diagrams defining $\pi\pi$ scattering are shown in figure 3.

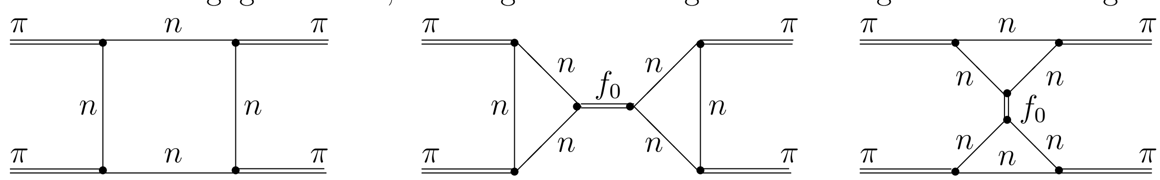

Fig.3

$$I^{\pi\pi}_{box}(s,t,u) = I_{box}(s,t,u,m_\pi^2,m_\pi^2,m_\pi^2,m_\pi^2,\Lambda_n,\Lambda_n,\Lambda_n,\Lambda_n) \quad (26)$$

$$S^{\pi\pi}(s,t,u) = F^2_{S\pi\pi}(s)\left(\frac{cos^2\delta_s}{\Pi_s(s)-\Pi_s(m_1^2)} + \frac{sin^2\delta_s}{\Pi_s(s)-\Pi_s(m_2^2)}\right) +$$
$$+F^2_{S\pi\pi}(t)\left(\frac{cos^2\delta_s}{\Pi_s(t)-\Pi_s(m_1^2)} + \frac{sin^2\delta_s}{\Pi_s(t)-\Pi_s(m_2^2)}\right) \quad (27)$$

$$F_{S\pi\pi}(x) = F_{SPP}(x,m_\pi^2,m_\pi^2,\Lambda_n,\Lambda_n,\Lambda_n)$$

m_1- $f_0(500)$ mass , m_2- $f_0(980)$ mass.

B. $K\pi$-scatterig

The total isospin of $K\pi$ scattering is equal $I = \frac{1}{1}, \frac{3}{3}$. The amplitude of this process is defined by (4).Box diagramm and diagrams with intermediate scalar mesons are shown in figure 4.

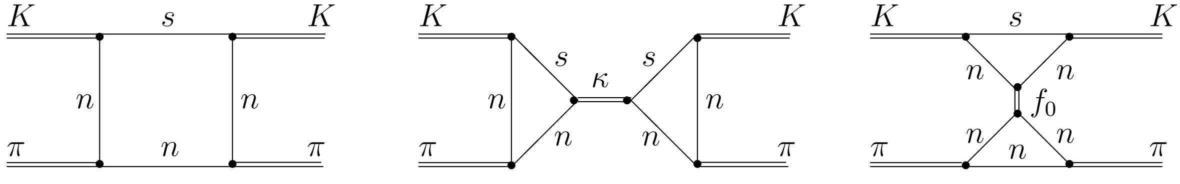

Fig.4

Corresponding terms in (4) are:

$$I_{box}^{K\pi}(s,t,u) = I_{box}(s,t,u,m_K^2,m_\pi^2,m_K^2,m_\pi^2,\Lambda_s,\Lambda_n,\Lambda_n,\Lambda_n) \tag{28}$$

$$S^{K\pi}(s,t,u) = F_{\kappa K_\pi}^2(s)\frac{1}{\sqrt{2}}\frac{1}{\Pi_s(s)-\Pi_s(m_\kappa^2)} +$$
$$+F_{f_0KK}F_{f_0\pi\pi}(t)(\frac{cos^2\delta_s}{\Pi_s(t)-\Pi_s(m_1^2)} + \frac{sin^2\delta_s}{\Pi_s(t)-\Pi_s(m_2^2)}) \tag{29}$$
$$F_{\kappa K\pi}(x) = F_{SPP}(x,m_K^2,m_\pi^2,\Lambda_s,\Lambda_n,\Lambda_n)$$
$$F_{f_0\pi\pi}(x) = F_{SPP}(x,m_\pi^2,m_\pi^2,\Lambda_n,\Lambda_n,\Lambda_n)$$
$$F_{f_0KK}(x) = F_{SPP}(x,m_K^2,m_K^2,\Lambda_n,\Lambda_n,\Lambda_s)$$

VII. ESTIMATION OF MASSES OF INTERMEDIATE SCALARS

One can see from (27) and (29) the numerical value of the contribution of diagrams with intermediate scalar meson to be dependent on the mass of $f_0(500)$ and $\kappa(700)$ meson, the value of which is unknown at present and has been intensively discussed in the literature [21]-[24],[1].

It turned out that in order to the obtained the numerical values of $\pi\pi$-scattering lengths a_0^0, a_0^2,$a_0^{\frac{3}{2}}$ and $a_0^{\frac{1}{2}}$ does not contradict the experimental data the mass of the intermediate $f_0(500)$ meson has to be chosen in the range of $500 \div 515$ MeV while the mass of the intermediate $\kappa(700)$ meson has to be chosen in the range of $650 \div 710$ MeV.

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