Hardronic Interactions of Strange Mesons

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Nonleptonic kaon decays $K \to \pi\pi$ and the matrix element of the $K^0 - \bar{K}^0$ transition were studied within the framework of the Quark Confined Model using effective four-quark Hamiltonians. The role of intermediate hadronic states in the description of these processes was studied. It is shown that correct consideration of the intermediate scalar state $f_0(600)$ allows one to obtain the relation $\gamma_{+-} = (\frac{\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K^0 \to \pi^+\pi^0)} = 433.84$, which is close to the experimental value, which makes it possible to explain the $\Delta I = \frac{1}{2}$ rule. Account of intermediate scalar, pseudo scalar and axial vector states made it possible to obtain the mass difference K_L^0 and K_S^0 $\Delta m_{LS} = 3.25 \times 10^{-15} GeV$, which is in good agreement with the experimental value.

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I. INTRODUCTION

Study of kaon decays has attracted the attention of researchers for decades. The reason is that kaon decays involve an intricate interplay between weak, electromagnetic and strong interactions.This decays are of extraordinary interest as a source of information about a New Physics beyond Standard Model.

The study of nonleptonic decays of kaons makes it possible to study the relationship between weak and strong interactions of quarks. One of the unsolved problems is the description of decays with a change in strangeness $|\Delta S| = 1$. The problem is that transitions with a change in isospin I by $\frac{3}{2}$ are significantly suppressed compared to transitions with $\Delta I = \frac{1}{2}$. Experimentally, this phenomenon manifests itself in the fact that the measured ratio [?]

$$\gamma_{+-} = (\frac{\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K^0 \to \pi^+\pi^0)} \approx 463$$

, which contradicts the estimate $\gamma_{+-} \sim O(1)$ obtained from perturbative calculations in the electroweak theory. Usually, states containing two π mesons in the final state are parametrized in terms of isotopic amplitudes A_I ($I = 0; 1$- isospin of the final state) [?]. Experimental data indicate that

$$Re|\frac{A_0}{A_2}| \approx 22$$

This relationship is called the $\Delta I = \frac{1}{2}$ rule, the nature of which has remained a mystery for almost sixty years.

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Along with the study of processes in which the strangeness S changes by 1, it seems interesting to study processes with $|\Delta S| = 2$, namely $K^0 - \bar{K}^0$ transitions. The standard six-quark scheme [?],[?]fails to explain the experimental mass splitting of $K_L^0 - K_S^0$ mesons.

The calculation of hadronic matrix elements in the most of theoretical approaches needs a great number of additional parameters and model assumptions . Kaon decays have been treated in several reviews and lecture notes during the past 30 years [?].

The purpose of this work is theoretical study nonleptonic interactions of kaons within the framework of the quark model, namely, to take into account the "large distances"contributions to the matrix elements of processes with $|\Delta S| = 1; 2$ and to obtain the decay parameters $K \to \pi\pi$ and $K^0 - \bar{K}^0$ transitions.

II. QUARK-MESON INTERACTIONS

The hadronic interactions is described in the QCM [?]. The hadron fields are assumed to arise after integration over gluon and quark variables in the QCM generating function. The transition of hadrons to quarks and vice versa is given by the interaction Lagrangian:

$$\mathcal{L}_M = \frac{g_M}{\sqrt{2}} M \bar{q}^a \Gamma \lambda^m q^a \tag{1}$$

where Γ- Dirak matrix,λ^m - is a corresponding SU(3)-matrix,q- quark vector

$$q_j^a = \begin{pmatrix} u^a \\ d^a \\ s^a \end{pmatrix}$$

The properties of scalars are not well established and its description needs an additional assumptions. We use the Lagrangian with additional interaction with derivative [?]:

$$L_S = \frac{g_s}{\sqrt{2}} s(x) \overline{q}(x) (I - i\frac{H}{\Lambda}(\overleftarrow{\hat{\partial}} - \overrightarrow{\hat{\partial}})) \lambda_S q(x) \tag{2}$$

with

$$\lambda_S = \begin{array}{c} diag(1, -1, 0) \Rightarrow a_0(980) \\ diag(\cos\delta_s, \cos\delta_s, -\sqrt{2}\sin\delta_s) \Rightarrow f_0(600) \\ diag(-\sin\delta_s, -\sin\delta_s, -\sqrt{2}\cos\delta_s) \Rightarrow f_0(980) \end{array}$$

We use the values of additional parameters H, δ_s fixed in [?]:

$$H = 0.54; \quad \delta_S = 17^\circ \tag{3}$$

The coupling constants g_M for meson-quark interaction are defined from so-called compositeness condition. It us convenient to use interaction constant in a form:

$$h_M = \frac{3g_M^2}{4\pi^2} = -\frac{1}{\tilde{\prod}'_M(m_M)} \tag{4}$$

instead of g_M in the further calculations. All hadron-quark interactions are described by quark diagrams induced by S matrix averaged over vacuum backgrounds.

The confinement ansatz in the case of one-loop quark diagrams consists in following replacement:

$$\int d\sigma_{VAC} Tr|M(x_1)S(x_1, x_2|B_{VAC})...M(x_n)S(x_n, x_1|B_{VAC})| \longrightarrow$$
$$\int d\sigma_v Tr|M(x_1)S_v(x_1 - x_2)...M(x_n)S_v(x_n - x_1)|, \tag{5}$$

where

$$S_v(x_1 - x_2) = \int \frac{d^4p}{i(2\pi)^4} e^{-ip(x_1-x_2)} \frac{1}{v\Lambda_q - \hat{p}} \tag{6}$$

The parameter Λ_q characterizes the confinement rang of quark with flavor number $q = u, d, s$. The measure $d\sigma_v$ is defined as:

$$\int \frac{d\sigma_v}{v - \hat{z}} = G(z) = a(-z^2) + \hat{z}b(-z^2) \tag{7}$$

The function $G(z)$ is called the confinement function. $G(z)$ is independent on flavor or color of quark. $G(z)$ is an entire analytical function on the z-plane.$G(z)$ decreases faster then any degree of z in Euclidean region.The choice of $G(z)$,or as the same of $a(-z^2)$ andq $b(-z^2)$, is one of model assumptions.I The parameter Λ_q characterizes the confinement rang of quark with flavor number $q = u, d, s$. We put $\Lambda_u = \Lambda_d = \Lambda_n$ in the most of decays. Parametre Λ_q has been fixed by fitting the decay constant of light mesons.

$$\Lambda_n = 460\ MeV \Lambda_s = 506\ MeV \tag{8}$$

III. THE EFFECTIVE WEAK INTERACTIONS

The quark weak interaction is described by effective Lagrangian $\mathcal{L}_w^{eff}$ for $\Delta S = 1$ - transitions . This Lagrangian is a sum of usual four-quark operators [**?**] :

$$\mathcal{L}_w^{eff} = \frac{G_F}{2\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{6} c_i O_i \tag{9}$$

where four-quark local operators O_i are chosen in following way:

$$\begin{aligned} O_1 &= (\bar{d}O_L^\mu s)(\bar{u}O_L^\mu u) - (\bar{d}O_L^\mu u)(\bar{u}O_L^\mu s) \\ O_2 &= (\bar{d}O_L^\mu u)(\bar{u}O_L^\mu s) + (\bar{d}O_L^\mu s)(\bar{u}O_L^\mu u) + \\ &\quad 2(\bar{d}O_L^\mu s)(\bar{d}O_L^\mu d) + 2(\bar{d}O_L^\mu s)(\bar{s}O_L^\mu s) \\ O_3 &= (\bar{d}O_L^\mu u)(\bar{u}O_L^\mu s) + (\bar{d}O_L^\mu s)(\bar{u}O_L^\mu u) - (\bar{d}O_L^\mu s)(\bar{s}O_L^\mu s) \\ O_4 &= (\bar{d}O_L^\mu u)(\bar{u}O_L^\mu s) + (\bar{d}O_L^\mu s)(\bar{u}O_L^\mu u) - (\bar{d}O_L^\mu s)(\bar{d}O_L^\mu d) \\ O_5 &= (\bar{d}O_L^\mu \lambda^a s) \sum_{q=u,d,s} (\bar{q}O_R^\mu \lambda^a q) \\ O_5 &= (\bar{d}O_L^\mu s) \sum_{q=u,d,s} (\bar{q}O_R^\mu q) \end{aligned} \tag{10}$$

Here $O_{R,L}^\mu = \gamma^\mu(1 \pm \gamma^5)$, λ^a-Gell-Mann matrices, acting in colour space. The numerical values of c_i depend on QCD parameters μ_s α_s [**?**]. In this note we use the set of coefficients c_i corresponding $\mu_s = 0.25$ GeV, $\alpha_s = 0.45$:

$$c_1 = -1.97 \quad c_2 = 0.12 \quad c_3 = 0.093 \quad c_4 = 0.47 \quad c_5 = -0.036 \tag{11}$$

The Hamiltonian with $\Delta S = 2$ is defined as [**?**],[**?**] :

$$H_w^{\Delta S=2} = \frac{G_F}{16\pi^2} V_{ud}^2 (V_{us}^*)^2 m_c^2 \eta O^{\Delta S=2} \tag{12}$$

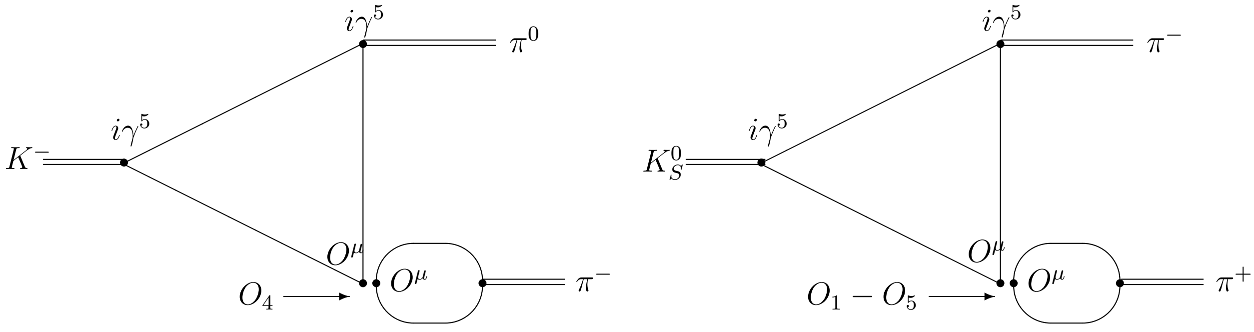
where

$$O^{\Delta S=2} = (\bar{s}O_L^\mu d)\,(\bar{s}O_L^\mu d) \tag{13}$$

Numerical value of parameter η, corresponding $\mu_s = 0.25$ GeV, $\alpha_s = 0.45$ is $\eta = 0,78$.

IV. $K \to \pi\pi$ DECAYS

The diagrams defining the decays $K \to \pi\pi$ Decays are shown in Fig.1 and Fig.2.



a **b**

Fig.1

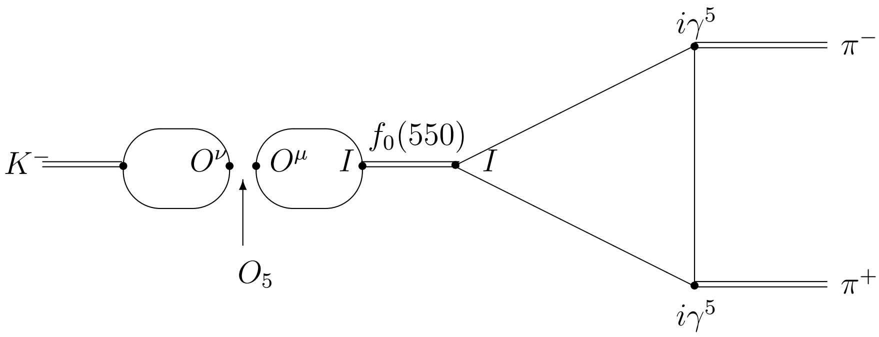


Fig.2

The corresponding amplitudes are written as:

$$A_0 = \frac{G_F}{2\sqrt{2}} V_{ud} V^*_{us} \{(c_1 + 2c_2 + 2c_3 + 2c_4) T^1_{K\pi\pi} + (c_5 + \frac{3}{16} c_6) T^5_{K\pi\pi} + (c_5 + \frac{3}{16} c_6) T^5_{Kf_0} D_{f_0)} (m^2_K g_{f_0\pi\pi}(m^2_K)\} \tag{14}$$

$$A_2 = \frac{G_F}{2\sqrt{2}} V_{ud} V^*_{us} c_4 T^1_{\pi\pi} \tag{15}$$

Here the notation is introduced:

$$T^i_{K\pi\pi} = \int dx_1 dx_2 dx_3 dy e^{ip_1x_1 + ip_2x_2 + ip_3x_3} \langle 0|T(L_\pi(x_1) L_\pi(x_2 L_K(x_3)) O^i(y))|0\rangle \tag{16}$$

$T^5_{Kf_0}$,$D_{f_0}(m^2_K$ are defined by (**??**) and (**??**), $g_{f_0\pi\pi}(m^2_K)$ is a form factor of $f_0(600) \to \pi\pi$ calculated with $m^2_{f_0} = m^2_K$ [**?**].

The matrix elements of the decays with $\Delta I = \frac{1}{2}$ ($K^0_S\pi^+\pi^-$,$K^0_S\pi^0\pi^0$) defined by both the contact diagrams (Fig 1b) and the pole one with intermediate $f_0(600)$ meson (Fig 2). The relative contributions from different diagrams to $K^0_S \to \pi^+\pi^-$ amplitude are given in Table 1.

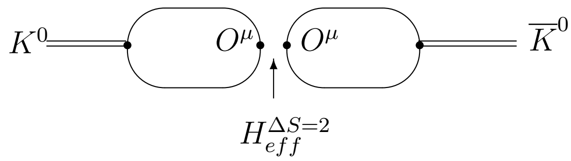
Table 1

$M(K^0_S \to \pi^+\pi^-)$	$\frac{M_{O_1-O_4}}{M}$	$\frac{M_{O_5}}{M}$	$\frac{M^{f_0(600)}_{O_5}}{M}$
$38,2 \times 10^{-8}$GeV	$0,228$	$0,016$	$0,756$

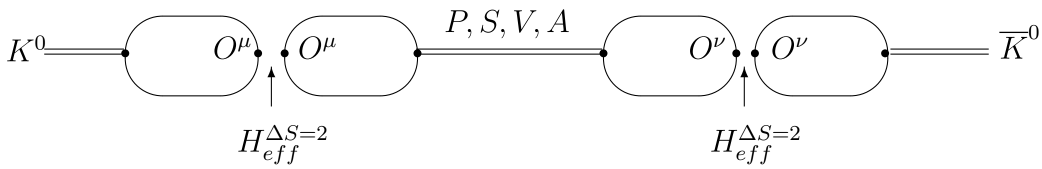
Table 1 showes that the contribution of an intermediate scalar state plays important role in the explanation of the $\Delta I = \frac{1}{2}$ rule.

V. $K_L^0 - K_S^0$ TRANSITION

The matrix element of the $K_0 - \overline{K}_0$ transition is determined by the graphs shown in Fig. 3, where $H_w^{\Delta S=2}$ and $H_w^{\Delta S=1}$ are determined by(**??**) , (**??**).



a



b

Fig.3

The matrix element defined by the diagram shown in Fig. 3a is written as:

$$M_{SD}(K^0 \to \overline{K}^0) = \frac{G_F}{16\pi^2} V_{ud}^2 (V_{us}^*)^2 m_c^2 \eta T^{\Delta S=2}_{K_0 \overline{K}_0} \tag{17}$$

where

$$T^{\Delta S=2}_{K^0\overline{K}^0} = \int dx_1 dx_2 dy e^{ipx_1 + ipx_2} \langle 0|T(L_{K^0}(x_1) L_{\overline{K}^0}(x_2) O^{\Delta S=2}(y))|0\rangle \tag{18}$$

Using the Fierz transformation, after standard calculations we obtain:

$$T^{\Delta S=2}_{K^0\overline{K}^0} = \frac{8}{3} m_K^2 f_K^2 \tag{19}$$

The contribution of "large" distances M^{LD} can be calculated by considering diagrams with all possible single-particle intermediate states (Fig. 3b).

The matrix element corresponding to the contribution of "large" distances can be written in the form:

$$M_{LD}(K^0 \to \overline{K}^0) = \sum_{M=P,S,A} D_M(m_K^2) \left[M(K^0 \to M)\right]^2 \tag{20}$$

where $M(K^0 \to M)$- matrix element of $K^0 \to M$ transition:

$$M(K^0 \to M) = \frac{G_F}{2\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{6} c_i T^i_{KM} \tag{21}$$

Here

$$T^i_{KM} = \int dx_1 dx_2 dy e^{ipx_1 + ipx_2} \langle 0|T(L_K(x_1) L_M(x_2) O^i(y))|0\rangle \tag{22}$$

$D_M(m_K^2)$-the propagator of the intermediate meson, which can be written in the form:

$$D_M(p^2) = \frac{1}{\Pi_M(p^2) - \Pi_M(m_M^2) + im_M\Gamma_M} \tag{23}$$

$\Pi_M(x)$-mass operator for pseudo scalar and scalar mesons and the transverse part of a polarization operator in the case of axial vector mesons.

Thus, the $K^0 - \overline{K}^0$ matrix element can be written as:

$$M_{12} = M_{SD}(K^0 \to \overline{K}^0) + M_{LD}(K^0 \to \overline{K}^0) \tag{24}$$

The $K_L^0 - K_0^S$ mass $K_L^0 - K_0^S$ difference is related with real part of M_{12} as:

$$\Delta m_{LS} = \frac{ReM_{12}}{2m_K} \tag{25}$$

$$ReM_{12} = M_{SD}(K^0 \to \overline{K}^0) + ReM_{LD}(K^0 \to \overline{K}^0) = \tag{26}$$

$$= M_{SD} + \sum_{M=P,S,A} \left[M(K^0 \to M)\right]^2 \frac{\Pi_M(p^2) - \Pi_M(m_M^2)}{\left(\Pi_M(p^2) - \Pi_M(m_M^2)\right)^2 + m_M^2\Gamma_M^2} \tag{27}$$

According to (**??**) $\Delta m_{LS} = \Delta m^{SD} + \Delta m^{LD}$, where Δm^{SD} is the contribution of "small" distances (by means of first term in (**??**), while the Δm^{LD} describes the contribution from "large" distances (by means of taken into account all possible intermediate one particle states).

As a result we have

$$\Delta m^{SD} = 2.01 \times 10^{-15} Gev \tag{28}$$

that is near 58 % of the experimental one [**?**].

$$\Delta m^{LD} = 1,21 \times 10^{-15} GeV \tag{29}$$

The relative contributions of the intermediate mesons to Δm^{SD} are shown in Table 2:

Table 2

Meson	π	η	η'	$f_0(600)$	$f_0(980)$	$a_0(980)$	$a_1(1260)$	$f_1(1285)$	$f_1(1420)$
	$-0,186$	$0,452$	$0,007$	$0,169$	$0,291$	$0,085$	$0,075$	$0,055$	$0,085$

Summing up (**??**) and (**??**)we finally have :

$$\Delta m_{LS} = 3,25 \times 10^{-15} GeV \tag{30}$$

Experimental value is [**?**]:

$$\Delta m_{LS}^{exp} = (3,484 \pm 0,006) \times 10^{-15} GeV \tag{31}$$

VI. CONCLUSION

Analytic expressions for the matrix elements of the nonlepton decays $K_S^0 \to \pi^+\pi^-$,$K^+ \to \pi^+\pi^0$ are obtained within the framework of the effective four-quark Hamiltonian.It turned out that the $K^+ \to \pi^+\pi^0$ decay with a change in isospin I by $\frac{3}{2}$ is described only by the operator O_4, while the decay of $K_S^0 \to \pi^+\pi^-$ with $\Delta I = \frac{1}{2}$ is described by all operators $O_1 - O_6$. It turned out that the amplification of the decay amplitudes with

$\Delta I = \frac{1}{2}$ is due to the influence of the operator O_5, but it is not sufficient to explain the amplification of the amplitudes by more than two orders of magnitude.

In the framework of the four-quark effective Hamiltonian with $\Delta S = 2$, the transition matrix element $K^0 - \bar{K}^0$ is obtained. It is shown that the account of strong and electroweak interactions only at small distances leads to the value of the mass difference K^0_L and K^0_S, which is only 58% of the experimental value.

It turned out that correct consideration of the intermediate scalar state $f_0(600)$ allows one to obtain the relation

$$\gamma_{+-} = (\frac{\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K^0 \to \pi^+\pi^0)} = 433.84$$

, which is close to the experimental one (**??**), which makes it possible to explain the rule $\Delta I = \frac{1}{2}$.

Account of intermediate scalar, pseudoscalar and axial vector states made it possible to obtain the mass difference K^0_L and K^0_S

$$\Delta m_{LS} = 3.25 \times 10^{-15} GeV$$

, which is in good agreement with the experimental value.

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