Charged Current Deep Inelastic Polarized *ep* Scattering

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The possible at electron-proton collider processes of the charged current deep inelastic ep scattering are suggested for the investigation of the proton spin structure. Several various approaches are proposed for the determination of the contribution to the proton spin the separate quark flavors, the valence quarks and the distributions of the polarized valence quarks by a help of the set of the observable quantities.

The complete electroweak radiative corrections of $O(\alpha^3)$ to the charged current deep inelastic polarized ep scattering in the framework of the quark parton model and the electroweak standard theory are calculated. The renormalization scheme on mass shell and Feynman gauge are used.

The numerical calculations of observable quantities (cross sections and the longitudinal polarization asymmetries) with allowance electroweak corrections at HERA energies have been made.

1. INTRODUCTION

Great successes have been achieved in the recent experiments on deep inelastic scattering (DIS) of polarized electrons and muons on the polarized fixed targets [1,2]. All present data are in excellent mutual agreement between the different experiments. The results of these experiments show violation of Ellis-Jaffe sum rule and confirmation of Bjorken sum rule by the data at the 10 % level. With the exception of E142, all data lead to the conclusion that the total quark contribution is small, $\Delta\Sigma \approx 0.2$, and that there is a small but significant negative contribution from the strange sea, $\Delta s \approx -0.1$. However, the origin of the nucleon spin has not been ascertained finally.

This call for the necessity of a new generation of experiments for the investigation of the nucleon spin structure. For this aim a series of experiments of SMC at CERN [3,4] and E154, E155 at SLAC [5] using improved experimental techniques is being prepared. The HERMES [6] and HELP [7] experiments using the pure polarized gas targets in electron-proton storage ring of HERA and LEP and the polarized electron beams with the energies 30 GeV and 46 GeV respectively may be interesting. The investigation of other DIS-processes for example νN – DIS [8] and the experiments on the colliders with both polarized beams [2] can be of significant interested for spin physics.

In the foreseeable future, the HERA collider may be able to accelerate polarized protons (see [9-13] and references therein). The important physics results at HERA is the determination of the small-x behavior of $g_1^p(x, Q^2)$, for which various, significantly different predictions exist.

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The above discussed opportunities of the polarized HERA are mainly connected with the neutral current processes. In addition to them there will be the charged weak current reactions, that is

$$e^{-(+)} + p \xrightarrow{W} \overset{(-)}{\nu_e} + X. \tag{1}$$

Certainly, since the W-boson mass (M_W) is very large, the cross sections of the processes (1) at small Q^2 are smaller than reactions with the neutral electromagnetic current. However, the results of the experiments on unpolarized ep – DIS investigation at HERA show that for $Q^2 \sim M_W^2$ the value of the processes (1) cross sections is compared to that of the processes with the neutral current, that is the strength of the weak interactions can be compared to that for the electromagnetic interactions for large Q^2. Therefore the investigation of the processes (1) with charged weak current is of interest as source of the independent data on the spin structure of nucleons.

2. ASYMMETRIES and PROTON SPIN STRUCTURE

The differential cross section of the polarized $e^{\pm}p$-DIS (1) can be written as the sum of the unpolarized cross section and the polarization contribution in the case of the longitudinal polarization of the electron or positron and proton .

$$\left.\frac{d^2\sigma}{dxdy}\right|_{e^{-(+)}} = \left.\frac{d^2\sigma}{dxdy}\right|^{a}_{e^{-(+)}} + \left.\frac{d^2\sigma}{dxdy}\right|^{p}_{e^{-(+)}}, \tag{2}$$

where $x = \frac{Q^2}{2p\cdot q}$ $y = \frac{p\cdot q}{p\cdot k_1}$ are the well-known dimensionless variables; $Q^2 = -q^2$, $q = k_1 - k_2$; $k_1(k_2), p$ are the 4-momenta of the incoming (outgoing) lepton and proton respectively. The expressions for $\left.\frac{d^2\sigma}{dxdy}\right|^{p}_{e^{-(+)}}$ can be obtained from the results of paper [8]. We find them using in formula (6) from [8] the replacements

$$\sigma_0 \to \chi_W = \frac{G^2 S_N}{2\pi}\left(\frac{1}{1+Q^2/M_W^2}\right)^2, \; g_i^{\nu,\bar\nu}(x) \to g_i^{e^-,e^+}(x),$$

where G is the Fermi constant, $S_N = 2pk_1$.

Then

$$\left.\frac{d^2\sigma}{dxdy}\right|^{p}_{e^{-(+)}} = p_N x \chi_W [(1+y_1^2) g_6^{e^{-(+)}}(x) \mp \frac{1}{2}(1-y_1^2) g_1^{e^{-(+)}}(x)], \tag{3}$$

where $y_1 = 1 - y$, p_N is the degree of longitudinal proton polarization.

By assuming that $p_N = \pm 1$ we obtain the polarization asymmetries

$$A_{\pm}(x,y) = \frac{\left(\left.\frac{d^2\sigma}{dxdy}\right|^{\downarrow\uparrow}_{e^-} \pm \left.\frac{d^2\sigma}{dxdy}\right|^{\uparrow\uparrow}_{e^+}\right) - \left(\left.\frac{d^2\sigma}{dxdy}\right|^{\downarrow\downarrow}_{e^-} \pm \left.\frac{d^2\sigma}{dxdy}\right|^{\uparrow\downarrow}_{e^+}\right)}{\left(\left.\frac{d^2\sigma}{dxdy}\right|^{\downarrow\uparrow}_{e^-} \pm \left.\frac{d^2\sigma}{dxdy}\right|^{\uparrow\uparrow}_{e^+}\right) + \left(\left.\frac{d^2\sigma}{dxdy}\right|^{\downarrow\downarrow}_{e^-} \pm \left.\frac{d^2\sigma}{dxdy}\right|^{\uparrow\downarrow}_{e^+}\right)}, \tag{4}$$

$$A_{e^-,e^+}(x,y) = \frac{\left(\left.\frac{d^2\sigma}{dxdy}\right|^{\downarrow\uparrow,\uparrow\uparrow}_{e^-,e^+}\right) - \left(\left.\frac{d^2\sigma}{dxdy}\right|^{\downarrow\downarrow,\uparrow\downarrow}_{e^-,e^+}\right)}{\left(\left.\frac{d^2\sigma}{dxdy}\right|^{\downarrow\uparrow,\uparrow\uparrow}_{e^-,e^+}\right) + \left(\left.\frac{d^2\sigma}{dxdy}\right|^{\downarrow\downarrow,\uparrow\downarrow}_{e^-,e^+}\right)}. \tag{5}$$

The first arrow corresponds to the direction of the electron ($\downarrow$) or positron ($\uparrow$) spin, the second arrow corresponds to the direction of the proton spin: $\uparrow$ ($p_N = +1$) and $\downarrow$ ($p_N = -1$).

Substituting in (4), (5) the expressions (2) and (3), we obtain for the asymmetries

$$A_{\pm}(x,y) = \frac{x\chi_W[(1+y_1^2)(g_6^{e^-}(x) \pm g_6^{e^+}(x)) - \frac{1}{2}(1-y_1^2)(g_1^{e^-}(x) \mp g_1^{e^+}(x))]}{\frac{d^2\sigma}{dxdy}\Big|_{e^-}^{a} \pm \frac{d^2\sigma}{dxdy}\Big|_{e^+}^{a}}, \quad (6)$$

$$A_{e^-,e^+}(x,y) = \frac{x\chi_W[(1+y_1^2)(g_6^{e^-,e^+}(x)) \mp \frac{1}{2}(1-y_1^2)(g_1^{e^-,e^+}(x))]}{\frac{d^2\sigma}{dxdy}\Big|_{e^-,e^+}^{a}}. \quad (7)$$

The structure functions (SFs) $g_i^{e^-,e^+}(x)$ of the processes (1) are connected with the neutrino SFs we have obtained [8] in the Quark Parton Model (QPM) (formulas (4)) as follows

$$g_i^{e^-,e^+}(x) = g_i^{\bar{\nu},\nu}(x) \quad (8)$$

and, hence, they are equal

$$g_1^{e^-,e^+}(x) = -2(\sum_q \Delta q(x) + \sum_{\bar{q}} \Delta\bar{q}(x)),$$

$$g_6^{e^-,e^+}(x) = \sum_q \Delta q(x) - \sum_{\bar{q}} \Delta\bar{q}(x). \quad (9)$$

Here $q = u,c,t$ (d,s,b), $\bar{q} = \bar{d},\bar{s},\bar{b}$ $(\bar{u},\bar{c},\bar{t})$ for the electron (positron) and $\Delta q(x)$ $(\Delta\bar{q}(x))$ designate the distribution functions of the polarized (anti)quarks.

With the help (9) and neglecting the contributions of the heavy quarks (c,b,t) the asymmetries (6) can be rewritten in QPM in the form

$$A_+(x,y) = \frac{2x\chi_W[\Delta u_v(x) + y_1^2\Delta\bar{d}_v(x)]}{\frac{d^2\sigma}{dxdy}\Big|_{e^-}^{a} + \frac{d^2\sigma}{dxdy}\Big|_{e^+}^{a}}, \quad (10)$$

$$A_-(x,y) = \frac{2x\chi_W[\Delta u(x) + \Delta\bar{u}(x) - y_1^2(\Delta d(x) + \Delta\bar{d}(x) + \Delta s(x) + \Delta\bar{s}(x))]}{\frac{d^2\sigma}{dxdy}\Big|_{e^-}^{a} - \frac{d^2\sigma}{dxdy}\Big|_{e^+}^{a}}, \quad (11)$$

where $\Delta q_v(x) = \Delta q(x) - \Delta\bar{q}(x)$ are the distribution functions of the polarized valence quarks.

Integrating (11) over x we obtain

$$\Delta u - y_1^2(\Delta d + \Delta s) = \int_0^1 \frac{dx}{2x\chi_W} A_-(x,y)\left(\frac{d^2\sigma}{dxdy}\Big|_{e^-}^{a} - \frac{d^2\sigma}{dxdy}\Big|_{e^+}^{a}\right) \quad (12)$$

where $\Delta q = \int_0^1 [\Delta q(x) + \Delta\bar{q}(x)]dx$.

So, we got the equation (12) that connects the quark flavors contributions to the proton spin with the observables $A_-(x,y)$ and $\frac{d^2\sigma}{dxdy}\Big|_{e\pm}^{a}$. If the couple of supplementary observable quantities e.g. a_3 and a_8 [1,2] which are in the QPM

$$a_3 = \Delta u - \Delta d, \; a_8 = \Delta u + \Delta d - 2\Delta s, \quad (13)$$

are used, the equations (12),(13) allow to determine the quark flavors contributions $\Delta u, \Delta d, \Delta s$ to the proton spin.

The investigation of the polarization effects in the processes (1) may be substantially expanded if reasonable suppositions regarding the polarized quark sea are used. In particular, it allows to decrease the number of supplementary observables that are used for the investigation of the proton spin and allows to get information about the spin distribution functions $\Delta u_v(x)$ and $\Delta d_v(x)$ and to estimate directly the valence quark contributions Δu_v and Δd_v.

3. ELECTROWEAK CORRECTION to POLARIZED ep – DIS with the CHARGED WEAK CURRENT

In this section the influence of radiative effects on polarized asymmetries (4) and (5) in the framework of the electroweak standard theory has been studied.

The electroweak corrections (EWC) to the processes (1) with unpolarized protons have been calculated with reliability by the different methods [14-17]. Here we calculate EWC to inclusive cross sections of longitudinally polarized electrons (positrons) on longitudinally polarized protons DIS in the case of charged current. As in [14], in this case the renormalization scheme on mass shell and Feynman gauge are used.

The cross sections (2) of the processes (1) in order $O(\alpha^3)$ will be obtained within the QPM by the cross sections of the subprocesses of longitudinally polarized electrons (positrons) on longitudinally polarized (anti)quarks scattering

$$e^{-(+)}(k_1, m_e) + \overset{(-)}{q}_f(p_1, m_f) \to \overset{(-)}{\nu}(k_2, 0) + \overset{(-)}{q}_{f'}(p_2, m_{f'}) \qquad (14)$$

and the corresponding parton distributions (in the brackets 4-momenta and masses of the particle are given).

The terms in (2) were obtained in the form:

$$\frac{d^2\sigma}{dxdy}\bigg|^m_{e^{-(+)}} = \frac{d^2\sigma^B}{dxdy}\bigg|^m_{e^{-(+)}} + \frac{d^2\sigma^H_R}{dxdy}\bigg|^m_{e^{-(+)}} + \frac{d^2\sigma^F_R}{dxdy}\bigg|^m_{e^{-(+)}}, \qquad (15)$$

where $m = a, p$.

In the first term of (15) one-loop correction δ_{1-loop} and the corrections relating to the bremsstrahlung of the soft photons δ^λ_R and the part of hard photons δ^H_1 factorize in front of Born cross section. The term δ^λ_R has infrared divergence, which was cancelled in the sum with the correction δ_{1-loop}

$$\delta_{1-loop} + \delta^\lambda_R = \delta^F_{1-loop} = \delta_{1-loop}(\lambda^2 \to \tilde{z}^2),$$

where $\tilde{z} = \sqrt{S_N}\frac{yx_1}{yx_1+x}$, $x_1 = 1 - x$.

For $\frac{d^2\sigma^B}{dxdy}\big|^m_{e^{-(+)}}$ we obtain

$$\frac{d^2\sigma^B}{dxdy}\bigg|^m_{e^{-(+)}} = \frac{\pi\alpha^2}{2s_w^4 S_0 Y_W^2}\Big\{\sum_{f=u(d),..} (B^m_{e^{-(+)}q})_0 F^m(x)(1 + \delta^F_{1-loop} + \delta^H_1) +$$
$$+ \sum_{f=\bar{d}(\bar{u}),...} (B^m_{e^{-(+)}\bar{q}})_0 F^m(x)(1 + \delta^F_{1-loop} + \delta^H_1)\Big\}, \qquad (16)$$

where $s_w = \sqrt{1 - c_w^2}$ is the sine of the weak mixing angle, $c_w = M_W/M_Z$, $F^{a/p}(z) = f^+(z) +/- f^-(z)$ are the parton distributions functions, $Y_W = Y + M^2_W$, $Y = Q^2$,

$$B^a_{e^-q} = B^a_{e^+\bar{q}} = S^2, B^a_{e^-\bar{q}} = B^a_{e^+q} = X^2,$$

$$B^p_{e^-q} = -B^p_{e^+\bar{q}} = p_N S^2, B^p_{e^+q} = -B^p_{e^-\bar{q}} = p_N X^2,$$

where $S = \xi S_N$, $X = \xi S_N y_1$; M_Z is the Z-boson mass, ξ is the QPM-parameter. The index "0" means the replacement $\xi \to x$.

The one-loop correction δ_{1-loop} from (16) is [14]

$$\delta_{1-loop} = \delta_W + \delta_{Vl} + \delta_{Vf} + \delta_{Sl} + \delta_{Sf} + \delta_{BW\gamma,f} + \delta_{BZW,f}. \qquad (17)$$

It includes W-boson, lepton, quark self-energy contributions (δ_W, δ_{Sl}, δ_{Sf}), leptonic and quark vertex corrections (δ_{Vl}, δ_{Vf}) and γW, ZW box contributions ($\delta_{BW\gamma,f}$ and $\delta_{BZW,f}$). The expressions for terms (17) in the case $e^-q(e^+\bar{q})$-scattering are given in [14,18]. The expressions for the $e^-\bar{q}(e^+q)$ - scattering can be obtained from the formulas for the $e^-q(e^+\bar{q})$-scattering by replacing: $S \leftrightarrow -X$, $f \leftrightarrow f'$. We denote this procedure by " ^ ".

The correction δ_1^H is: $\delta_1^H = -\frac{\alpha}{\pi}\int\limits_0^{v_{max}}\frac{dv}{v}(J(Y,v) - J(Y,0))$,

where $v_{max} = S_N x_1 y$,

$$J(Y,v) = Q_e^2 + c_q Q_e Q_f L_X - c_q Q_e Q_{f'}\frac{X}{S-Y}L_A + Q_f^2 - Q_f Q_{f'}\frac{Y}{S_X}L_u + Q_{f'}^2\frac{m_{f'}^2}{\tau},$$

$$L_X = ln\frac{S^2}{m_e^2 m_f^2}, L_A = ln\frac{(Y-S)^2}{m_e^2\tau}, L_u = ln\frac{S_X^2}{m_f^2\tau},$$

$$\tau = v_- + m_f^2, v = v_- + m_f^2 - m_{f'}^2, v_- = S - X - Y,$$

Q_j is the charge of the fermion j in the proton charge unit,
$c_q = +1(-1)$ for $e^-q, e^+\bar{q}$ ($e^-\bar{q}, e^+q$)-scattering.

The second and the third terms in (15) are the finite contributions of bremsstrahlung of real photon. In the process of calculation of this part of cross section the integration was conducted over the whole phase space of an unobserved real photon. This procedure was made analytically. We can not given the expressions for these terms in the framework this paper.

4. DISCUSSION OF NUMERICAL RESULTS AND CONCLUSIONS

To estimate the scale of radiative effects and it's influence on measurable observables in the processes (1), the numerical calculations of EWC $\delta^{a,p}_{e^-(+)}$ to the cross section (2) at typical for the collider HERA quantity $S_N = 10^{5\ 2}$ (Born cross section is denoted by index "0")

$$\left.\frac{d^2\sigma}{dxdy}\right|_{l^-(+)} = \left.\frac{d^2\sigma_0}{dxdy}\right|^a_{l^-(+)}(1 + \delta^a_{e^-(+)}) + \left.\frac{d^2\sigma_0}{dxdy}\right|^p_{l^-(+)}(1 + \delta^p_{e^-(+)}), \tag{18}$$

and the longitudinal polarization asymmetries $A_\pm(x,y)$ (4) and $A_{e^-,e^+}(x,y)$ (5) with allowance EWCs have been made.

We used the following standard set of electroweak parameters: $\alpha = 1/137.036$, $M_W = 80.0$ GeV, $M_Z = 91.0$ GeV, $M_H = 300.0$ GeV the fermion masses: $m_u = m_d = 30$ MeV, $m_s = 150$ MeV, $m_c = 1.5$ GeV, $m_b = 4.5$ GeV, $m_t = 170.0$ GeV and the parton distributions [19].

We calculated numerically only the lowest-order EWCs $\delta^p_{e^-(+)}$ to the polarization part of cross sections (2), since as has been noted in sect. 3 EWC $\delta^a_{e^-(+)}$ has been studied well enough.

The correction δ_{1-loop} gives the dominant contribution to EWC $\delta^p_{e^-(+)}$. Its biggest parts are given by the quark vertex corrections

$$\frac{\alpha}{4\pi}Q_f Q_{f'}(ln^2\frac{Y}{m_f^2} + ln^2\frac{Y}{m_{f'}^2}),$$

and γW-boxes

$\frac{\alpha Q_e}{4\pi}(Q_f C_f(S) - Q_{f'}C_{f'}(X)),\ C_f(s) = -(ln^2\frac{m^2}{s} + ln^2\frac{m_f^2}{s})$ (more than half of its whole magnitude over the entire kinematical range). The one-loop correction is positive everywhere, it grows weakly with the decrease of x and the growth of y both for e^-p-DIS and e^+p-DIS.

The contribution of bremsstrahlung in EWC $\delta^p_{e^-(+)}$ for $x \geq 0.1$ is not larger than $\sim \frac{1}{3}$ of its magnitude, but with the decrease of x it becomes of the same order as δ_{1-loop} but negative.

The numerical analysis has shown the asymmetries A_{e^+,e^-} depend poorly on y. The influence of EWC on the born asymmetry A^0_{e+} does not practically depend on x and y, and the absolute correction ΔA_{e^+} ($\Delta A = |A - A^0|$) does not exceed 1% anywhere. The magnitude ΔA_{e^-} at small x can reach 5%, but already at $x \sim 0.1$ it amounts to $\sim 1\%$. At middle and large x it does not exceed 1% over the whole range of y.

The behavior of the asymmetries $A_{-,+}$ and corrections to these is the following. The magnitude of corrections grows with decreasing x and y for A_-. The magnitude ΔA_- at $x = 0.1$ and $y = 0.05$ reaches 35%, but at $x = 0.5$ and $y = 0.05$ it amounts to less than 9%. At middle and large y it does not exceed 1% over the whole range of x. The correction ΔA_+ does not exceed 4% anywhere and it is most essential at small x and in the region $x \geq 0.5$, $y < 0.6$.

So, in this paper we propose using the possible at electron-proton collider processes of the charged *ep*-DIS for the investigation of the proton spin structure. Several various approaches are proposed for the determination of the contribution to the proton spin both the separate quark flavors and the valence quarks with the help of the set of the observable quantities and the distributions of the polarized valence quarks.

The EWCs to the differential cross sections of the processes (1) are calculated. The analysis of the numerical results shows that the corrections $\delta^p_{e^-(+)}$ to the cross sections may reach a few tens of per cent and the one-loop correction is the basic contribution to $\delta^p_{e^-(+)}$. The influence of EWC on the observable polarized asymmetries is most essential in the region of small x. As this same kinematical region will be of the utmost interest in the future polarized experiments at HERA, the procedure of radiative correction will occupy an important place in these experiments.

REFERENCES

1. Voss R. // CERN-PPE/95-131, 1995.
2. Hughes V.W. // CERN-PPE/95-178, 1995.
3. Nappi E. et al. // CERN/SPSLC95-27, 1995;
4. Nappi E. et al. // The Spin Muon Collaboration (SMC). CERN/SPS LC 95-28, 1995.
5. Arnold R.G. et al. // SLAC proposals E154 and E155, 1993.
6. Rith K. et al. // HERMES proposal. DESY. 1990.
7. Dick L. // Proc. 2nd Adriatico Research Conf. 1992. P.34.
8. Kuzhir P.P., Timoshin S.I, Shumeiko N.M. // Sov.J.Nucl.Phys. 1995. V.58. P.507.
9. Gehrman T., Stirling W.J. // DTP/95/82, Durham, 1995.
10. Bourrely C. et al. // CPT-96/PE. 3327, DSF-T-96/17, hep-ph/9604 204. Marseille, 1996.
11. Forte S. // CERN-TH/95-305, hep-ph/9511345. 1995.
12. Stratmann M., Vogelsang W. DO-TH 96/10, RAL-TR-96-033.1996.
13. Stratmann M. et al. RAL-TR-95-043(Tech.Rep.).1995.
14. Böhm M., Spiesberger H.// Nucl.Phys.1987.V.B304.P.749.
15. Bardin D.Yu. et al. Z.Phys.1989.V.C44.P.149.
16. Spiesberger H. DESY 89-175. 1989.
17. Spiesberger H. et al. CERN-TH.6447/92. 1992.
18. Böhm M. et al. // Forschr.Phys. 1986. V.34. P.687.
19. Bartelski J., Tatur S. // Acta Phys.Pol. 1996. V.B27. P4