Nonleptonic Decays of B- Mesons

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Abstract

We consider nonleptonic $B \to PP$ decays in the frame work of low-energy effective Hamiltonian. It turned out that the numerical values of branching rations strongly depend on the parameters of the effective Hamiltonian. It turned out that the decays, which are dominated by Penguin diagrams are unsatisfactorily described in the case of $N_c = 3$.

1 Introduction

Weak B decays play a very valuable role as they provide insights into the standard model (SM) world of CP violation, testing the Kobayashi-Maskawa (KM) mechanism, which allows CP violation in the SM of electroweak interactions, determining the CKM unitarity angles and exploring models beyond the SM. The two B-factories BaBar at SLAC and Belle at KEK have started a new era in the exploration of CP violation. The main objective of these factories is to test the SM and also to look for possible signals of new physics (NP). They are providing us with huge data and we expect many exciting years in B physics and CP violation. With this in mind, we see that B physics has many important contributions to particle physics. These topics are of great interest in particle physics and the knowledge of them will be improved with forthcoming experiments at Large Hadron Collider (LHC) [2].Two-body B decays have been considered one of the premier places to understand the interplay of QCD and electroweak interactions, to look for CP violation and over constrain the CKM parameters in the Standard Model. And indeed, exclusive modes

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$B \to PP, PV$ and VV , which have been extensively discussed in the literature have committed such expectations.Some of these channels provide methods for determining the angles of the unitarity triangle, allow to study the role of QCD and test some QCD- motivated models (see for example some recent reviews in [1]).The theoretical analysis of nonleptonic B decays requires the use of a low-energy effective Hamiltonian which is the starting point for the phenomenology of weak charmless two-body nonleptonic B decays can be described by a tree-level spectator diagram or a one-loop penguin diagram. Therefore, such decays are usually divided into three classes: (a) decays having both tree and penguin (QCD and EW) contributions, (b) decays having only tree contributions, and (c) decays having only penguin contributions. These decays can also have contributions from W-exchange, annihilation and vertical loop processes but their contributions are expected to be small. Present QCD technology, however, does not allow to undertake a complete calculation of the exclusive non-leptonic decay rates from first principles, such as provided by the lattice-QCD approach, as this requires the knowledge of the hadronic matrix elements $< M_1 M_2 | Heff | B >$.This matrix elements are usually approximated by product of two matrix elements of single currents.The corresponding form factors were treated in different approaches, for example The ISGW model [3],the WSB model [4],the CLFA model [6] and other. In this work we use relativistic quark model [5].

2 Effective Hamiltonian

The effective Hamiltonian describing the nonleptonic decays of B- mesons is given by [7]

$$H_{eff}(\Delta B = 1) = \frac{G_F}{\sqrt{2}}(V_{ub}V_{uq}^*(C_1(\mu)O_1^u(\mu) + C_2(\mu)O_2^u(\mu)) - \\ - V_{ub}V_{uq}^* \sum_{i=3}^{10} C_i(\mu)O_i(\mu)) + h.c., \quad (1) \\ q = d, s$$

where $O_i(\mu)$ $(i = 1, \dots, 10)$- are local four-quark operators, arising from different graphs :

- Current- current (tree) operators

$$O_1^u = (\overline{u}b)_{V-A}(\overline{q}u)_{V-A}$$
$$O_2^u = (\overline{u}_\alpha b_\beta)_{V-A}(\overline{q}_\beta u_\alpha)_{V-A}$$

- QCD- penguin operators

$$O_3 = (\overline{q}b)_{V-A}\sum_{q'}(\overline{q'}q')_{V-A}$$
$$O_4 = (\overline{q}_\alpha b_\beta)_{V-A}\sum_{q'}(\overline{q'_\beta}q'_\alpha)_{V-A}$$
$$O_5 = (\overline{q}b)_{V-A}\sum_{q'}(\overline{q'}q')_{V+A}$$
$$O_6 = (\overline{q}_\alpha b_\beta)_{V-A}\sum_{q'}(\overline{q'_\beta}q'_\alpha)_{V+A}$$

- Electro- weak penguin operators

$$O_7 = \frac{3}{2}(\overline{q}b)_{V-A}\sum_{q'} e_{q'}(\overline{q'}q')_{V+A}$$
$$O_8 = \frac{3}{2}(\overline{q}_\alpha b_\beta)_{V-A}\sum_{q'} e_{q'}(\overline{q'_\beta}q'_\alpha)_{V+A}$$
$$O_9 = \frac{3}{2}(\overline{q}b)_{V-A}\sum_{q'} e_{q'}(\overline{q'}q')_{V-A}$$
$$O_{10} = \frac{3}{2}(\overline{q}_\alpha b_\beta)_{V-A}\sum_{q'} e_{q'}(\overline{q'_\beta}q'_\alpha)_{V-A}$$

where α, β -color indices, q'- active quark in the loop at the scale m_b. Left- handed and right- handed currents are denoted as:

$$(\overline{q'}_\alpha q'_\beta)_{V-A} = \overline{q'}_\alpha \gamma_\mu (1-\gamma_5) q'_\beta$$
$$(\overline{q'}_\alpha q'_\beta)_{V+A} = \overline{q'}_\alpha \gamma_\mu (1+\gamma_5) q'_\beta$$

3 Factorization of matrix elements

The hadronic matrix elements $\langle O(\mu)\rangle$ are conventionally evaluated under the factorization hypothesis [8] so that $\langle O(\mu)\rangle$ is factorized into the product of two matrix elements of single currents, governed by decay constants and form factors.But this approach immediately leads to the following problem. Hadronic matrix element under factorization is renormalization scale μ independent as the vector or axial vector current is partially conserved.Consequently, the amplitude $C_i(\mu)\langle O\rangle_{fact}$ is not truly physical as the scale dependence of Wilson coefficients $C_i(\mu)$ does not get compensation from the matrix elements. The solution of this scale problem is to extract this μ dependence from the matrix element $\langle O(\mu)\rangle$ and and combine it with μ - dependent Wilson coefficient function to form μ - independent effective coefficient [9]

$$C(\mu)\langle O(\mu)\rangle = C(\mu)g(\mu)\langle O\rangle_{tree} = C^{eff}\langle O\rangle_{tree} \tag{2}$$

So, finally one arrives at the form

$$\begin{aligned}\langle M_1M_2|H_{eff}|B\rangle = Z_1\langle M_1|j^{\mu}|0\rangle\langle M_2|j_{\mu}|B\rangle +\\ +Z_2\langle M_1|j'^{\mu}|0\rangle\langle M_2|j'_{\mu}|B\rangle\end{aligned} \tag{3}$$

where j_μ and j'_μ are the corresponding (neutral or charged) $V-A$ currents.Z_1, Z_2 involve the C_i^{eff}, CKM factors $V_{qq'}$ and G_F.The C_i^{eff} coefficients enter in combination

$$\begin{aligned}a_i \equiv C_i^{eff} + \frac{1}{N_c}C_{i+1}^{eff}, (i = odd)\\ a_i \equiv C_i^{eff} + \frac{1}{N_c}C_{i-1}^{eff}, (i = even)\end{aligned} \tag{4}$$

All dynamical details are coded in this quantities (4).

4 Classification of factorized amplitudes

The generic decay amplitude can be written as

$$T + P + P_{EW} \tag{5}$$

where T, P and P_{EW} denote contribution from tree (O_1, O_2), QCD penguin (O_3-O_6) and electro-weak penguin(O_7-O_{10}) operators. So the amplitudes can be classified by domination of corresponding parts of (5).

Classes I-III- tree (T) dominated amplitudes.

- Class I. Only a charged meson can be generated directly from a singlet current. The relevant coefficient is a_1.

- Class II. Only a neutral meson can be generated directly from a singlet current.The relevant coefficient is a_2.

- Class III. Both charged and neutral mesons can be generated through the currents, involved in H_{eff}. The relevant coefficient is $a_1 + ra_2$, where r is decay dependent.

Classes IV,V- penguin dominated($T + P + P_{EW} \cong P + P_{EW}$) amplitudes.

- Class IV. Decays whose amplitudes involve one or more of dominant penguin coefficients a_4, a_6, a_9 and have the generic form:

$$M(B^0 \to h_1^{\pm} h_2^{\mp}) \cong \alpha_1 a_1 + \sum_{i=4,6,9} \alpha_i a_i$$

$$M(B^0 \to h_1^0 h_2^0) \cong \alpha_2 a_2 + \sum_{i=4,6,9} \alpha_i a_i$$

$$M(B^{\pm} \to h_1^{\mp} h_2^0) \cong \alpha_1 (a_1 + ra_2) + \sum_{i=4,6,9} \alpha_i a_i$$

- Class V . Decays whose amplitudes involve penguin coefficients a_3, a_5, a_7, a_{10} interfering significantly with one of dominant penguin coefficients a_4, a_6, a_9.

Decay modes classified according above scheme, are listed in table 1.

Table 1

class	Decay Mode
I	$B^0 \to \pi^-\pi^+, B^0 \to \rho^-\pi^+, B^0 \to \pi^-\rho^+, B^0 \to \rho^-\rho^+, B^0 \to \rho^- K^+$
II	$B^0 \to \pi^0\pi^0$ and other from set of $\pi^0, \eta, \eta', \rho^0, \omega$
III	$B^+ \to (\pi^+, \rho^+)(\pi^0, \eta, \eta', \rho^0, \omega)$
IV	$B^+ \to K^+\pi^0, B^+ \to K^+\eta^{(\prime)}, B^0 \to K^0\pi^0$
IV pure penguin	$B^+ \to K^0\pi^+, B^+ \to K^+\overline{K^0}, B^0 \to K^0\overline{K^0}$

5 Amplitudes of $B \to PP$ decays.

The factorized amplitudes are defined by (3), so one has to calculate matrix elements $\langle M_1|j^\mu|0\rangle$ and $\langle M_2|j_\mu|B\rangle$. In the case of $B \to PP$ decays the first one is defined by pseudoscalar decay constant:

$$\langle P(p)|\bar{q}\gamma^\mu(1-\gamma_5)q|0\rangle = if_P p^\mu \quad (6)$$

The second one can be written in the standard form [5]:

$$\langle P(p_2)|\bar{q}\gamma^\mu q|B(p_1)\rangle = F_+(q^2)(p_1+p_2)^\mu + F_-(q^2)q^\mu \quad (7)$$

where $q^\mu = p_1^\mu - p_2^\mu$.

So, matrix element can be written as:

$$\langle P_1(p_1)P_2(p_2)|H_{eff}|B(p_B)\rangle = i\frac{G_F}{\sqrt{2}}C(a_i, V_{qa'}, m_q)f_{P_2}((m_B^2 - m_1^2)F_+(m_2^2) + \\ + m_2^2 F_-(m_2^2)) + (1 \leftrightarrow 2) \quad (8)$$

where $C(a_i, V_{qb}, V_{qq'})$- is certain combination of effective coefficients a_i, defined by (4), CMK-matrix elements $V_{qq'}$ and quark masses $m_q, q = u, d, s, b$. Matrix elements for different channels of $B \to PP$ decay have the following expressions

$$M(\overline{B^0} \to \pi^-\pi^+) = -i\frac{G_F}{\sqrt{2}}f_\pi\left((m_B^2 - m_\pi^2)F_+^\pi(m_\pi^2) + m_\pi^2 F_-^\pi(m_\pi^2)\right) \times \\ \times \left\{V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^*\left[a_4 + a_{10} + (a_6 + a_8)\frac{2m_\pi^2}{(m_b - m_u)(mu + md)}\right]\right\} \quad (9)$$

$$M(B^- \to \pi^-\pi^0) = -i\frac{G_F}{\sqrt{2}}f_\pi\left((m_B^2 - m_\pi^2)F_+^\pi(m_\pi^2) + m_\pi^2 F_-^\pi(m_\pi^2)\right) \times \\ \times \left\{V_{ub}V_{ud}^*(a_1 + a_2) - V_{tb}V_{td}^*\frac{3}{2}\left[a_9 + a_{10} - a_7 + a_8\frac{2m_\pi^2}{(m_b - m_d)(md + md)}\right]\right\} (10)$$

$$M(\overline{B^0} \to \pi^0\pi^0) = -i\frac{G_F}{\sqrt{2}}f_\pi\left((m_B^2 - m_\pi^2)F_+^\pi(m_\pi^2) + m_\pi^2 F_-^\pi(m_\pi^2)\right) \times \\ \times \left\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\left[a_4 - \frac{1}{2}a_{10} + \frac{3}{2}a_7 - \frac{3}{2}a_{10} + \frac{\left(a_6 - \frac{1}{2}a_8\right)2m_\pi^2}{(m_b - m_d)(md + md)}\right]\right\} (11)$$

$$M(B^- \to \pi^- K^0) = i\frac{G_F}{\sqrt{2}} f_K \left((m_B^2 - m_\pi^2)F_+^\pi(m_K^2) + m_K^2 F_-^\pi(m_K^2)\right) \times$$
$$\times V_{tb}V_{td}^* \left[a_4 - \frac{1}{2}a_{10} + \frac{3}{2}a_7 - \frac{3}{2}a_9 + (a_6 - \frac{1}{2}a_8)\frac{2m_\pi^2}{(m_b - m_d)(md + ms)}\right] \quad (12)$$

$$M(B^- \to K^- K^0) = i\frac{G_F}{\sqrt{2}} f_K \left((m_B^2 - m_K^2)F_+^K(m_K^2) + m_K^2 F_-^K(m_K^2)\right) \times$$
$$\times V_{tb}V_{td}^* \left[a_4 - \frac{1}{2}a_{10} + \frac{3}{2}a_7 - \frac{3}{2}a_9 + (a_6 - \frac{1}{2}a_8)\frac{2m_\pi^2}{(m_b - m_d)(md + ms)}\right] \quad (13)$$

6 Numerical results for branching rates of $B \to PP$ decays.

The form factors $F_\pm^{\pi,K}$ entering (9)-(13) were calculated in the quark model [5].This form factors were interpolated by the function

$$F(q^2) = \frac{F(0)}{1 - as + bs^2}, s = \frac{q^2}{m_B^2} \quad (14)$$

The values of $F(0)$ and parameters a, b for $F_\pm^{\pi,K}$ form factors are listed in table 2.

Table 2

	F_+^π	F_-^π	F_+^K	F_-^K
$F(0)$	0.24	-0.24	0.29	-0.28
a	1.87	1.97	1.85	1.95
b	0.93	1.04	0.96	1.09

For quark masses we use following numerical values [11]:

m_u(GeV)	m_d(GeV)	m_s(GeV)	m_b(GeV)
0.0025	0.054	0.1	4.19

The CKM matrix is parameterized in standard form [12]

$$CKM = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \overline{\rho} - i\overline{\eta}) & -A\lambda^2 & 1 \end{pmatrix}$$

with $\overline{\rho} = \rho(1 - \frac{1}{2}\lambda^2)$, $\overline{\eta} = \eta(1 - \frac{1}{2}\lambda^2)$.

We take for the Wolfenstein parameters the central values from the global fit [11]:

$$\lambda = 0.2259, \quad A = 0.82, \quad \rho = 0.158, \quad \eta = 0.343 \tag{15}$$

The decay rate for $B \to PP$ is defined in standard way:

$$\Gamma(B \to P_1 P_2) = \frac{p_c}{8\pi m_B^2} |M(B \to P_1 P_2)|^2 \tag{16}$$

where p_c - is the c.m. momentum of the decay particles.

$$p_c = \frac{\sqrt{m_B^2 - (m_1 + m_2)^2 (m_B^2 - (m_1 - m_2)^2)}}{2m_B} \tag{17}$$

We use the numerical values of a_i calculated with different sets of C_i^{eff} from [7], [9]and [10]. Our results for branching ratios of $B \to PP$ are listed in tables 3-5.

Table 3. Branching ratios for $B \to PP$(in units 10^{-6}) with a_i calculated with set of C_i^{eff} from [7] with $N_c = 2, 3, \infty$:

Channel	Class	Exp. [11]	$N_c = 2$	$N_c = 3$	$N_c = \infty$
$B^0 \to \pi^+\pi^-$	I	5.13 ± 0.31	7.12	7.04	9.36
$B^0 \to \pi^0\pi^0$	II	1.62 ± 0.31	1.13	0.064	1.71
$B^+ \to \pi^+\pi^0$	III	5.7 ± 0.5	4.9	3.88	2.20
$B^+ \to K^+\pi^0$	IV	12.9 ± 0.6	9.46	0.97	12.3
$B^0 \to K^+\pi^-$	IV	19.4 ± 0.6	3.75	1.06	4.96
$B^0 \to K^0\pi^0$	IV	9.5 ± 0.8	4.65	0.0621	7.34
$B^+ \to \pi^+K^0$	IV	23.1 ± 1.0	13.2	0.0615	2.00
$B^+ \to K^+\overline{K}^0$	IV	1.36 ± 0.27	15	1.01	22.6
$B^0 \to K^0\overline{K}^0$	IV	$0.96^{+0.20}_{-0.18}$	142	9.29	234

Table 4. Branching ratios for $B \to PP$(in units 10^{-6}) with a_i calculated with set of C_i^{eff} from [9] with $N_c = 2, 3, \infty$:

Channel	Class	Exp. [11]	$N_c = 2$	$N_c = 3$	$N_c = \infty$
$B^0 \to \pi^+\pi^-$	I	5.13 ± 0.31	5.9	6.64	8.19
$B^0 \to \pi^0\pi^0$	II	1.62 ± 0.31	1.05	0.341	1.49
$B^+ \to \pi^+\pi^0$	III	5.7 ± 0.5	4.62	3.65	2.10
$B^+ \to K^+\pi^0$	IV	12.9 ± 0.6	7.31	8.13	9.83
$B^0 \to K^+\pi^-$	IV	19.4 ± 0.6	27.5	30.5	36.8
$B^0 \to K^0\pi^0$	IV	9.5 ± 0.8	3.49	4.17	5.66
$B^+ \to \pi^+K^0$	IV	23.1 ± 1.0	10.2	12.0	16.1
$B^+ \to K^+\overline{K}^0$	IV	1.36 ± 0.27	1.01	0.976	1.36
$B^0 \to K^0\overline{K}^0$	IV	$0.96^{+0.20}_{-0.18}$	1.06	0.969	1.49

Table 5. Branching ratios for $B \to PP$(in units 10^{-6}) with a_i calculated with set of C_i^{eff} from [10] with $N_c = 2, 3, \infty$:

Channel	Class	Exp. [11]	$N_c = 2$	$N_c = 3$	$N_c = \infty$
$B^0 \to \pi^+\pi^-$	I	5.13 ± 0.31	6.11	6.93	8.66
$B^0 \to \pi^0\pi^0$	II	1.62 ± 0.31	1.44	0.081	2.11
$B^+ \to \pi^+\pi^0$	III	5.7 ± 0.5	4.59	3.63	2.08
$B^+ \to K^+\pi^0$	IV	12.9 ± 0.6	2.75	3.63	5.69
$B^0 \to K^+\pi^-$	IV	19.4 ± 0.6	24.5	28.9	38.5
$B^0 \to K^0\pi^0$	IV	9.5 ± 0.8	8.34	9.11	10.7
$B^+ \to \pi^+K^0$	IV	23.1 ± 1.0	12.2	13.8	17.2
$B^+ \to K^+\overline{K}^0$	IV	1.36 ± 0.27	0.803	0.861	1.45
$B^0 \to K^0\overline{K}^0$	IV	$0.96^{+0.20}_{-0.18}$	0.831	0.855	1.64

The results displayed in tables 3-5 demonstrate the strong dependence on the number of colors.This applies in particular to the $B^0 \to \pi^0\pi^0$ decay.

It is interesting to compare the results obtained for different sets of C_i^{eff} at the same N_c.

Table 6. Branching ratios for $B \to PP$(in units 10^{-6}) with a_i calculated with sets of C_i^{eff} from [7], [9]and [10] with $N_c = 3$:

Channel	Class	Exp. [11]	[7]	[9]	[10]
$B^0 \to \pi^+\pi^-$	I	5.13 ± 0.31	7.04	6.64	6.93
$B^0 \to \pi^0\pi^0$	II	1.62 ± 0.31	0.0649	0.341	0.808
$B^+ \to \pi^+\pi^0$	III	5.7 ± 0.5	3.88	3.65	3.63
$B^+ \to K^+\pi^0$	IV	12.9 ± 0.6	0.97	8.13	3.63
$B^0 \to K^+\pi^-$	IV	19.4 ± 0.6	10.6	30.5	28.9
$B^0 \to K^0\pi^0$	IV	9.5 ± 0.8	0.0621	4.17	9.11
$B^+ \to \pi^+K^0$	IV	23.1 ± 1.0	0.615	12	13.8
$B^+ \to K^+\overline{K}^0$	IV	1.36 ± 0.27	1.01	0.976	0.861
$B^0 \to K^0\overline{K}^0$	IV	$0.96^{+0.2}_{-0.18}$	0.929	0.969	0.855

Table 7. Branching ratios for $B \to PP$(in units 10^{-6}) with a_i calculated with sets of C_i^{eff} from [7], [9]and [10] with $N_c = \infty$:

Channel	Class	Exp. [11]	[7]	[9]	[10]
$B^0 \to \pi^+\pi^-$	I	5.13 ± 0.31	9.36	8.19	8.66
$B^0 \to \pi^0\pi^0$	II	1.62 ± 0.31	1.71	1.49	2.11
$B^+ \to \pi^+\pi^0$	III	5.7 ± 0.5	3.88	2.1	2.08
$B^+ \to K^+\pi^0$	IV	12.9 ± 0.6	12.3	9.83	5.69
$B^0 \to K^+\pi^-$	IV	19.4 ± 0.6	49.6	36.8	38.5
$B^0 \to K^0\pi^0$	IV	9.5 ± 0.8	7.34	5.66	10.7
$B^+ \to \pi^+K^0$	IV	23.1 ± 1.0	20	16.1	17.2
$B^+ \to K^+\overline{K}^0$	IV	1.36 ± 0.27	22.6	1.36	1.45
$B^0 \to K^0\overline{K}^0$	IV	$0.96^{+0.2}_{-0.18}$	23.4	1.49	1.64

It should be noted that the numerical values for the decays of type IV differ greatly from the experimental data.Perhaps it is a signal the presence of some additional interactions.

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