

# Low Energy Hadronic Interactions of Scalar Mesons

*E. Avakyan\**, *S. Avakyan†*

*Gomel State Technical University, Gomel*

## Abstract

The  $\eta, \eta'$  decays with the nonabelian anomaly ( $P \rightarrow V\gamma, V \rightarrow P\gamma, \eta \rightarrow \pi\pi\gamma$ ) are under investigation. The transition form factors of  $\eta, \eta'$  mesons are received. It is shown, that the account of the graphs with intermediate vector mesons leads to good agreement with experimental data.

## 1 Introduction

The world of light quarks and hadrons is very reach with interesting phenomena. At short distances there are only free quarks and gluons. They are governed by Quantum Chromodynamics. At large distances there are only hadrons. This point-like particles are described by the standard quantum field equations. And intermediate distances color confinement and hadronization take place. From the physical point of view, this is a low energy region of hadronic physics where physical processes with the liberated energy  $1 \div 2$  GeV proceed. The investigation of simple quark-antiquark systems such as pseudoscalar mesons  $\pi^0, \eta, \eta' \dots$  is of extraordinary interest as a source of information about structure of hadrons. The  $\eta, \eta'$  mesons plays a special role in understanding low energy QCD. The basic facts about  $\eta, \eta'$  mesons from the PDG report [1] are summarized in Table 1.

Table 1

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\*E-mail:mikot@tut.by

†E-mail:avakyan@tut.by

$m_\eta = 547.51 \pm 0.18 \text{ MeV}$	$m_{\eta'} = 957.78 \pm 0.14 \text{ MeV}$
$\Gamma_\eta = 1.30 \pm 0.07 \text{ KeV}$	$\Gamma_{\eta'} = 0.203 \pm 0.016 \text{ MeV}$
$\eta \rightarrow \gamma\gamma$ 39%	$\eta' \rightarrow \pi^+\pi^-\eta$ 44%
$\eta \rightarrow \pi^0\pi^0\pi^0$ 32%	$\eta' \rightarrow \rho^0\gamma$ 29%
$\eta \rightarrow \pi^+\pi^-\pi^0$ 23%	$\eta' \rightarrow \pi^0\pi^0\eta$ 21%
$\eta \rightarrow \pi^+\pi^-\gamma$ 5%	$\eta' \rightarrow \omega\gamma$ 3%
	$\eta' \rightarrow \gamma\gamma$ 2%

There are two classes of decays: radiative decays and hadronic decays into three pseudoscalars. It's well known that first one are connected with triangle and box anomalies; the second class provide the source of information on low energy scalar interaction.

SU(3)-symmetry predicts the existence of massless pseudoscalar octet  $\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - s\bar{s})$  and massive singlet  $\eta_0 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$ . Physical states  $\eta, \eta'$  are the mixture of  $\eta_8$  and  $\eta_0$ . The study of  $\eta - \eta'$  mixing is very important for understanding of basic properties of quark - hadron matter. This phenomena was considered in different approaches [2]. Recent CLEO [3] and L3[4]experiments call the additional interest to this problem.

Theoretical analysis is model dependent and mixing angle  $\theta$  prediction varies from  $-12^\circ$ [5] to  $-20^\circ$  [7]. Recently another scheme with two mixing angles was proposed by [6]. Also there exists approach connected with quark basis  $q\bar{q} = \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}, s\bar{s}$ [5]. In this case decay constant mixing is considers to be the same as for meson states. The analysis of mentioned schemes can be performed by study two-photon decays of  $\pi^0, \eta, \eta'$  mesons.

The finite size of hadron, which in electromagnetic interactions is revealed as electromagnetic structure of hadrons, is phenomenologically described by one ore more functions of one variable (the four-momentum transfer squared  $t = q^2 = -Q^2$  of a virtual photon), called the EM form factors (FFs). If in the vertex with three lines appear one virtual photon and two identical strongly interacting particles, one speaks about the elastic electro magnetic form factors of hadron. If there is one virtual photon and two different hadrons (or one hadron and one real photon) we speak about the transition form factors. Since the form factors are sensitive to the decay constants they also provide a crucial test of the quark- favor mixing scheme. The transition form factors between pseudoscalar mesons and photons at large momentum transfer are subject of intense theoretical interest, see e.g. [8], [9], [10],[11]. A number of experiments have been performed to measure these transition form factors. Nevertheless, exist-

ing data in the low and intermediate regions are quite poor. The CELLO collaboration at PETRA has measured in the space-like region at large momentum transfers using the reaction  $e^+e^- \rightarrow e^+e^-P$  [12]. In this experiment, two photons are radiated virtually by the colliding  $e^+e^-$  beams. One of the virtual photons is close to real and the other has a larger  $q^2$  and is tagged by the detection of an  $e^+$  or  $e^-$ . The low energy experiments are planned [13],[14].

Another problem is the evaluation of the hadronic matrix elements. We perform the calculations in the Quark Confinement Model (QCM) [15]. This model based on the certain assumptions about nature of quark confinement and hadronization allows to describe the electromagnetic, strong and weak interactions of light (nonstrange and strange) mesons from a unique point of view.

## 2 Quark Confinement Model

The hadronic interactions will be described in the QCM. This model is based on the following assumptions [15]:

The hadron fields are assumed to arise after integration over gluon and quark variables in the QCM generating function. The transition of hadrons to quarks and vice versa is given by the interaction Lagrangian. In particular necessary interaction Lagrangians for  $\pi^\pm, \eta$  and  $\eta'$  mesons look like:

$$\mathcal{L}_M = \frac{g_M}{\sqrt{2}} M \bar{q}^a \Gamma \lambda^m q^a \quad (1)$$

where  $\Gamma$ - Dirac matrix,  $\lambda^m$  - is a corresponding SU(3)-matrix,  $q$ - quark vector

$$q_j^a = \begin{pmatrix} u^a \\ d^a \\ s^a \end{pmatrix}$$

The coupling constants  $g_M$  for meson-quark interaction are defined from so-called compositeness condition. It is convenient to use interaction constant in a form:

$$h_M = \frac{3g_M^2}{4\pi^2} = -\frac{1}{\prod'_M(m_M)} \quad (2)$$

instead of  $g_M$  in the further calculations. All hadron-quark interactions are described by quark diagrams induced by  $S$  matrix averaged over vacuum backgrounds.

The second QCM assumption is that the quark confinement is provided by nontrivial gluon vacuum background. The averaging of quark diagrams generated by  $S$ -matrix over vacuum gluon fields  $B_{VAC}$  is suggested to provide quark confinement and to make the ultraviolet finite theory. The confinement ansatz in the case of one-loop quark diagrams consists in following replacement:

$$\int d\sigma_{VAC} Tr |M(x_1)S(x_1, x_2|B_{VAC}) \dots M(x_n)S(x_n, x_1|B_{VAC})| \longrightarrow \int d\sigma_v Tr |M(x_1)S_v(x_1 - x_2) \dots M(x_n)S_v(x_n - x_1)|, \quad (3)$$

where

$$S_v(x_1 - x_2) = \int \frac{d^4 p}{i(2\pi)^4} e^{-ip(x_1 - x_2)} \frac{1}{v\Lambda_q - \hat{p}} \quad (4)$$

The parameter  $\Lambda_q$  characterizes the confinement rang of quark with flavor number  $q = u, d, s$ . The measure  $d\sigma_v$  is defined as:

$$\int \frac{d\sigma_v}{v - \hat{z}} = G(z) = a(-z^2) + \hat{z}b(-z^2) \quad (5)$$

The function  $G(z)$  is called the confinement function.  $G(z)$  is independent on flavor or color of quark.  $G(z)$  is an entire analytical function on the  $z$ -plane.  $G(z)$  decreases faster than any degree of  $z$  in Euclidean region. The choice of  $G(z)$ , or as the same of  $a(-z^2)$   $b(-z^2)$ , is one of model assumptions. In the notes [15], [16]  $a(-z^2)$  and  $b(-z^2)$  are chosen as:

$$\begin{aligned} a(u) &= a_0 e^{-u^2 - a_1 u} \\ b(u) &= b_0 e^{-u^2 - b_1 u} \end{aligned} \quad (6)$$

The request of satisfaction of Ward anomaly identity in QCM gives the additional correlation between  $a(0)$  and  $b(0)$ :  $b(0) = -a'(0)$ ,  $a(0) = 2$ . Using  $a(u)$  and  $b(u)$  as (6), one can receive:  $a_0 = 2$ ,  $a_1 = \frac{b_0}{4}$ . So, the free parameters of the model are  $\Lambda_q, b_0, b_1$ . The model parameters were fixed in the [16] by fitting the well-established constants of low-energy

physics. ( $f_\pi, f_K, g_{\rho\gamma}, g_{\pi\gamma\gamma}, g_{\omega\pi\gamma}, g_{\rho\pi\pi}, g_{K^*\pi\gamma}$ )

$$\begin{aligned}\Lambda_u &= 460 \text{ MeV} \\ \Lambda_s &= 506 \text{ MeV} \\ b_0 &= 2 \quad b_1 = 0.2 \\ a_0 &= 2 \quad a_1 = 0.5\end{aligned}\tag{7}$$

We put  $\Lambda_u = \Lambda_d$  in the most of decays.

### 3 The description of $\eta, \eta'$ mixing

In order to quantify the mixing in the  $\eta, \eta'$  system, one have to define appropriate mixing parameters, which can be related to physical observables. In this work we use so-called mixing in the quark basis(QBM). The two independent axial-vector currents are taken as [5]

$$\begin{aligned}J_{\mu 5}^q &= \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d) \\ J_{\mu 5}^s &= \bar{s}\gamma_\mu\gamma_5 s\end{aligned}\tag{8}$$

In this scheme  $\eta, \eta'$  mixing is defined as

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}\tag{9}$$

where  $\eta_q = \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}, \eta_s = s\bar{s}$ .

The numerical value for mixing angle  $\varphi$  varies as  $\varphi = 30^\circ \div 45^\circ$  in different approaches [2]. In the [17] the best agreement with experimental data was achieved with the

$$\varphi = 39.3^\circ\tag{10}$$

Let us calculate the constants of  $V \rightarrow P\gamma$  and  $P \rightarrow V\gamma$  ( $P \equiv \eta, \eta'$ ;  $V \equiv \rho, \omega, \phi$ ) decays using the mixing parameters received in previous section.

The decay amplitudes are defined as

$$A(V \rightarrow P\gamma) = eg_{VP\gamma}\epsilon^{\mu\nu\alpha\beta}\epsilon^\mu(p_V)\epsilon^\nu(q_\gamma)q_\gamma^\alpha p_V^\beta\tag{11}$$

$$A(P \rightarrow V\gamma) = eg_{PV\gamma}\epsilon^{\mu\nu\alpha\beta}\epsilon^\mu(q_\gamma)\epsilon^\nu(p_V)q_\gamma^\alpha p_V^\beta\tag{12}$$

and the corresponding decay widths are

$$W(P \rightarrow V\gamma) = m_V^3 \alpha g_{PV\gamma}^2 \quad (13)$$

$$W(V \rightarrow P\gamma) = \frac{1}{3} m_P^3 \alpha g_{VP\gamma}^2 \quad (14)$$

The following expressions for decay constants were received

$$g_{\eta\rho\gamma} = \sqrt{h_\rho h_\eta(\varphi)} \cos \varphi F_{PVV}(m_\eta^2, m_\rho^2, 0, \Lambda_n) \quad (15)$$

$$g_{\eta'\rho\gamma} = \sqrt{h_\rho h_{\eta'}(\varphi)} \sin \varphi F_{PVV}(m_{\eta'}^2, m_\rho^2, 0, \Lambda_n) \quad (16)$$

$$g_{\eta\omega\gamma} = \sqrt{h_\omega h_\eta(\varphi)} \cos \varphi \frac{1}{3} F_{PVV}(m_\eta^2, m_\omega^2, 0, \Lambda_n) \quad (17)$$

$$g_{\eta'\omega\gamma} = \sqrt{h_\omega h_{\eta'}(\varphi)} \sin \varphi \frac{1}{3} F_{PVV}(m_{\eta'}^2, m_\omega^2, 0, \Lambda_n) \quad (18)$$

$$g_{\eta\phi\gamma} = \sqrt{h_\phi h_\eta(\varphi)} \sin \varphi \frac{2}{3} F_{PVV}(m_\eta^2, m_\phi^2, 0, \Lambda_s) \quad (19)$$

$$g_{\eta'\phi\gamma} = \sqrt{h_\phi h_{\eta'}(\varphi)} \cos \varphi \frac{2}{3} F_{PVV}(m_{\eta'}^2, m_\phi^2, 0, \Lambda_s) \quad (20)$$

The structure integral  $F_{PVV}(m^2, 0, 0, \Lambda)$  in (15)-(20) is written as

$$F_{PVV}(p^2, q_1^2, q_2^2, \Lambda) = \frac{1}{\Lambda} \int_0^1 \{d^3\alpha\} a(-Q) \quad (21)$$

where  $a(-Q)$  is confinement function defined by (6) with  $Q = \frac{p^2\alpha_1\alpha_3 + q_1^2\alpha_2\alpha_3 + q_2^2\alpha_1\alpha_2}{\Lambda^2}$ .

The expression for coupling constants  $h_M(\varphi)$  one can find in [17].

The numerical values for  $V \rightarrow P\gamma$  and  $P \rightarrow V\gamma$  decays are given in tables 2 and 3.

Table 2

$g_{V\eta\gamma}(\text{GeV}^{-1})$	Experiment [1]	QBM $\varphi = 39.3^\circ$
$g_{\rho\eta\gamma}$	$1.47 + 0.25 - 0.28$	1.49
$g_{\omega\eta\gamma}$	$0.5 \pm 0.04$	0.496
$g_{\phi\eta\gamma}$	$0.69 \pm 0.02$	0.715

Table 3

$g_{V\eta'\gamma}(\text{GeV}^{-1})$	Experiment[1]	QBM $\varphi = 39.3^\circ$
$g_{\rho\eta'\gamma}$	$1.31 \pm 0.06$	1.065
$g_{\omega\eta'\gamma}$	$0.45 \pm 0.03$	0.353
$g_{\phi\eta'\gamma}$	$1.00 \pm 0.28$	0.761

## 4 The electromagnetic form factors of $\eta$ and $\eta'$ mesons.

The electromagnetic form factors of  $\eta$  and  $\eta'$  mesons are described by graphs displayed in fig.1.

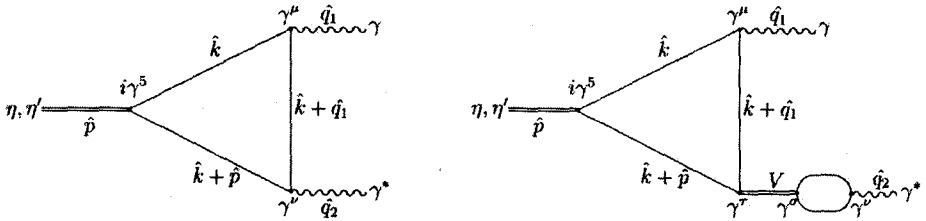


Fig. 1

and can be written as:

$$F_{P\gamma\gamma^*}(q^2) = F_{P\gamma\gamma^*}^{\text{dir}}(q^2) + \sum_{\rho,\omega,\varphi} g_{PV\gamma}(q^2)G_V(q^2)g_{V\gamma}(q^2) \quad (22)$$

The first term in (26) corresponds to the direct graph of  $\eta(\eta') - \gamma\gamma^*$  and is expressed by  $F_{PVV}$  in following way:

$$F_{\eta\gamma\gamma^*}^{\text{dir}}(\varphi, q^2) = \frac{\sqrt{3h_\eta(\varphi)}}{\pi} \left( \frac{5}{9\sqrt{2}} F_{PVV}(m_\eta^2, 0, q_2^2, \Lambda_n) \cdot \cos \varphi - \frac{1}{9} F_{PVV}(m_\eta^2, 0, q_2^2, \Lambda_s) \cdot \sin \varphi \right) \quad (23)$$

$$F_{\eta'\gamma\gamma^*}^{\text{dir}}(\varphi) = \frac{\sqrt{3h_{\eta'}(\varphi)}}{\pi} \left( \frac{5}{9\sqrt{2}} F_{PVV}(m_{\eta'}^2, 0, q_2^2, \Lambda_n) \cdot \sin \varphi - \frac{1}{9} F_{PVV}(m_{\eta'}^2, 0, q_2^2, \Lambda_s) \cdot \cos \varphi \right) \quad (24)$$

for  $\eta$  and  $\eta'$ .

The second term in (26) accumulates the contribution from intermediate vector mesons. The form factors  $g_{PV\gamma}$  are looked like:

$$\begin{aligned} g_{\eta\rho\gamma}(\varphi, q_2^2) &= \sqrt{h_\eta(\varphi)} \cdot \cos \varphi \cdot F_{PVV}(m_\eta^2, 0, q_2^2, \Lambda_n); \\ g_{\eta\omega\gamma}(\varphi, q_2^2) &= \sqrt{h_\eta(\varphi)} \cdot \frac{1}{3} \cos \varphi \cdot F_{PVV}(m_\eta^2, 0, q_2^2, \Lambda_n); \\ g_{\eta\varphi\gamma}(\varphi, q_2^2) &= \sqrt{h_\eta(\varphi)} \cdot \frac{2}{3} \sin \varphi \cdot F_{PVV}(m_\eta^2, 0, q_2^2, \Lambda_n) \end{aligned} \quad (25)$$

$$\begin{aligned} g_{\eta'\rho\gamma}(\varphi, q_2^2) &= \sqrt{h_{\eta'}(\varphi)} \cdot \sin \varphi \cdot F_{PVV}(m_{\eta'}^2, 0, q_2^2, \Lambda_n); \\ g_{\eta'\omega\gamma}(\varphi, q_2^2) &= \sqrt{h_{\eta'}(\varphi)} \cdot \frac{1}{3} \sin \varphi \cdot F_{PVV}(m_{\eta'}^2, 0, q_2^2, \Lambda_n); \\ g_{\eta'\varphi\gamma}(\varphi, q_2^2) &= \sqrt{h_{\eta'}(\varphi)} \cdot \frac{2}{3} \cos \varphi \cdot F_{PVV}(m_{\eta'}^2, 0, q_2^2, \Lambda_n) \end{aligned} \quad (26)$$

$G_V(q^2)$  is connected with the vector mesons propagators,  $g_V(q^2)$  - is the form factor of  $V \rightarrow \gamma$  transition. Both  $G_V(q^2)$  and  $g_V(q^2)$  are given in [16].

For comparison of our results with experimental value the following formula have been used [18]

$$F_{P\gamma\gamma}^2(Q^2) \frac{\pi\alpha^2 m_P^3}{4} = \frac{\Gamma(P \rightarrow \gamma\gamma)}{\left(1 + \frac{Q^2}{\Lambda_P^2}\right)^2}, \quad (27)$$

where parameters  $\Lambda_P$  had been reported by different experimental groups:

	Lepton-G [19]	TCP/2 $\gamma$ [20]	CLEOI [21]	CLEO II[18]
$\Lambda_\eta$ (MeV)	$720 \pm 90$	$700 \pm 80$	$839 \pm 63$	$787 \pm 23$
$\Lambda_{\eta'}$ (MeV)		$850 \pm 70$	$794 \pm 44$	$851 \pm 15$

Fig.2,3 represents the transition form factors for  $\eta$  and  $\eta'$  mesons. The dotted line denotes the form factor with only direct graph, the dashed line- the result form factor with intermediate vector mesons, the solid line- the experimental curve has been received by averaging the experimental data [18]-[21].



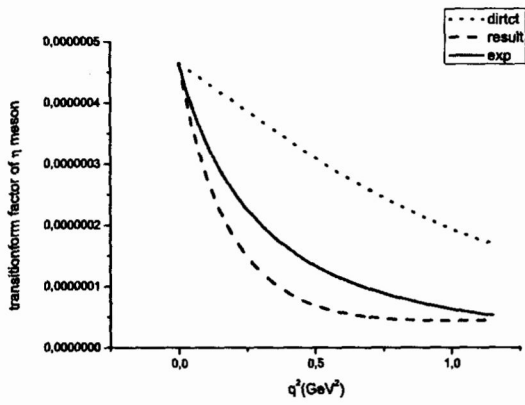


Fig. 2

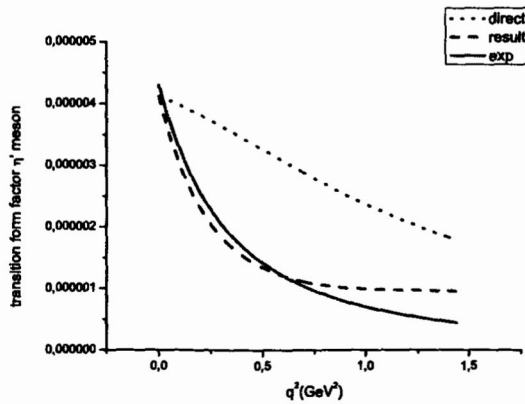


Fig.3

## 5 THE $\eta \rightarrow \pi\pi\gamma$ decay.

A major role in  $\eta, \eta'$  physics is the study of the nonabelian anomaly [22]. The review for anomalous  $\eta$  processes one can find, for example, in [23].

The  $\eta \rightarrow \pi\pi\gamma$  decay is one of such "anomalous" decays. The matrix element of this decay is defined by "box" graphs shown in Fig. 4.

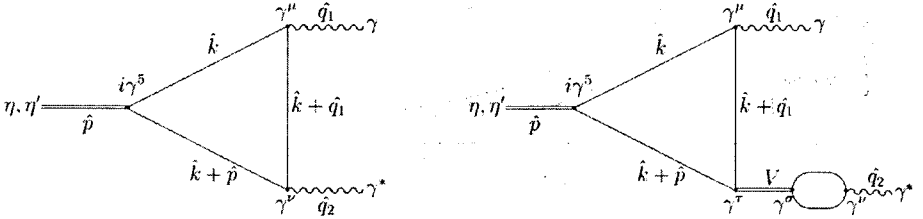


Fig. 4

The corresponding loop integral is written in QCM as:

$$I_{PPP\gamma}^\mu = \int \frac{d^4k}{4\pi^2 i} \text{Tr} \{ i\gamma^5 S(\hat{k}) i\gamma^5 S(\hat{k} + \hat{p}_1) i\gamma^5 S(\hat{k} + \hat{p} - \hat{q}) \gamma^\mu S(\hat{k} + \hat{p}) \} \quad (28)$$

where  $\hat{p}, \hat{q}, \hat{p}_{1,2}$  - are the four- momenta of  $\eta, \gamma$  and final  $\pi^-$  mesons. The invariant variables  $s_1, s_2, s_3$  are defined in following way:

$$s_1 = (p_\eta - p_{\pi^+})^2$$

$$s_2 = (p_\eta - q)^2$$

$$s_3 = (p_\eta - p_{\pi^-})^2$$

The matrix element for  $\eta \rightarrow \pi\pi\gamma$  decay have been received as:

$$M^\mu(\eta \rightarrow \pi\pi\gamma) = e\epsilon^{\mu\nu\alpha\beta} p_{\pi^+}^\alpha q^\beta p_\eta^\nu A(s_1, s_2) \quad (29)$$

where

$$A(s_1, s_2) = \frac{\pi}{\sqrt{3}\Lambda^3} h_\pi \sqrt{h_\eta} \cos \varphi \cdot F_{PPP\gamma}(s_1, s_2, m_\eta^2, m_{\pi^\pm}^2) \quad (30)$$

$F_{PPP\gamma}(s_1, s_2, m_\eta^2, m_{\pi^\pm}^2)$  in (30) is the result of evaluation of loop integral (28) under QCM rules.

$$F_{PPP\gamma}(s_1, s_2, p^2, p_1^2, p_2^2) = \int_0^1 \{d^4\alpha\} (a'(Q_1) + a'(Q_2) + a'(Q_3)),$$

where

$$Q_1 = \frac{1}{\Lambda^2} \{ \alpha_1 \alpha_4 p^2 + \alpha_3 \alpha_4 p_1^2 + \alpha_2 \alpha_3 p_2^2 + \alpha_1 \alpha_3 s_1 + \alpha_2 \alpha_4 s_2 \},$$

$$Q_2 = \frac{1}{\Lambda^2} \{ \alpha_4 (\alpha_1 + \alpha_2) p^2 + \alpha_2 (\alpha_3 + \alpha_4) p_1^2 + \alpha_2 (\alpha_1 + \alpha_4) p_2^2 - \alpha_2 \alpha_4 s_1 + -(\alpha_2 \alpha_4 - \alpha_1 \alpha_3) s_2 \},$$

$$Q_3 = \frac{1}{\Lambda^2} \{ \alpha_4 (\alpha_1 + \alpha_3) p^2 + \alpha_4 (\alpha_2 + \alpha_3) p_1^2 + \alpha_3 (\alpha_1 + \alpha_4) p_2^2 + (\alpha_1 \alpha_2 - \alpha_3 \alpha_4) s_1 - \alpha_3 \alpha_4 s_2 \}.$$

The width of  $\eta \rightarrow \pi\pi\gamma$  decay calculated by standard formulas is written as:

$$\Gamma(\eta \rightarrow \pi\pi\gamma) = \frac{\pi\alpha}{8m_\eta^3} \int_{4m_\pi^2}^{(m_\eta - m_\pi)^2} ds_2 \int_{s_1^-}^{s_1^+} ds_1 f(s_1, s_2, m_\eta^2, m_{\pi^\pm}^2) A^2(s_1, s_2) \quad (31)$$

where

$$f(s_1, s_2, m_\eta^2, m_{\pi^\pm}^2) = \frac{m_\eta^6}{4} \left( \left(1 - \frac{s_2}{m_\eta^2}\right) \left(\frac{s_1}{m_\eta^2} - \frac{m_\pi^2}{m_\eta^2}\right) + \frac{s_1}{m_\eta^2} \left(1 - \frac{s_2}{m_\eta^2}\right)^2 + \frac{s_2}{m_\eta^2} \left(\frac{s_1}{m_\eta^2} - \frac{m_\pi^2}{m_\eta^2}\right)^2 \right)$$

$$s_1^\pm = 2m_\pi^2 - \{(s_2 - m_\eta^2 + m_\pi^2)s_2 \mp \lambda^{\frac{1}{2}}(s_2, m_\eta^2, m_\pi^2)\lambda^{\frac{1}{2}}(s_2, m_\pi^2, m_\pi^2)\}$$

where  $\lambda(x, y, z)$  is standard kinematical function:

$$\lambda(x, y, z) = (x - y - z)^2 - 4yz \quad (32)$$

So the numerical value of  $\Gamma(\eta \rightarrow \pi\pi\gamma)$  is

$$\Gamma(\eta \rightarrow \pi\pi\gamma) = 6.23 \times 10^{-2} \text{KeV} \quad (33)$$

## 6 Conclusion

The  $\eta, \eta'$  interactions can be successfully described in the quark basis(QBM) with mixing angle  $\varphi = 39.3^\circ$ . The numerical values for constants of  $V \rightarrow P\gamma$  and  $P \rightarrow V\gamma$  ( $P \equiv \eta, \eta'$ ;  $V \equiv \rho, \omega, \phi$ ) decays are in a good agreement with experimental data. The numerical value of the width of  $\eta \rightarrow \pi^+\pi^-\gamma$  decay is also in sufficient agreement with experimental one. The account of intermediate vector mesons leads to the good agreement of transition form factors for  $\eta, \eta'$  mesons with experimental curves.

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