Target mass corrections in deep inelastic scattering

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Abstract

A method of studying target mass effects based on the Jost–Lehmann–Dyson integral representation for structure functions of the inelastic lepton-hadron scattering is elaborated. It is shown that, in accordance with general principles of local quantum field theory, expressions obtained for the physical structure functions depending on the target mass have a correct spectral property.

1 Introduction

The cross-section of the inclusive lepton-hadron scattering (see Fig. 1) is parameterized by structure functions $F_i(x, Q^2)$ which connected with the hadronic tensor $W_{\mu\nu}$ as follows

$$W_{\mu\nu}(q,P) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) + \frac{1}{(q\cdot P)}\left(P_\mu - q_\mu \frac{(q\cdot P)}{q^2}\right) \times \left(P_\nu - q_\nu \frac{(q\cdot P)}{q^2}\right) F_2(x,Q^2) - \frac{i}{2(q\cdot P)}\varepsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta F_3(x,Q^2)\,. \tag{1}$$

The inclusive cross section for inelastic lepton-hadron scattering is expressed as the Fourier transform of the expectation value of the current product $J(z)J(0)$ in the target state. The operator product expansion (OPE) is a powerful tool to study inelastic scattering processes. This method has been applied to define the contribution of target mass terms to the structure functions in paper [1]. Within this method the structure functions are parameterized by the quark distribution function $F(x)$ that appears with the argument

$$\xi = \frac{2x}{1+\sqrt{1+4x^2\epsilon}}, \tag{2}$$

where scaling variable (2) is usually called the Nachtmann variable [2], x is the Bjorken scaling variable

$$x = \frac{Q^2}{2\nu} = \frac{Q^2}{2(q\cdot P)}, \tag{3}$$

and ϵ is expressed through the target mass M and the transfer momentum Q as

$$\epsilon = \frac{M^2}{Q^2}, \qquad Q^2 = -q^2. \tag{4}$$

All definitions used here can be understood from Fig. 1.

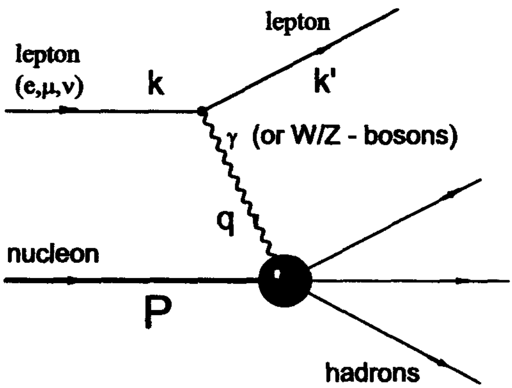

Figure 1: *Diagram of the inelastic lepton-nucleon scattering.*

This ξ-approach leads to expressions for the physical structure functions which conflict with general spectral condition at $x = 1$. At the same time the target mass corrections become to be large at large x. This trouble with the ξ-scaling has widely been discussed in the literature (see, for example, [3, 4, 5, 6]).

We will use the Jost–Lehmann–Dyson (JLD) [7, 8] representation for the structure function which reflects general principles of the theory [9]. We argue that in this case it is possible to get an expressions for the structure functions in terms of the quark distribution incorporating the target mass effects and having the correct spectral property. Our investigation of the inelastic lepton-hadron scattering based on the JLD representation has been started in [10] and continued in [11, 12]. This paper belongs to this series.

2 Method

The situation that an approximation can be in a confrontation with general properties is not rare event in quantum field theory. It is well known, for example, that when the renormalization group equation for the running coupling is solved directly, there arise unphysical singularities of the ghost pole type. One has to apply some additional requirements to correct this trouble. The analytic approach proposed in [13] gives a possible resolution of the ghost pole problem.[1] This method combines the renormalization invariance and the Q^2-analyticity of the Källén–Lehmann type has revealed new important properties of the analytic coupling.

The integral representation for the analytic running coupling is

$$\bar{a}_{\rm an}(Q^2) = \frac{1}{\pi}\int_0^\infty d\sigma \frac{\rho(\sigma)}{\sigma + Q^2 - i\epsilon}, \tag{5}$$

where the spectral function $\rho(\sigma)$ can be found via a discontinuity of the perturbative running coupling on the physical cut. In the leading order the analytic running coupling has the form

$$\bar{a}^{(1)}_{\rm an}(Q^2) = \frac{1}{\beta_0}\left[\frac{1}{\ln Q^2/\Lambda^2} + \frac{\Lambda^2}{\Lambda^2 - Q^2}\right], \tag{6}$$

where β_0 is the one loop coefficient of the renormalization group β-function.

The analytic coupling has no ghost pole at $Q^2 = \Lambda^2$. The first term on the of right-hand side (6) preserves the standard ultraviolet behavior of the invariant coupling. The second term, which comes from the representation (5) and enforces the proper analytic properties, compensates the ghost pole at $Q^2 = \Lambda^2$. This term gives no contribution to the perturbative expansion. We note also that unlike in electrodynamics, the asymptotic

[1]See also [10, 14] for reviews and [15, 16, 17], where a new comparative analysis of the analytic perturbation theory and ordinary one has been performed.

freedom property in QCD has the effect that such nonperturbative contributions show up in the effective coupling function already in the domain of low energies and momentum transfers reachable in realistic experiments, rather than at unrealistically high energies.

The invariant analytic formulation maintaining the asymptotic freedom ultraviolet properties essentially modifies a behavior of the analytic running coupling in the infrared region by making it stable with respect to higher-loop corrections. This is radically different from the situation encountered in the standard renormalization-group perturbation theory, which is characterized by strong instability with respect to the next-loop corrections in the domain of small energy scale. The analytic perturbation theory results are much less sensitive to the choice of the renormalization scheme than those in the standard approach that allows us to reduce theoretical uncertainties drastically.

Thus, the causality and spectrality principles expressed in the form of Q^2-analyticity, send us the message that perturbation theory is not the whole story. The requirement of proper analytic properties leads to the appearance of contributions given by powers of Q^2 that cannot be seen in the original perturbative expansion.

Our consideration of the inelastic lepton-hadron process based on the JLD representation for structure functions. The structure functions depend on two arguments, and the corresponding representation that accumulates the fundamental properties of the theory (such as relativistic invariance, spectrality, and causality) have a more complicated form in our analysis than in the representation of the Källén–Lehmann type for functions of one variable.[2] Applications of the JLD representation to automodel asymptotic have been considered in [18].

For the function $W(\nu, Q^2)$ satisfying conditions of the covariance, spectrality, reality, Hermiticity and causality there exists a real moderately growing distribution $\psi(\mathbf{u}, \lambda^2)$ such that the JLD integral representation in the nucleon rest frame can be written as [18]

$$W(\nu, Q^2) = \varepsilon(q_0) \int d\mathbf{u}\, d\lambda^2\, \delta\left[q_0^2 - (M\mathbf{u} - \mathbf{q})^2 - \lambda^2\right] \psi(\mathbf{u}, \lambda^2) . \quad (7)$$

[2]The 4-dimensional integral representation has been proposed by Jost and Lehmann in [7] for the so-called symmetric case. A more general case has been considered by Dyson [8].

The weight function $\psi(\mathbf{u},\lambda^2)$ is supported in

$$\rho = |\mathbf{u}| \leqslant 1, \qquad \lambda^2 \geqslant \lambda^2_{\min} = M^2\bigl(1-\sqrt{1-\rho^2}\,\bigr)^2. \tag{8}$$

The physical values of ν and Q^2, for the process under consideration, are positive. One can neglect the sign factor $\varepsilon(q_0)$ and keep the same notation for $W(\nu,Q^2)$. Taking into account that the weight function $\psi(\mathbf{u},\lambda^2) = \psi(\rho,\lambda^2)$ is radial-symmetric we write down the JLD representation for W in the following covariant form,

$$W(\nu,Q^2) = \int\limits_0^1 d\rho\,\rho^2 \int\limits_{\lambda^2_{\min}}^{\infty} d\lambda^2 \int\limits_{-1}^{1} dz \tag{9}$$
$$\times\,\delta\bigl(Q^2 + M^2\rho^2 + \lambda^2 - 2z\rho\sqrt{\nu^2+M^2Q^2}\,\bigr)\psi(\rho,\lambda^2)\,.$$

A 'natural' target mass dependent variable coming from the JLD representation which is different from the ξ-variable (2) is

$$s = x\sqrt{\frac{1+4\epsilon}{1+4x^2\epsilon}}\,. \tag{10}$$

In terms of the s-variable there arises the dispersion relation for the forward Compton scattering amplitude $T(\nu,Q^2)$ [10]

$$T(\nu,Q^2) = \frac{2}{\pi}\int_0^1 \frac{ds_1}{s_1}\frac{1}{1-(s_1/s)^2}\,W(\nu_1,Q^2)\,. \tag{11}$$

The structure of this dispersion integral is similar to the structure which is appear for the x-variable. The s-moments of the structure function are the analytic functions in the complex Q^2-plane with a cut along a negative part of the real axis.

Eq. (11) can be used to expand the Compton amplitude in the inverse powers of s. If the operator basis is chosen such that an arbitrary contraction of the tensor $\langle P|\widehat{O}_{\mu_1\ldots\mu_n}|P\rangle$ with the nucleon momentum P_{μ_i} vanishes, then the operator product expansion leads to a power series for the forward Compton scattering amplitude with the expansion parameter $q^\mu q^\nu(P_\mu P_\nu - g_{\mu\nu}P^2)/(q^2)^2$, which corresponds to expanding dispersion integral (11) in powers of $1/s^2$. This relation between the analytic s-moments and the structure of the operator product expansion has been found in [10]. It should be stressed that the orthogonality requirement of the symmetric tensor $\langle P|\widehat{O}_{\mu_1\ldots\mu_n}|P\rangle$ to the nucleon momentum P_{μ_i} determines its Lorentz structure unambiguously.

3 Target mass effects

A physical structure function $W(x, Q^2)$ it is convenient to represent, as in [11, 12], in the form

$$W(x, Q^2) = W_0(x, Q^2) + w(x, Q^2) . \tag{12}$$

The function $W_0(x, Q^2)$ for any physical structure function is expressed via the corresponding parton distribution $F(x)$ as follows

$$W_0(x, Q^2) = \begin{cases} F(\beta_-) - F(1) , & \text{if} \quad 0 \leqslant x < \tilde{x} , \\ F(\beta_-) - F(\beta_+) , & \text{if} \quad \tilde{x} \leqslant x \leq 1 . \end{cases} \tag{13}$$

Here

$$\beta_\pm = \frac{x\sqrt{1+4\epsilon x^2}}{1+4\epsilon x^2 + 4\epsilon^2 x^2} \left[1 + 2\epsilon \pm 2\epsilon \sqrt{\frac{1-x^2}{1+4\epsilon x^2}} \right] \tag{14}$$

and

$$\tilde{x} = \frac{1}{\sqrt{1+4\epsilon^2}} . \tag{15}$$

The function $w(x, Q^2)$ can be represent in the form

$$w(x, Q^2) = \int_0^1 d\beta \, \theta[f(\beta; x, \epsilon)] \, \phi(\beta; x, \epsilon) , \tag{16}$$

where

$$f(\beta; x, \epsilon) = \frac{\beta}{s}\sqrt{1+4\epsilon} - 1 - 2\epsilon(1 - \sqrt{1-\beta^2}) . \tag{17}$$

For the scalar quark currents function $\phi(\beta; x, \epsilon)$ in Eq. (16) is [11]

$$\phi(\beta; x, \epsilon) = \frac{1}{4\sqrt{\tau}} \theta(\tau) \, \theta(1-\tau) \frac{\partial}{\partial(\sqrt{\tau})} \left[\sqrt{\tau} F(\sqrt{\tau}) \right] , \tag{18}$$

where

$$\tau \equiv \tau(\beta; x, \epsilon) = \frac{1}{\epsilon} \left(\frac{\beta}{s} \sqrt{1+4\epsilon} - 1 \right) . \tag{19}$$

The analysis of combination of the physical structure functions $F_1(x, Q^2)$ and $F_2(x, Q^2)$ which is taken in the form

$$2x \left(2W_T - W_L \right) = 6xF_1 - \left(1 + 4x^2\epsilon \right) F_2 \tag{20}$$

is similar to the scalar case.

The ξ-scaling method leads to the expression

$$2x\,(2W_T - W_L) = \frac{2x^2}{\sqrt{1+4x^2\epsilon}}\,F(\xi)\,. \tag{21}$$

The defect of this equation is that there is a mismatch at $x = 1$. The combination physical structure functions in the left hand side of Eq. (21) vanishes at $x = 1$, at the same time the right hand side does not because at $x = 1$ the variable ξ remains less than unity. Within the method based on the JLD representation it is possible to get a correct result which obeys the spectral condition. In this case we have to substitute into Eqs. (13) and (18) instead $F(x)$ the function $2x^2F(x)$.

For the structure function $F_2 = F_2^{(0)} + f_2$ the function $F_2^{(0)}$ is restored via the parton distribution as in (13). For the function $f_2(x, Q^2)$ we have

$$\begin{aligned} f_2(x,Q^2) = {} & \frac{\epsilon\,\eta}{16}\left[3\int_{z_-}^{1} dz\,\phi_1(z)\int_{z}^{1} dy\,y\,F(y) + \int_{z_-}^{1} dz\,\phi_2(z)\,F(z)\right] \\ & + \frac{\epsilon\,\eta}{16}\left[\phi_3(z)F(z) + \phi_4(z)F'(z) + \phi_5(z)F''(z)\right]\Big|_{z_-}^{1}\,. \end{aligned} \tag{22}$$

Here $z_- \le 1$ and is defined as $z_-{}^2 = \tau(\beta_-; x, \epsilon)$. The variable η is

$$\eta = \frac{x}{\sqrt{1+4\epsilon x^2}}\,. \tag{23}$$

The functions ϕ_k are

$$\begin{aligned} \phi_1(z) &= 1 - 6\left(\frac{\beta}{z}\right)^2 + 15\left(\frac{\beta}{z}\right)^4, \\ \phi_2(z) &= \frac{3}{z^2}\left[-3\eta^4\epsilon^4 z^8 + 2\eta^2\epsilon^2(1-10\eta^2\epsilon)z^6\right. \\ &\quad \left. +(1+12\eta^2\epsilon - 90\eta^4\epsilon^2)z^4 + 2\eta^2(13-34\eta^2\epsilon)z^2 + 5\eta^4\right], \\ \phi_3(z) &= \frac{1}{z}\left[-3\eta^4\epsilon^4 z^8 + 6\eta^2\epsilon^2(3-2\eta^2\epsilon)z^6\right. \\ &\quad \left. +(1+30\eta^4\epsilon^2 + 52\eta^2\epsilon)z^4 - 6\eta^2(1-14\eta^2\epsilon)z^2 + 45\eta^4\right], \\ \phi_4(z) &= 6\eta^4\epsilon^4 z^8 - 4\eta^2\epsilon^2(1-8\eta^2\epsilon)z^6 \\ &\quad - 2(1+8\eta^2\epsilon - 30\eta^4\epsilon^2)z^4 - 12\eta^2(1-4\eta^2\epsilon)z^2 + 14\eta^4, \\ \phi_5(z) &= z^5\left[1-\left(\frac{\beta}{z}\right)^2\right]^2 \end{aligned} \tag{24}$$

with

$$\beta = \eta(1 + \epsilon z^2).$$

For the structure function xF_3 represented in the form $xF_3(x,Q^2) = W_3^{(0)}(x,Q^2) + w_3(x,Q^2)$ we have

$$w_3(x,Q^2) = \int_0^1 d\beta\, \theta[f(\beta;x,\epsilon)]\, \theta(z)\, \theta(1-z)\, \Phi_3(\beta;x,\epsilon), \tag{25}$$

where

$$\Phi_3(\beta;x,\epsilon) = \frac{1}{8}\left[3\left(1+5\frac{\beta^2}{z^2}\right) z \left(\frac{F(z)}{z^2}\right)\right. \tag{26}$$
$$\left. -3\left(1-3\frac{\beta^2}{z^2}\right) z^2 \left(\frac{F(z)}{z^2}\right)' - \left(1-\frac{\beta^2}{z^2}\right) z^3 \left(\frac{F(z)}{z^2}\right)''\right].$$

We take the input form of the parton distribution as follows [19]

$$x\,F_3(x) = \sqrt{x}\,(1-x)^3. \tag{27}$$

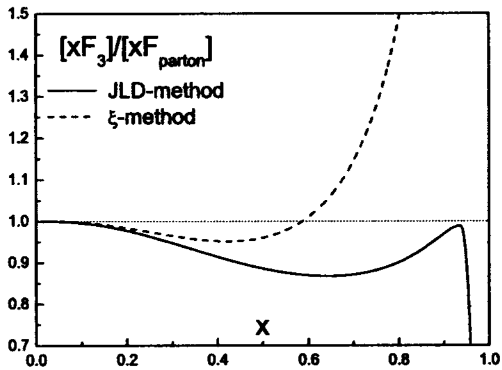

Figure 2: *Ratios of xF_3 structure function with target mass correction to the parton distribution for the JLD-method (solid line) and the ξ-approach (dashed curve).*

In Fig. 2 we plot ratios of the structure function xF_3 to the parton distribution (27) for $\epsilon = 1/2$. The solid line corresponds to incorporating the target mass effects by using the JLD approach, the dashed curve reflects the method of ξ-scaling.

4 Conclusions

We have argued that the approach based on the Jost-Lehmann-Dyson representation gives the self-consistent method of incorporating the target mass dependence into the structure function and does not lead to the conflict with the spectral condition. The corresponding expressions for the physical structure functions F_1, F_2 and F_3 have been presented.

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