Low Energy Hadron Interactions

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ABSTRACT

Electromagnetic, rare semileptonic decays of kaons and low energy hadronic decays ofτ-lepton are treated in the framework of Quark Confinement Model with the effective Hamiltonian with $\Delta S = 1$. The account of intermediate vector, pseudoscalar and axial vector states turns out to be extremely important for description of this processes. The obtained numerical values are in a good agreement with current experimental data

1 Introduction

The world of light quarks and hadrons can be represented as follows. At short distances there are only free quarks and gluons.They are governed by Quantum Chromodynamics.At large distances there are only hadrons.This point-like particles are described by the standard quantum field equations. At intermediate distances color confinement and hadronization take place. From the physical point of view, this is a low energy region of hadronic physics where physical processes with the liberated energy 1-2 GeV proceed.

The physics of kaons is extremely rich in interesting physical phenomena. In the 90-s an extensive research is performed at modern accelerators (BNL, FNAL, KEK, TRIUMF,$DA\Phi NE$) to study rare kaon decays: $K \to \pi\ell^+\ell^-$, $K \to \pi\nu\bar{\nu}$ and etc. [1]. This decays are of extraordinary interest as a source of information about a New Physics beyond Standard Model.For example,statistics at $DA\Phi NE$ should be sufficient to improve the measurement of the width and of lepton spectrum in $K^+ \to \pi^+e^+e^-$ and to detect the decay $K^+ \to \pi^+\mu^+\mu^-$ [2] . From this point of view it is

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very important to have trustworthy quantitative estimations of parameters of mentioned decays in the framework of Standard model. The problem is that calculation of hadronic matrix elements in the most of theoretical approaches needs a great number of additional parameters and model assumptions [3], [2].
The electroweak meson decays are of extraordinary interest as a source of information about structure of hadrons. Among a great number of this processes the leptonic and semileptonic decays are especially important, because leptons are known as "ideal" probes for study the hadronic matter.
Since the time it was found in 1975 τ-lepton is known as a very important one for investigations of fundamental properties of electroweak interactions. Because of τ-mass value , the hadronic , namely, mesonic decays are allowed. This fact gives one the opportunity to use the hadronic decays of τ-lepton as additional and very powerful tool for study both strong and electroweak phenomena.

The calculation of hadronic matrix elements in this paper are performed in the Quark Confinement Model (QCM) [6]. This model based on certain assumptions about the nature of quark confinement and hadronization allows to describe the electromagnetic,strong and weak interactions of light (nonstrange and strange)mesons from a unique point of view. We considere basic low-energy properties of kaons in QCM [6]. The undoubtful merit of the model is that further study of hadron decays doesn't need any more additional assumptions and more additional free parameters.
One have to stress the very important role the intermediate mesonic states plays in the description of the low energy hadronic processes.

2 Quark Confinement Model

Quark Confinement Model (QCM) is based on the following assumptions [6]:

- The hadron fields are assumed to arise after integration over gluon and quark variables in the QCD generating function. The transition of hadrons to quarks and vice versa is given by the interaction Lagrangian:

$$\mathcal{L}_M = \frac{g_M}{\sqrt{2}} M^i \bar{q}^a_m \Gamma_\mu \lambda_i^{mn} q^a_n \tag{1}$$

Here

$$q_j^a = \begin{pmatrix} u^a \\ d^a \\ s^a \end{pmatrix}$$

quark fields, M_i is Euclidian fields connected with the physical one in a standard manner [6], $\lambda_{i=1,...,8}$ is Gell-Mann flavor matrices, Γ_μ is Dirac matrices: $i\gamma_5$ in the case of the pseudoscalar mesons $P(\pi, K, \eta, \eta')$,γ_μ the vector ones $V(\rho, K^*, \omega, \varphi)$,$\gamma_\mu\gamma_5$ the axial ones $A(a_1, k_1, f_1)$,$I - iH\partial_\mu\gamma_\mu/\Lambda$ the scalar ones $S(a_0, k_0, f_0, \varepsilon)$; a is color index.

The mixing angles of the octet and singlet states are defined as follows [7]

$$(\eta', \omega, \varepsilon) \to \cos\delta \frac{\overline{u}u + \overline{d}d}{\sqrt{2}} - \sin\delta\overline{s}s$$

$$(\eta, \varphi, f_0) \to -\sin\delta \frac{\overline{u}u + \overline{d}d}{\sqrt{2}} - \cos\delta\overline{s}s$$

$$\delta = \theta - \theta_I, \theta_I = \arcsin\frac{1}{\sqrt{3}}$$

The coupling constants g_M for meson-quark interaction are defined from so-called compositeness condition [8]

$$Z_M = 1 + \frac{3g_M^2}{4\pi^2}\tilde{\prod}'_M(m_M) = 0 \tag{2}$$

where $\tilde{\prod}'_M(P)$ is the derivative of the corresponding meson mass operator.

All hadron-quark interactions are described by quark diagrams induced by S matrix averaged over vacuum backgrounds:

$$S = \int d\sigma_{VAC} Texp\{i\int dx\mathcal{L}_{int}\} \tag{3}$$

The quark propagator has the following form:

$$S(x_1, x_2|B_{VAC}) = \langle 0|T(q(x_1)\overline{q}(x_2))|0\rangle = i(\hat{p} + \hat{B}_{VAC})^{-1}\delta(x_1 - x_2) \tag{4}$$

- The second QCM assumption is that the quark confinement is provided by nontrivial gluon vacuum background. The averaging of quark diagrams generated by S-matrix (3) over vacuum gluon fields $\hat{B}_{VAC}$ is suggested to provide quark confinement and to make the ultraviolet finite theory. The confinement ansatz in the case of one-loop quark diagrams lies in the following replacement:

$$\int d\sigma_{VAC} Tr|M(x_1)S(x_1,x_2|B_{VAC})...M(x_n)S(x_n,x_1|B_{VAC})| \longrightarrow$$
$$\int d\sigma_v Tr|M(x_1)S_v(x_1-x_2)...M(x_n)S_v(x_n-x_1)|, \quad (5)$$

where

$$S_v(x_1-x_2) = \int \frac{d^4p}{i(2\pi)^4} e^{-ip(x_1-x_2)} \frac{1}{v\Lambda_q - \hat{p}} \quad (6)$$

The parameter Λ_q characterizes the confinement rang of quark with flavor number $q = u, d, s$. The measure $d\sigma_v$ is defined as:

$$\int \frac{d\sigma_v}{v-\hat{z}} = G(z) = a(-z^2) + \hat{z}b(-z^2) \quad (7)$$

The function $G(z)$ is called the confinement function. $G(z)$ is independent on flavor or color of quark. $G(z)$ is an entire analytical function on the z-plane.$G(z)$ decreases faster then any degree of z in Euclidean region. The choice of $G(z)$,or as the same of $a(-z^2)$ and $b(-z^2)$, is one of model assumptions. In the note [6] $a(-z^2)$ and $b(-z^2)$ are chosen as:

$$a(u) = a_0 e^{-u^2-a_1 u}$$
$$b(u) = b_0 e^{-u^2-b_1 u} \quad (8)$$

The request of satisfaction of Ward anomaly identity in QCM gives the additional correlation between $a(0)$ and $b(0)$: $b(0) = -a'(0)$, $a(0) = 2$. Using $a(u)$ and $b(u)$ as (8), one can receive: $a_0 = 2$, $a_1 = \frac{b_0}{4}$. So, the free parameters of the model are Λ_q, b_0, b_1. The model parameters were fixed in the [6] by fitting the well-established constants of low-energy physics.($f_\pi, f_K, g_{\rho\gamma}, g_{\pi\gamma\gamma}, g_{\omega\pi\gamma}, g_{\rho\pi\pi}, g_{K^*\pi\gamma}$)

$$\Lambda_u = \Lambda_d = 460\ MeV$$
$$\Lambda_s = 506\ MeV$$
$$b_0 = 2 \qquad b_1 = 0.2$$
$$a_0 = 2 \qquad a_1 = 0.5 \quad (9)$$

The scattering matrix (3) describes all possible processes of hadronic interactions. There are hadron and quark fields in the representation (3), but quarks in the form of the free asymptotic fields are absent due to the confinement ansatz.

The QCM describes strong interactions, therefore, the perturbative theory over the coupling constant is not applicable. It is to be noted that the chain approximation is assumed to be valid for the compositeness condition (2). Therefore, our calculations will also be based on approximations which are connected with the summing up of the same class of diagrams for the hadron Green functions. This approximation will be done in the framework of the $1/N_c$ expansion. As follows from the compositeness condition (2)the effective strong coupling constant is:

$$h_M = 4N_c(\frac{g_M^2}{4\pi})^2 \tag{10}$$

where $N_c = 3$ is the number of colours and 4 is the number of quark spinor indexes.

Therefore, the series of the perturbative theory can be represented over two parameters h_M and

$$\lambda = \frac{1}{4N_c} = \frac{1}{12} \tag{11}$$

The chain approximation corresponds to the zero degree of the parameter λ.One can obtained expressions for the Green function of pseudoscalar,scalar, vector and axial mesons are summarized in the table 1.

Table 1

Meson	h_J	$h_J D_J(P)$
$M = S, P$	$h_M = -\frac{1}{\Pi'_M(m^2)}$	$h_M D_M(p^2) = \frac{1}{\Pi_M(p^2) - {}_M(m_M^2)}$
$W = V, A$	$h_W = -\frac{1}{\Pi'_{1W}(m^2)}$	$h_W D_W^{\mu\nu}(p^2) = \frac{1}{\Pi_{1W}(p^2) - {}_{1W}(m_W^2)}\{-g^{\mu\nu} + p^\mu p^\nu \frac{{}_{2W}(p^2)}{\Pi_{1W}(p^2) - \Pi_{1W}(m_W^2) + p^2 {}_{2W}(p^2)}\}$

3 ELECTROMAGNETIC KAON INTERACTIONS

Let us pass to the investigation of electromagnetic kaon interactions. One have to stress the QCM allows one to receive the momentum dependence of physical matrix elements.

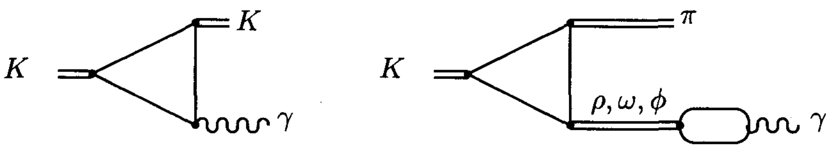


Fig. 1

The Lagrangian of electromagnetic interactions can be written in conventional form as:

$$\mathcal{L}_{em} = eA_\mu(\bar{q}_i^a Q_{ij}\gamma^\mu q_j^a + \bar{l}\gamma^\mu l), \tag{12}$$

where A_μ is the electromagnetic field, $Q = diag\left\{\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right\}$ is the charged matrix.

3.1 Electromagnetic Radii of Kaons

The electromagnetic form factors of the charged and neutral kaons are defined by the diagrams of Fig.1 and written in the form:

$$\begin{aligned} &F_{K^+}(t) = F_\triangle^+(t) + F_\rho(t) + F_\omega(t) - F_\phi(t), \\ &F_{K^0}(t) = F_\triangle^0(t) - F_\rho(t) + F_\omega(t) - F_\phi(t), \\ &t = q^2 \end{aligned} \tag{13}$$

where $F_\triangle(t)$- contribution to the form factor from triangle diagrams, $F_{\rho,\omega,\phi}(t)$ — contribution from diagrams with intermediate vector meson. The electromagnetic radii were calculated by standard formula:

$$\langle r_k^2\rangle = 6\frac{dF_K(t)}{dt}\Big|_{t=0} \tag{14}$$

The received values are shown in Table 2.The contributions from separate diagrams are also given in the Table 2. One can see the main contribution to the K^0 - meson electromagnetic radii to arise from the diagram with intermediate ρ- meson. This fact is in agreement with the results of Vector Dominance Model.

3.2 K_{l_3} Decay

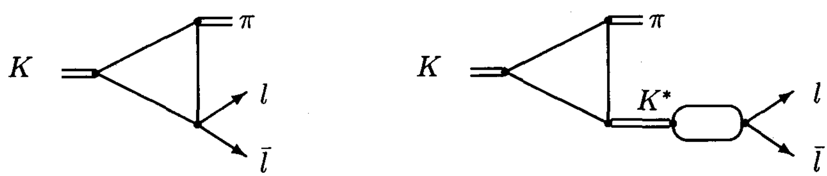
The matrix element of K_{l_3}: $K \to \pi e \nu$-decay is defined by diagrams from Fig.2 and is written as:

$$M^{\mu}(p_1, p_2) = F_+(t)(p_1 + p_2)^{\mu} + F_-(t)(p_1 - p_2)^{\mu}, \tag{15}$$

where

$$F_+(t) = F_+^a(t) + F_+(t), F_-(t) = F_-^a(t) + F_-(t), t = (p_1 - p_2)^2 \tag{16}$$

Indexes a and b define the contributions from diagrams of Fig.2a and Fig.2b.



a. b.

Fig. 2

Usually the form factors $F_{\pm}(t)$ are written in the following parameterizations :

$$F_{\pm}(t) = F_{\pm}(0)(1 + \frac{t}{m_{\pi}^2}\lambda_{\pm} \tag{17}$$

The experimentally measured values are

$$\lambda_{\pm} = \frac{m_{\pi}^2}{F_{\pm}(0)} / F_{\pm}(0)\xi(0) = \frac{F_-(0)}{F_+(0)} \tag{18}$$

Their numerical values calculated in QCD are given in Table 2.

Table 2.

Value	QCM					Experiment[11]
	The contributions from separate diagrams					
	Δ	ρ	ω	ϕ	sum	
$\langle r^2_{k^+}\rangle,^2$	0,1	0,078	0,026	-0,027	0,176	0,26±0,07
$\langle r^2_{k^0}\rangle,^2$	0,009	-0,078	0,026	-0,027	-0,07	-0,05±0,26
λ_+	0,037					0,029±0,04
λ_-	0,003					0
$\xi(0)$	-0,39					-0,35±0,14

4 Rare Semileptonic Decays of Charged Kaons

We shall describe the quark weak interaction by effective Lagrangian $\mathcal{L}_w^{eff}$ for $\Delta S = 1$ - transitions (the $K^+ \to \pi^+\ell^+\ell^-$ decay is of this type). This Lagrangian is a sum of usual four-quark operators [5] :

$$\mathcal{L}_w^{eff} = \frac{G_F}{2\sqrt{2}} \sin\theta_C \cos\theta_C \sum_{i=1}^{6} c_i O_i \tag{19}$$

Where θ_C is Cabbibo angle.

The four quark operators are defined as:

$$
\begin{array}{lll}
O_1 = (\bar{d}s)^L_\mu (\bar{u}u)^L_\mu - (\bar{d}u)^L_\mu (\bar{u}s)^L_\mu & & \Delta I = \frac{1}{2} \\
O_2 = (\bar{d}u)^L_\mu (\bar{u}s)^L_\mu - (\bar{d}s)^L_\mu (\bar{u}u)^L_\mu + 2(\bar{d}s)^L_\mu (\bar{d}d)^L_\mu & & \\
+ 2(\bar{d}s)^L_\mu (\bar{s}s)^L_\mu & & \Delta I = \frac{1}{2} \\
O_3 = (\bar{d}u)^L_\mu (\bar{u}s)^L_\mu - (\bar{d}s)^L_\mu (\bar{u}u)^L_\mu + 2(\bar{d}s)^L_\mu (\bar{d}d)^L_\mu & & \\
- (\bar{d}s)^L_\mu (\bar{s}s)^L_\mu & & \Delta I = \frac{1}{2} \\
O_4 = (\bar{d}u)^L_\mu (\bar{u}s)^L_\mu + (\bar{d}s)^L_\mu (\bar{u}u)^L_\mu & & \\
- (\bar{d}s)^L_\mu (\bar{d}d)^L_\mu & & \Delta I = \frac{3}{2} \\
O_5 = (\bar{d}\lambda^A s)^L_\mu \quad \sum_{q=u,d,s} (\bar{q}\lambda^A q)^R_\mu & & \Delta I = \frac{1}{2} \\
O_6 = (\bar{d}s)^L_\mu \quad \sum_{q=u,d,s} (\bar{q}q)^R_\mu & & \Delta I = \frac{1}{2}
\end{array}
$$

$$
\begin{aligned}
(\bar{q}_1 q_2)^L_\mu &= \bar{q}_1 \gamma_\mu (1-\gamma_5) q_2 \\
(\bar{q}_1 q_2)^R_\mu &= \bar{q}_1 \gamma_\mu (1+\gamma_5) q_2
\end{aligned}
$$

The numerical values of c_i depend on QCD parameters μ_s and α_s [5]:

$$c_1 = -\chi_1^{4/b}(0.98\chi_2^{0.42} + 0.01\chi_2^{0.80}) + 0.04\chi_1^{-2/b}(\chi_2^{0.42} - \chi_2^{-0.30})$$

$$c_2 = 0.20\chi_1^{-2/b}(0.96\chi_2^{-0.30} + 0.03\chi_2^{-0.12}) - \\ -0.02\chi_1^{4/b}(\chi_2^{0.42} - \chi_2^{-0.30})$$

$$c_5 = 10^{-2}\chi_1^{4/b}(3.3\chi_2^{0.42} + 0.3\chi_2^{-0.30} - 3.9\chi_2^{0.80} + 0.3\chi_2^{-0.12}) + \\ +10^{-2}\chi_1^{-2/b}(-0.1\chi_2^{0.42} - 2.9\chi_2^{-0.30} - 1.4\chi_2^{0.80} - 1.4\chi_2^{-0.12})$$

$$c_6 = 10^{-2}\chi_1^{4/b}(4.8\chi_2^{0.42} - 0.6\chi_2^{-0.30} - 2.9\chi_2^{0.80} - 1.3\chi_2^{-0.12}) + \\ +10^{-2}\chi_1^{-2/b}(-0.2\chi_2^{0.42} - 5.8\chi_2^{-0.30} - 1.0\chi_2^{0.80} + 7.0\chi_2^{-0.12})$$

$$\begin{pmatrix} c_3 \\ c_4 \end{pmatrix} = \chi_2^{-2/9}\chi_1^{-2/b} \begin{pmatrix} 2/15 \\ 2/3 \end{pmatrix}$$

where

$$\chi_1 = 1 + b \, \frac{\bar{g}^2(m_c)}{16\pi^2} \, \ln \frac{m_W^2}{m_c^2}$$

$$b = 11 - \frac{2}{3} N$$

$$\chi_2 = 1 + 9 \, \frac{\bar{g}^2(m)}{16\pi^2} \, \ln \frac{m_c^2}{m^2}$$

The CP conserving Dalitz pair decays of charged kaons are dominated by virtual one photon exchange. The $K^+ \to \pi^+\ell^+\ell^-$ decay is described by the diagrams represented in Fig.3. Let us write the matrix element corresponding $K^+ \to \pi^+\ell^+\ell^-$ decay in a form analogously to those given in [3]

$$M(K^+ \to \pi^+\ell^+\ell^-) = \frac{G_F}{2\sqrt{2}} \sin\theta_C \cos\theta_C$$

$$\times e\ F_+(q^2, m_K^2, m_\pi^2)\quad p^\mu\quad \frac{(-i)g^{\mu\nu}}{q^2 + i\epsilon}\ (-ie)\ \bar{l}(k)\gamma^\nu l(k') \tag{20}$$

where $F_+(q^2, m_K^2, m_\pi^2) = \sqrt{h_K h_\pi}\ \frac{3\Lambda^2}{8\sqrt{2\pi}}\ \Phi(\frac{q^2}{\Lambda^2}, \frac{m_K^2}{\Lambda^2}, \frac{m_\pi^2}{\Lambda^2})$

It is obvious,that the component along q^μ gives no contribution to the amplitude,so this term in (20) is omitted.

One must notice that for simplification we consider $\Lambda_u = \Lambda_s =\ 460\ MeV$.

Parameter Λ was shown in [6] to characterize the degree of chiral invariance breaking in the QCM, so the matrix element corresponding $K^+ \to \pi^+\ell^+\ell^-$ decay is proportional to Λ^2.

The total decay rate for $K^+ \to \pi^+e^+e^-$ is given by:

$$\Gamma(K^+ \to \pi^+e^+e^-) = \frac{C^2}{128 m_K^3 \pi^3} \int\limits_{4m_e^2}^{(m_K - m_\pi)^2} dq^2 q^2$$

$$\times\lambda^{\frac{3}{2}}(1, \frac{m_K^2}{q^2}, \frac{m_\pi^2}{q^2})\lambda^{\frac{1}{2}}(1, \frac{m_e^2}{q^2}, \frac{m_e^2}{q^2})(1 + \frac{2m_e^2}{q^2})\left|\Phi(\frac{q^2}{\Lambda^2}, \frac{m_K^2}{\Lambda^2}, \frac{m_\pi^2}{\Lambda^2})\right|^2 \tag{21}$$

where $C = G_F \sin\theta_C \cos\theta_C \sqrt{h_K h_\pi}\frac{3\alpha}{2\sqrt{2\pi}}$, $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$

In [9], [10] it was shown that for correct calculation of hadron matrix elements was necessary to take into account the contributions from intermediate meson states.

In the case of $K^+ \to \pi^+\ell^+\ell^-$ -decay it is necessary to account the intermediate axial $(a_1(1260))$, $(K_1(1270))$, pseudoscalar π^+ and vector $(\rho(770))$ - mesons. The account of axial and pseudoscalar mesons means the appearance of additional diagrams,represented in Fig.4.

The propagators of intermediate mesons in the one-loop approximation are written in the Table 1. It is important to note that the gradient invariance was controlled directly in every step of our calculations.

The account of intermediate vector $\rho(770)$ - meson is the multiplication of all matrix elements by value:

$$\frac{m_\rho^2 \tilde{\prod}_{VV}(m_\rho^2)}{m_\rho^2 \tilde{\prod}_{VV}(m_\rho^2) - q^2 \tilde{\prod}_{VV}(q^2)} \tag{22}$$

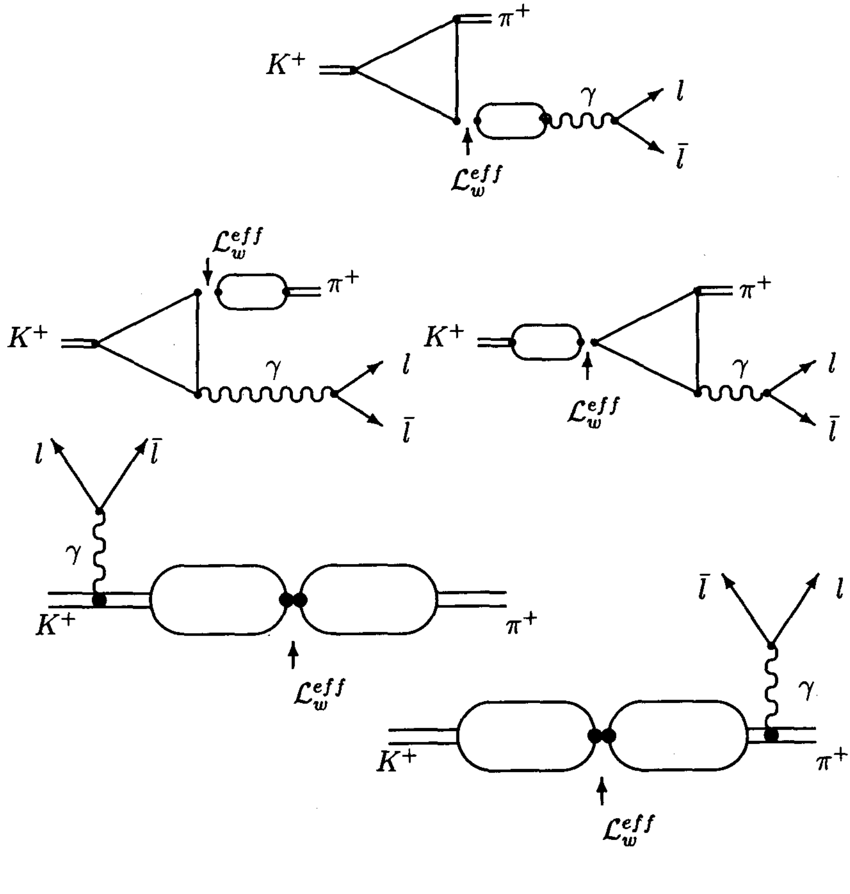


Fig. 3

It is worth noting that the function $\Phi(\frac{q^2}{\Lambda^2}, \frac{m_K^2}{\Lambda^2}, \frac{m_\pi^2}{\Lambda^2})$ () contains coefficients c_i that depend on QCD parameters. The explicit form of this dependence is given in [5]. The numerical value for $Br(K^+ \to \pi^+ e^+ e^-)$ was calculated with different values of α_s and μ_s

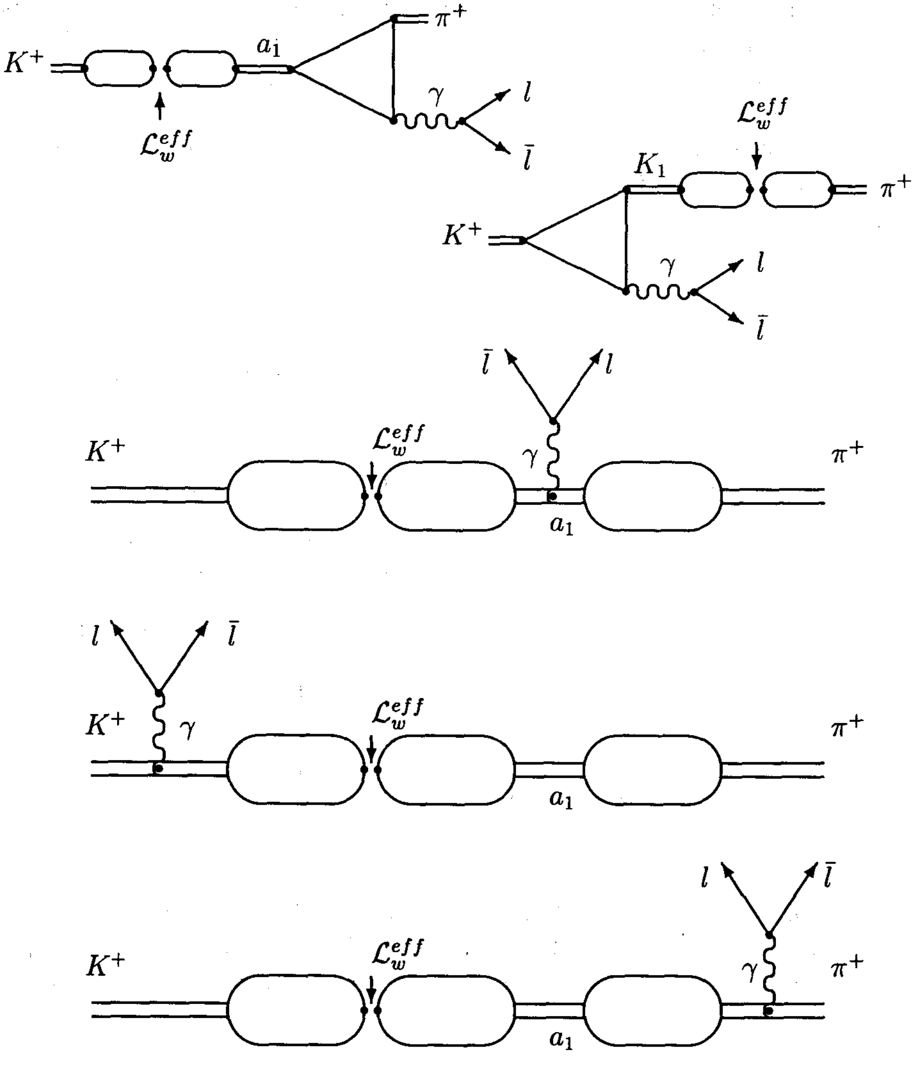


Fig. 4

The numerical results for $Br(K^+ \to \pi^+ e^+ e^-)$ with only diagrams represented in Fig. 3(Br_1) and with the account of intermediate axial, pseudoscalar and vector mesons are displayed in the Table 3. The displayed values are the average of those calculated with different α_s and μ_s.

One can see from the Table 3 that Br_1 obtained without intermediate meson states turns out ≈ 1.8 time greater than experimental one. This correlates well with the results obtained in the chiral perturbation theory [3]. The account of intermediate mesons leads to the value which is in a better agreement with experimental data.

The branching ratio for $K^+ \to \pi^+ \mu^+ \mu^-$ has been calculated. The result is given in the Table 3.

Table 3

Decay	$K^+ \to \pi^+ e^+ e^-$	$K^+ \to \pi^+ \mu^+ \mu^-$
$Br_{EXP} \times 10^{-7}$ [11]	2.99 ± 0.22	< 2.30
$Br \times 10^{-7}$ *without int. mesons*	5.58 ± 0.56	1.15 ± 0.12
$Br \times 10^{-7}$ *with int. mesons*	3.23 ± 0.56	0.73 ± 0.14

The performed investigation proves that the "long distance " contribution to the matrix elements of low-energy hadron processes can be described by means of additional quark diagrams with intermediate meson states.

5 Low Energy Hadronic Decays of τ-lepton

The electroweak meson decays are of extraordinary interest as a source of information about structure of hadrons. Among a great number of this processes the leptonic and semileptonic decays are especially important, because leptons are known as "ideal" probes for study the hadronic matter.
Since the time it was found in 1975 τ-lepton is known as a very important one for investigations of fundamental properties of electroweak interactions. Because of τ-mass value , the hadronic , namely, mesonic decays are allowed. This fact gives one the opportunity to use the hadronic decays of τ-lepton as additional and very powerful tool for study both strong and electroweak phenomena.

5.1 Two Particle τ-Decays

5.1.1 $\tau \to \pi\nu_\tau$-Decay

The amplitude of the $\tau \to \pi\nu_\tau$-decay is described by the diagram represented in Fig.5a.

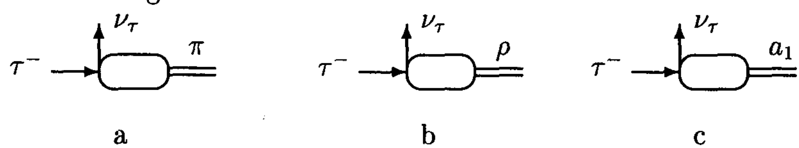


Fig. 5

It's analytical expression is written as:

$$M = \frac{G_F}{\sqrt{2}} f_\pi k^\alpha cos\theta_C \phi_\pi \bar{u}_{\nu_\tau} \gamma_\alpha (1+\gamma_5) u_\tau \tag{23}$$

where

$$f_\pi = \frac{\Lambda}{\pi} \frac{\sqrt{3} F_P(\mu_\pi^2)}{\sqrt{2 F_{PP}(\mu_\pi^2)}} \tag{24}$$

The functions $F_P(\mu_\pi^2)$ and $F_{PP}(\mu_\pi^2)$ have the following form:

$$F_P(x) = \int_0^\infty a(u)du + \frac{x}{4} \int_0^1 dua(-u\frac{x}{4})\sqrt{1-u}, \tag{25}$$

$$F_{PP}(x) = \int_0^\infty b(u)du + \frac{x}{4}\int_0^1 dub(-u\frac{x}{4})\frac{1-u/2}{\sqrt{1-u}} \qquad (26)$$

here $\mu_\pi^2 = \frac{m_\pi^2}{\Lambda^2}$
The total decay rate for the $\tau \to \pi\nu_\tau$-decay is:

$$\Gamma(\tau \to \pi\nu_\tau) = \frac{G_F^2 cos^2\theta_C f_\pi^2 m_\tau^3}{16\pi}(1 - \frac{m_\pi^2}{m_\tau^2})^2 \qquad (27)$$

The obtained numerical values for branching ratio

$$Br(\tau \to \pi\nu_\tau) = \frac{\Gamma(\tau \to \pi\nu_\tau)}{\Gamma_{tot}} = 11,25\% \qquad (28)$$

turned out to be in a good agreement with experimental data[11] :

$$Br^{exp}(\tau \to \pi\nu_\tau) = 11,31 \pm 0,15 \qquad (29)$$

5.1.2 $\tau \to \rho\nu_\tau$-Decay

The theoretical study of the τ-lepton decay into vector particles is very important because it is the main decay mode of heavy lepton. Also calculation of $\tau \to \rho\nu_\tau$-width is an additional test of model of strong interaction , pretending to describe matrix elements momentum dependence . One have to stress that calculation of this amplitudes in the another theoretical approaches , for example in chiral one [12] , needs the $\rho \to \gamma$ constant which is known to be calculated at zero momentum.
Matrix element of $\tau \to \rho\nu_\tau$-decay is defined by diagram represented in Fig.5b. Amplitude of this decay is obtained as:

$$M^{\mu\nu}(\tau \to \rho\nu_\tau) = [g^{\mu\nu}q^2 - q^\mu q^\nu]M_{\tau\to\rho\nu_\tau}(q^2), \qquad (30)$$

where $M_{\tau\to\rho\nu_\tau}(q^2)$ looks like:

$$M_{\tau\to\rho\nu_\tau}(q^2) = \frac{G_F}{\sqrt{2}}cos\Theta_C\sqrt{h_\rho}\frac{\sqrt{3}}{2\pi}\Lambda^2\Pi_V(q^2) \qquad (31)$$

Momentum dependent form factor $\Pi_V(q^2)$ is:

$$\Pi_V(x) = \frac{1}{3\Lambda^2}(\int_0^\infty b(u)du + \frac{x}{4}\int_0^1 dub(-u\frac{x}{4})\sqrt{1-u}) \qquad (32)$$

The decay width have been calculated in standard way with the account of (30), (31) is given by:

$$\Gamma(\tau \to \rho\nu_\tau) = \frac{1}{16\pi} G_F^2 cos^2\theta_C \frac{3h_\rho}{8\pi^2 m_\rho^2} \times$$
$$\times \Lambda^4 m_\tau^3 \left(1 - \frac{m_\rho^2}{m_\tau^2}\right)^2 \left(1 + \frac{2m_\rho^2}{m_\tau^2}\right) \Pi_V(m_\rho^2) \quad (33)$$

The calculated by (33)branching ratio value for $\tau \to \rho\nu_\tau$

$$Br(\tau \to \rho\nu_\tau) = \frac{\Gamma(\tau \to \rho\nu_\tau)}{\Gamma_{tot}} = 23,5\% \quad (34)$$

is in a good agreement with experimental data [11]

$$Br^{exp}(\tau \to \rho\nu_\tau) = 23,31 \pm 0,05 \quad (35)$$

5.1.3 $\tau \to a_1\nu_\tau$-Decay

The study of τ-lepton decay into axial - vector meson is important because of study of heavy lepton physics (study of decays into $(2n+1)$ π-mesons). Also it is important for the development of model of strong interactions. Last statement is connected with the problem of calculation of constant of mentioned decay in the most approaches. QCM allows one to calculate this matrix element without any additional assumptions and phenomenological parameters.
Diagram defines the matrix element of $\tau \to \rho\nu_\tau$-decay is shown in Fig.5c:

$$M^{\mu\nu}(\tau \to a_1\nu_\tau) = \frac{G_F}{\sqrt{2}} cos\Theta_C \sqrt{h_{a_1}} \frac{\sqrt{3}}{2\pi} \Lambda^2 [g^{\mu\nu} q^2 F_1^A(q^2) - q^\mu q^\nu F_2^A(q^2)], \quad (36)$$

where form-factors $F_1^A(q^2)$ and $F_2^A(q^2)$ can be represented as follows:

$$F_1^A(x) = -2. \int_0^\infty b(u) u du \quad (37)$$
$$- \frac{4}{3}\frac{x}{4} \left(\int_0^\infty b(u) du + \frac{x}{4} \int_0^1 du b(-u\frac{x}{4})(2u-1)\sqrt{1-u} \right) \quad (38)$$
$$F_2^A = \frac{1}{3\Lambda^2} \left(\int_0^\infty b(u) du + \frac{x}{4} \int_0^1 du b(-u\frac{x}{4}) \sqrt{1-u} \right) \quad (39)$$

The width of $\tau \to a_1\nu_\tau$ can be written as:

$$\Gamma(\tau \to a_1\nu_\tau) = \frac{1}{16\pi} G_F^2 cos^2\theta_C \frac{3h_{a_1}}{8\pi^2 m_{a_1}^2} \times$$
$$\times \Lambda^4 m_\tau^3 \left(1 - \frac{m_{a_1}^2}{m_\tau^2}\right)^2 \left(1 + \frac{2m_{a_1}^2}{m_\tau^2}\right) F_1^A(m_{a_1}^2) \tag{40}$$

Branching ratio numerical value calculated with(40) turned out to be:

$$Br(\tau \to a_1\nu_\tau) = \frac{\Gamma(\tau \to a_1\nu_\tau)}{\Gamma_{tot}} = 10,4\% \tag{41}$$

We have used the value of a_1 -meson mass equal $m_{a_1} = 1.275$ GeV.

5.2 $\tau \to P_1P_2\nu_\tau$-Decays

Decay $\tau \to \pi\pi\nu_\tau$ is one of the main decay modes of heavy charged lepton. So calculation of it's decay width is necessary for study of τ-lepton hadronic decays [13]. Besides this, study of $\tau \to \pi\pi\nu_\tau$ decay provides one with important information about nonstrange vector mesons properties. The $\tau \to \pi\eta\nu_\tau$,$\tau \to K\eta\nu_\tau$ decays are of extraordinary interest as a source of information about strong and electroweak interactions.
The amplitude of the decay is described by the diagrams represented in Fig.6.

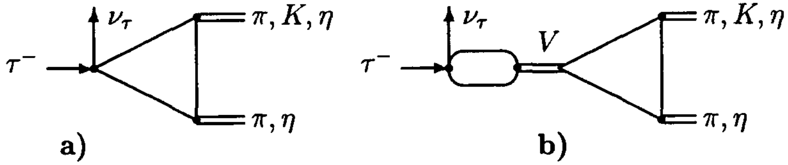


Fig. 6

$$M^\mu(\tau \to P_1P_2\nu_\tau) = M^\mu_{dir}(\tau \to P_1P_2\nu_\tau) + M^\mu_V(\tau \to P_1P_2\nu_\tau), \tag{42}$$

where M^μ_{dir} M^μ_V corresponds to the contribution of direct diagram (Fig.6a) and M^μ_V -contribution of diagram with intermediate vector state (Fig.6b).P_1 means π, or η meson,P_2 - πor$K$.

Matrix element for direct diagram (Fig.6a) was obtained as :

$$M^\mu_{dir} = -\frac{G_F}{\sqrt{2}}\sqrt{h_{\pi^\pm}}\sqrt{\eta}\frac{1}{2}SpJ_C[\lambda_{P_1}, \lambda_{P_2}]((q_1 - q_2)^\mu F_- + (q_1 + q_2)^\mu F_+) \tag{43}$$

where G_F-Fermi constant, $h_{P_{1,2}}$-quark-mesons interaction constants defined by (10). J_C-Cabibbo matrix, defined in standard way. Form factors F_- and F_+ have been received in QCM with account of Λ_u, Λ_d and Λ_s difference.

Matrix element corresponds the diagram with intermediate vector state can be written in following way:

$$\begin{aligned} M^{\mu}_{im} &= \frac{G_F}{\sqrt{2}}\sqrt{h_{P_1}}\sqrt{h_{P_2}}\frac{1}{2}Sp(J_C\lambda_V)Sp\lambda_V[\lambda_{P_1},\lambda_{P_2}] \\ &\times ((q_1+q_2)^{\mu}(q_1+q_2)^{\nu} - g^{\mu\nu}(q_1+q_2)^2)h_V G_V^{\nu\sigma}((q_1+q_2)^2) \\ &\times D_{VV}((q_1+q_2)^2)[(q_1-q_2)^{\sigma}F_- + (q_1+q_2)^{\sigma}F_+] \end{aligned} \tag{44}$$

Function $D_{VV}((q_1+q_2)^2)$ is a structure integral of the $\tau \to \nu_\tau V$ -decay matrix element(32). It was also evaluated with the account of Λ_u and Λ_d difference.

Finally, the expression (44) for matrix element corresponding diagram Fig.2 can be written as:

$$\begin{aligned} M^{\mu}_{im} &= \frac{G_F}{\sqrt{2}}\sqrt{h_{P_2}}\sqrt{h_{P_1}}\frac{1}{2}Sp(J_C\lambda_V)Sp\lambda_V[\lambda_{P_2},\lambda_{P_1}]D_{VV}((q_1+q_2)^2) \\ & [(q_1+q_2)^2(q_1-q_2)^{\sigma} - (q_1+q_2)^{\sigma}(q_1^2-q_2^2)]F_- \end{aligned} \tag{45}$$

Matrix element for three particle decay of heave lepton with η-meson was evaluated with (43) and(45):

$$M^{\mu}(\tau^- \to P_1P_2\nu_\tau) = \frac{G_F}{\sqrt{2}}\sqrt{h_{P_1}}\sqrt{h_{P_2}}(q_1-q_2)^{\mu}F_1 + (q_1+q_2)^{\mu}F_2 \tag{46}$$

where form factors F_1 and F_2 have the following form :

$$\begin{aligned} F_1 &= F_-[-\frac{1}{2}SpJ_C[\lambda_{P_1},\lambda_{P_2}] + \frac{1}{4}Sp(J_C\lambda_V)Sp\lambda_V[\lambda_{P_1},\lambda_{P_2}] \\ &\times D_{VV}((q_1+q_2)^2)(q_1+q_2)^2\frac{1}{D_{VV}(x)-D_{VV}(m_V^2)+im_V\Gamma_V}] \end{aligned} \tag{47}$$

$$\begin{aligned} F_2 &= -\frac{1}{2}SpJ_C[\lambda_{P_1},\lambda_{P_2}]F_+ - \frac{1}{4}Sp(J_C\lambda_V)Sp\lambda_V[\lambda_\eta,\lambda_P] \\ &\times D_{VV}((q_1+q_2)^2)\frac{q_1^2-q_2^2}{D_{VV}(x)-D_{VV}(m_V^2)+im_V\Gamma_V}F_- \end{aligned} \tag{48}$$

Decay width have been calculated in standard way :

$$\Gamma(\tau^- \to P_1 P_2 \nu_\tau) = \frac{G_F^2}{128 m_\tau \pi^3} h_{P_1} h_{P_2} \times$$
$$\times \int\limits_{(m_{P_1}+m_{P_2})^2}^{m_\tau{}^2} dq^2 q^4 \lambda(1, \frac{m_\tau^2}{q^2}, 0) \lambda^{\frac{1}{2}}(1, \frac{m_{P_2}^2}{q^2}, \frac{m_{P_1}^2}{q^2}) \times$$
$$\times \left\{ F_1^2 + 2F_2 F_1 \frac{m_{P_2}^2 - m_{P_1}^2}{q^2} + \right.$$
$$+ \frac{1}{3} F_2^2 \left[1 + \frac{2q^2}{m_\tau^2} - \frac{2}{q^2}(1 + \frac{2q^2}{m_\tau^2})(m_{P_2}^2 + m_{P_1}^2) + \right.$$
$$\left.\left. + \frac{2}{q^4}(m_{P_2}^2 - m_{P_1}^2)^2 (2 - \frac{q^2}{m_\tau^2}) \right] \right\} \tag{49}$$

where$\lambda(x, y, z)$ - is defined in standard way

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz) \tag{50}$$

Form factorsF_1, F_2 are defined by (47) and (48).
We have used the following values for ρ-meson parameters [11] :

$$m_\rho = 768,5 \pm 0,6 MeV, \qquad \Gamma_\rho^{full} = 150,7 \pm 1.2 MeV \tag{51}$$

The following branching ratio values were received:

$$\begin{array}{rcl} Br(\tau \to \pi\pi\nu_\tau) & = & 23{,}7\ \% \\ Br(\tau \to \pi\eta\nu_\tau) & = & 1,52 \times 10^{-4}\% \\ Br(\tau \to K\eta\nu_\tau) & = & 5,72 \times 10^{-4}\% \end{array}$$

The performed investigation proofs that the "long distance " contribution to the matrix elements is important for proper description of this kind of processes .

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