Target mass effects and integral representation for structure functions

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A method based on the Jost-Lehmann-Dyson integral representation is applied to study target mass effects in the inelastic lepton-hadron scattering. It is shown that expressions obtained for the structure functions have correct spectral properties.

Introduction. The inclusive cross section of inelastic lepton-hadron scattering is expressed as the Fourier transform of the expectation value of the current commutator $[J_\mu(z/2), J_\nu(-z/2)]$ in the target state. Structure functions of the nucleon $F_i(x,Q^2)$ parametrize the corresponding hadronic tensor $W_{\mu\nu}$ as follows

$$\begin{aligned} W_{\mu\nu}(q,P) &= \left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)F_1(x,Q^2)+\frac{1}{(q\cdot P)}\left(P_\mu-q_\mu\frac{(q\cdot P)}{q^2}\right) \qquad (1)\\ &\times \left(P_\nu-q_\nu\frac{(q\cdot P)}{q^2}\right)F_2(x,Q^2)-\frac{i}{2(q\cdot P)}\varepsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta F_3(x,Q^2)\,. \end{aligned}$$

We consider the structure functions by using an integral representation which accumulates general principles of local quantum field theory [1]. We apply here the method proposed in [2] which develops an idea of analytic approach in quantum chromodynamics [3, 4] which uses the Källén-Lehmann type analyticity. The structure functions are more complicated objects than the two-point functions, which are in one way or another related with the Källén-Lehmann representation. For these functions, the general quantum field theory principles, including covariance, Hermiticity, spectrality, and causality, are expressed by the Jost-Lehmann-Dyson (JLD) integral representation [5, 6].

The operator product expansion (OPE) is a powerful tool to study inelastic scattering processes. This method has been applied to define the contribution of target mass terms to the structure functions in paper [7]. The scheme that has been elaborated is the following. The first step is to organize the OPE by using the operators with definite twist and to take the leading twist contribution to get the free-field OPE. The second step is to collect the terms in the OPE of the form $(q \cdot P)^n$ and relate corresponding coefficients to the moments of the structure function. Then, one can restore physical structure functions by inverting the moments through the Mellin transformation. For F_2 this method gives

$$
\begin{aligned}
F_2(x,Q^2) &= \frac{x^2}{(1+4x^2\epsilon)^{3/2}}F(\xi) + 6\epsilon\frac{x^3}{(1+4x^2\epsilon)^2}\int_\xi^1 d\xi' F(\xi') \\
&+ 12\epsilon^2\frac{x^4}{(1+4x^2\epsilon)^{5/2}}\int_\xi^1 d\xi' \int_{\xi'}^1 d\xi'' F(\xi''),
\end{aligned} \tag{2}
$$

where $F(x)$ is the quark distribution function, $x = Q^2/2\nu = Q^2/2(q \cdot P)$ is the Bjorken scaling variable, M is the target mass, $\epsilon = M^2/Q^2$, and

$$
\xi = \frac{2x}{1+\sqrt{1+4x^2\epsilon}}, \tag{3}
$$

is the Nachtmann variable [8].

The defect of Eq. (2) is that there is a clear mismatch at $x = 1$. The physical structure function $F_2(Q^2, x)$ in the left hand side vanishes at $x = 1$, at the same time the right hand side does not. The trouble with the "ξ"-scaling has widely been discussed in the literature (see, for example, [9–12]).

The fact that an approximation can conflict with general principles of a theory is not rare event in quantum physics. For example, it is well known that when the renormalization group equation for the running coupling is solved directly, there arise unphysical singularities, for example, the ghost pole in the one-loop approximation, and they subsequently appear in physical quantities. This trouble can be resolved within the analytic approach proposed in [3, 4] and elaborated in [13–23]. This method combines the renormalization invariance and the Q^2-analyticity of the Källén–Lehmann type has revealed new important properties of the analytic coupling [3, 4, 20]. The invariant analytic formulation essentially modifies the behavior of the analytic running coupling in the infrared region by making it stable with respect to higher-loop corrections. This is radically different from the situation encountered in the standard renormalization-group perturbation theory, which is characterized by strong instability with respect to the next-loop corrections in the domain of small energy scale. The analytic perturbation theory leads to new non-power-series expansions with new nonsingular functions [20, 21]. Applying this algorithm to analyze the amplitudes of processes like the e^+e^--annihilation into hadrons [18], the inclusive τ-decay [14, 16, 22, 23], and the sum rules for the inelastic lepton-hadron scattering [19], it has been demonstrated that, in addition to loop stability, the analytic perturbation theory results are much less sensitive to the choice of the renormalization scheme than those in the standard approach. The three-loop level practically insures both the loop saturation and the scheme invariance of the relevant physical quantities in the entire energy or momentum range.

The method that will be considered here is a generalization of the idea used in the analytic approach to quantum chromodynamics. We base our consideration on the JLD representation for structure functions of the inelastic lepton-hadron process that has been suggested in [5, 6]. The structure functions depend on two arguments, and the corresponding representation that accumulates the fundamental properties of the theory (such as relativistic invariance, spectrality, and causality) have a more complicated form in our analysis than in the representation of the Källén–Lehmann type for functions of one variable. We use the 4-dimensional integral representation proposed by Jost and Lehmann [5] for the so-called symmetric case. A more general case has been considered by Dyson [6], and similar representation are therefore often called the Jost-Lehmann-Dyson representation. Applications of this representation to automodel asymptotic structure functions were considered by Bogoliubov, Vladimirov, and Tavkhelidze [24]; some of these results and notation will be used in what follows.

The Jost-Lehmann-Dyson representation. The proof of the JLD representation is based on the most general properties of the theory, such as covariance, Hermiticity, spectrality, and causality [1].

The <u>*covariance*</u> property means that a structure function $W(q, P)$ depend on two scalar arguments, which we choose as $\nu = q \cdot P$ and $Q^2 = -q^2$,

$$W(q, P) = W(\nu, Q^2) .$$

The <u>*spectrality*</u> property is written as

$$W(\nu, Q^2) = 0 \qquad \text{for} \qquad \frac{Q^2}{2\nu} = x > 1,$$

where we used the dimensionless Bjorken variable, which in the physical domain of the process for $(q + P)^2 > M^2$ is kinematically restricted by the interval $0 < x < 1$.

The structure function parametrizes the scattering cross-section and is real (the reality property),

$$W(\nu, Q^2) = W^*(\nu, Q^2)\,.$$

The Hermiticity of the current operator leads to the (anti-)symmetry property

$$W(-\nu, Q^2) = -W(\nu, Q^2)\,.$$

The vanishing of the current commutator at space-like intervals because of the local commutativity of currents gives the causality condition

$$\int dq\, \exp(-iqz)\, W(q, P) = 0 \qquad \text{for} \qquad z^2 < 0.$$

For the function $W(\nu, Q^2)$ satisfying all these conditions, there exists a real moderately growing distribution $\psi(\mathbf{u}, \lambda^2)$ such that the JLD integral representation holds; in the nucleon rest frame, this can be written as [24]

$$W(\nu, Q^2) \;=\; \varepsilon(q_0) \int d\mathbf{u}\, d\lambda^2\, \delta[q_0^2 - (M\mathbf{u} - \mathbf{q})^2 - \lambda^2]\, \psi(\mathbf{u}, \lambda^2) \qquad (4)$$

where the function $\psi(\mathbf{u}, \lambda^2)$ has a support for

$$\rho = |\mathbf{u}| \leq 1, \qquad \lambda^2 \geq \lambda^2_{\mathrm{min}} = M^2(1 - \sqrt{1 - \rho^2}\,)^2\,. \qquad (5)$$

For the process under consideration, the physical values of ν and Q^2 are positive. We, thus, can neglect the factor $\varepsilon(q_0) = \varepsilon(\nu)$ and keep the same notation for $W(\nu, Q^2)$. Taking into account that the weight function $\psi(\mathbf{u}, \lambda^2) = \psi(\rho, \lambda^2)$ is radial-symmetric, as follows from covariance, we write down the JLD representation for W in the covariant form,

$$\begin{aligned} W(\nu, Q^2) \;&=\; \int_0^1 d\rho\, \rho^2 \int_{\lambda^2_{\mathrm{min}}}^{\infty} d\lambda^2 \int_{-1}^{1} dz \\ &\times\; \delta(Q^2 + M^2\rho^2 + \lambda^2 - 2z\rho\sqrt{\nu^2 + M^2Q^2}\,)\psi(\rho, \lambda^2)\,. \qquad (6) \end{aligned}$$

As follows from representation (6), a natural scaling variable is given by [20, 25]

$$s \;=\; x\sqrt{\frac{1 + 4\epsilon}{1 + 4x^2\epsilon}}\,, \qquad (7)$$

which accumulates the root structure determined by the δ-function argument. At the same time, in the physical region of the process, the s variable

This expression determines simple properties of the amplitude $T(\nu, Q^2)$ in the complex x-plane and is convenient in the OPE.

In considering consequences of the JLD representation, as noted above, the natural scaling variable is given by s. In this case, there arises a similar structure of the dispersion integral

$$T(\nu, Q^2) = \frac{2}{\pi} \int_0^1 \frac{ds_1}{s_1} \frac{1}{1 - (s_1/s)^2} W(\nu_1, Q^2) \,. \tag{16}$$

The identity between the structures of the dispersion relations with respect to the variables x and s allows us to establish the relation of analytic moments to the operator product expansions of currents used in finding the Q^2-evolution of the structure functions of the moments. The moments associated with the Bjorken variable correspond to the case where only the Lorentz structures of the form $P_{\mu_1} \dots P_{\mu_n}$ are taken into account in matrix elements of the operator $\langle P|\widehat{O}_{\mu_1 \dots \mu_n}|P\rangle$. Then the application of the operator product expansion for the Compton amplitude leads to the expansion in powers of $(q \cdot P)/Q^2$, i.e., to the expansion in the inverse powers of x. A similar expansion in the inverse powers of x can also be done in dispersion integral (15). The coefficients are then determined by the x-moments. Comparing the two power series gives the sought relation between the x-moments and the operator product expansion.

In the general case, the symmetric matrix element $\langle P|\widehat{O}_{\mu_1 \dots \mu_n}|P\rangle$ contains the Lorentz structures given by $\{P_{\mu_1} \dots P_{\mu_n}\}$, $M^2 g_{\mu_i \mu_j} \{P_{\mu_1} \dots P_{\mu_{n-2}}\}$, etc. The moments with respect to the ξ variable correspond to choosing the operator basis where the expansion goes over traceless tensors, i.e., such that the contraction of $g_{\mu_i \mu_j}$ with $\langle P|\widehat{O}_{\mu_1 \dots \mu_n}|P\rangle$ vanishes for any two indices. It is then obvious that the Lorentz structure of the matrix element $\langle P|\widehat{O}_{\mu_1 \dots \mu_n}|P\rangle$ is fixed unambiguously.

Dispersion representation (16) allows us to expand the Compton amplitude in the inverse powers of s. If the operator basis is chosen such that an arbitrary contraction of the tensor $\langle P|\widehat{O}_{\mu_1 \dots \mu_n}|P\rangle$ with the nucleon momentum P_{μ_i} vanishes, then the operator product expansion leads to a power series for the forward Compton scattering amplitude with the expansion parameter $q^\mu q^\nu (P_\mu P_\nu - g_{\mu\nu} P^2)/(q^2)^2$, which corresponds to expanding dispersion integral (16) in powers of $1/s^2$. This relation between the analytic s-moments and the structure of the OPE has been found in [20]. It should be stressed that the orthogonality requirement of the symmetric tensor $\langle P|\widehat{O}_{\mu_1 \dots \mu_n}|P\rangle$ to the nucleon momentum P_{μ_i} determines its Lorentz structure unambiguously.

Target mass effects. Now we consider the method of incorporating the target mass corrections. To make our explanation more transparent and not to obscure an essence of the approach with details of technical character we start with the case of scalar currents.

Following the approach suggested in [7], consider the twist-two symmetrical local operators $\bar{\psi}\, \partial^{\mu_1} \cdots \partial^{\mu_{2N}}\, \psi$. For massless quarks

$$< P|O^{\mu_1 \cdots \mu_{2N}}|P >= O_{2N}\left\{P^{\mu_1} \cdots P^{\mu_{2N}}\right\} ,$$

where $\{P^{\mu_1} \cdots P^{\mu_{2N}}\}$ is a traceless combination of the products of vectors P^{μ_i}. By using the expression for the scalar combination of $\{P^{\mu_1} \cdots P^{\mu_{2N}}\}$ with the tensor $q_{\mu_1} \cdots q_{\mu_{2N}}$ and relating the parameters O_k according to [7] to the moments of the quark distribution function $F(x)$ of the parton language

$$O_k = \int_0^1 dx\, x^{k-2}\, F(x)\,, \tag{17}$$

for the moments of the 'physical' structure function $W(x, Q^2)$, we find

$$M_n(Q^2) = \int_0^1 dx\, x^{n-2}\, W(x, Q^2) = \frac{1}{n!} \sum_{m=0}^{\infty} \frac{(n+m)!}{m!}\, \epsilon^m\, O_{n+2m}\,. \tag{18}$$

The formal Mellin transformation of (18) gives

$$W(x, Q^2) = \frac{x}{\xi}\, \frac{F(\xi)}{1 + \epsilon\, \xi^2}\,. \tag{19}$$

This relation has obvious an trouble with the spectrality at $x = 1$ that has been mentioned above. This difficulty can be overcome by applying the JLD representation in a manner as the momentum analyticity is used for resolving the ghost pole problem.

The analytic moments can be written as follows

$$\mathcal{M}_n(Q^2) = \int_0^1 dx\, \frac{x^{n-2}}{(1 + \epsilon x^2)^{n+1}}\, F(x)\,. \tag{20}$$

The first step of our procedure is to find the weight function $U_n(\sigma)$ in the representation (8) for the analytic moments. As a result, we have

$$U_n(\sigma) = U_n(\infty) + \frac{\sigma^2}{n} \Phi_n^{'}(\sigma) - 2\sigma \frac{n-1}{n} \Phi_n(\sigma) - (n-1) \int_\sigma^\infty ds \Phi_n(s). \tag{21}$$

Here $U_n(\infty)$ is defined by the relation $\mathcal{M}_n(\infty) = U_n(\infty)/(n-1)$ and $\Phi_n(\sigma) = (\sigma/M^2)^{(n-3)/2}\, F(\sqrt{\sigma/M^2})$.

The weight functions $H(\beta,\sigma)$ in (9) and $U_n(\sigma)$ in the integral representation for the analytic moments (8) are related as follows

$$U_n(\sigma) = \int_0^1 d\beta\, \beta^{n-1}\, \tilde{H}(\beta,\sigma)\,, \tag{22}$$

where $\tilde{H}(\beta,\sigma) = H(\beta, \sigma - 2M^2(1-\sqrt{1-\beta^2}))$. Thus, the functions $U_n(\sigma)$ are the moments of the weight function $\tilde{H}(\beta,\sigma)$ and, therefore, $U_n(\sigma)$ can be restored by the Mellin transformation.

Then, we represent the function $H(\beta,\sigma)$ in the form $H(\beta,\sigma) = H_0(\beta) + h(\beta,\sigma)$, where the function H_0 is connected with the parton distribution function, and define the function $\tilde{h}(\beta,\sigma) = h(\beta,\sigma - 2M^2(1-\sqrt{1-\beta^2}))$, for which one can write

$$\tilde{h}(\beta,\sigma) = \frac{1}{2\pi i}\int\limits_{\gamma-i\infty}^{\gamma+i\infty} dn\, \beta^{-n}\, [U_n(\sigma) - U_n(\infty)]\,,$$

where the difference $U_n(\sigma) - U_n(\infty)$ is expressed via the parton distribution function as follows

$$\begin{aligned} U_n(\sigma) - U_n(\infty) &= \frac{1}{2M^2}\frac{\sigma^2}{n}\frac{\partial}{\partial\sigma}\left[\left(\frac{\sigma}{M^2}\right)^{(n-3)/2} F\left(\sqrt{\frac{\sigma}{M^2}}\right)\right] \\ &- \frac{\sigma}{M^2}\frac{n-1}{n}\left[\left(\frac{\sigma}{M^2}\right)^{(n-3)/2} F\left(\sqrt{\frac{\sigma}{M^2}}\right)\right] \\ &- \frac{n-1}{2M^2}\int_\sigma^\infty ds \left[\left(\frac{s}{M^2}\right)^{(n-3)/2} F\left(\sqrt{\frac{s}{M^2}}\right)\right]. \end{aligned} \tag{23}$$

Next, we represent the structure function as $W(x,Q^2) = W_0(x,Q^2) + w(x,Q^2)$, where $W_0(x,Q^2)$ corresponds to the weight function $H_0(\beta)$; and $w(x,Q^2)$, to $h(\beta,\sigma)$, and express $W_0(x,Q^2)$ in the form

$$\begin{aligned} W_0(x,Q^2) &= \int_0^1 d\beta\, \theta[f(\beta;x,\epsilon)]\, H_0(\beta)\,, \\ f(\beta;x,\epsilon) &= \frac{\beta}{s}\sqrt{1+4\epsilon} - 1 - 2\epsilon(1-\sqrt{1-\beta^2})\,. \end{aligned} \tag{24}$$

The variables β_- and β_+, if $x > \tilde{x} \equiv 1/\sqrt{1+4\epsilon^2}$,

$$\beta_\pm = \frac{x\sqrt{1+4\epsilon x^2}}{1+4\epsilon x^2 + 4\epsilon^2 x^2}\left[1 + 2\epsilon \pm 2\epsilon\sqrt{\frac{1-x^2}{1+4\epsilon x^2}}\right], \tag{25}$$

are the roots of the equation $f(\beta; x, \epsilon) = 0$. Thus, we have

$$W_0(x, Q^2) = \begin{cases} F(\beta_-) - F(1), & 0 \leq x < \tilde{x}, \\ \\ F(\beta_-) - F(\beta_+), & \tilde{x} \leq x \leq 1. \end{cases} \tag{26}$$

The spectral property of $W_0(x, Q^2)$, its vanishing at $x = 1$, comes from the relation $\beta_-(x = 1) = \beta_+(x = 1)$. The function $W_0(x, Q^2)$ is a continuous function at $x = \tilde{x}$ because $\beta_+(\tilde{x}) = 1$.

For the function $w(x, Q^2)$, one finds

$$w(x, Q^2) = \int_0^1 d\beta \, \theta[f(\beta; x, \epsilon)] \, \theta[g(\beta; x, \epsilon)] \, \phi(\beta; x, \epsilon) \,, \tag{27}$$

where $f(\beta; x, \epsilon)$ is defined in (24),

$$\begin{aligned} g(\beta; x, \epsilon) &= \left[(\beta/s)\sqrt{1 + 4\epsilon} - 1 \right] / \epsilon - \beta^2 \,, \\ \phi(\beta; x, \epsilon) &= \frac{1}{4\sqrt{\tau}} \theta(\tau) \, \theta(1 - \tau) \, \frac{\partial}{\partial(\sqrt{\tau})} \left[\sqrt{\tau} F(\sqrt{\tau}) \right] \,, \end{aligned}$$

with $\tau \equiv \tau(\beta; x, \epsilon) = \left[(\beta/s)\sqrt{1 + 4\epsilon} - 1 \right] / \epsilon$. The equation $\tau(\beta; x, \epsilon) = 1$ has the root $\beta_\tau = (1 + \epsilon)s/\sqrt{1 + 4\epsilon}$. The solutions of the equation $g(\beta; \eta, \epsilon) = 0$ are connected with the ξ-variable ($\xi_- = \xi$) and are of the form $\xi_\pm = (\sqrt{1 + 4\epsilon x^2} \pm 1)/2\epsilon x$.

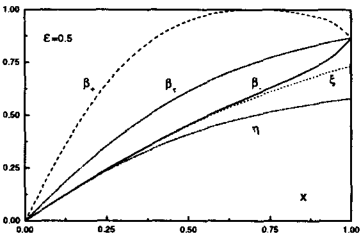

Fig. 1: *Behavior of functions $\beta_\pm$, β_τ, ξ, and η as function of x for $\epsilon = 0.5$.*

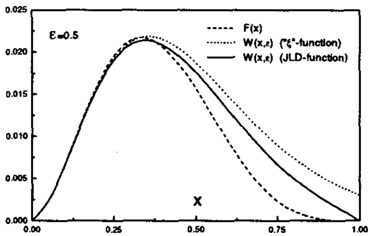

Fig. 2: *Behavior of structure functions for $\epsilon = 0.5$.*

The relative behavior of the functions $\beta_\pm$, β_τ, ξ, and $\eta = s/\sqrt{1 + 4\epsilon}$ as a function of x for $\epsilon = 0.5$ is shown in Fig. 1. This figure demonstrates that the ξ does not appear in the expression for the structure function, because the range of integration in Eq. (27) includes the interval from β_- to β_τ.

In Fig. 2, we plot the structure functions as functions of x for $\epsilon = 0.5$. The parton distribution is taken in the form $F(x) = x^2(1-x)^4$ (dashed curve). The physical structure functions, $W(x,\epsilon)$, that depend on the target mass are obtained in two ways: the dotted curve was constructed by the "ξ"-scaling expression (19), and the solid line was constructed by using the JLD representation. This figure demonstrates the difference between these methods. The structure function obtained by the JLD representation has the correct spectral behavior at $x = 1$ as compared with the "ξ"-scaling prediction.

Consider, as an example of the physical structure functions, the function F_3. In the leading twist approximation [7], for the corresponding amplitude T_3, one can write

$$T_3 = \sum_m \frac{1}{x^{2m}} \sum_{j=0}^{\infty} \epsilon^j \, \frac{(2m+j)!\, \mathcal{A}_{2m+2j}}{j!(2m-1)!(2m+2j)!}\,, \tag{28}$$

where, as usually, the moments $\mathcal{A}_n$ in (28) are defined via the quark distribution function $F(x)$

$$\mathcal{A}_n = \int_0^1 dy\, y^{n-2}\, F(y)\,. \tag{29}$$

For the x-moments of the physical structure function $xF_3(x,Q^2)$ one finds

$$\begin{aligned} M_n(Q^2) &= n \sum_{j=0}^{\infty} \epsilon^j \, \frac{(n+j)!}{j!\,n!} \, \frac{\mathcal{A}_{n+2j}}{(n+2j)} \\ &= n \int_0^1 dy \, \frac{F(y)}{y^2} \int_0^1 dz \, \frac{z^{n-1}}{(1-\epsilon z^2)^{n+1}}\,. \end{aligned} \tag{30}$$

The Mellin transformation gives the expression (cf. [7, 29])

$$\begin{aligned} x\, F_3(x,Q^2) &= -x^2 \frac{\partial}{\partial x} \int_\xi^1 dy \, \frac{F(y)}{y^2\,(1+\epsilon\xi^2)} \\ &= \frac{x^2\, F(\xi)}{\xi^2(1+4\epsilon x^2)} + \frac{2x^3 \epsilon}{(1+4\epsilon x^2)^{3/2}} \int_\xi^1 dy \, \frac{F(y)}{y^2}\,, \end{aligned} \tag{31}$$

where ξ is defined by Eq. (3). In the limit $\epsilon \to 0$ we get $x\, F_3(x,Q^2) \to F(x)$.

The analytic moments in this case have the form

$$\mathcal{M}_n(Q^2) = \int_0^1 dz \left[n(1-\epsilon z^2)^2 - 4\epsilon z^2\right] \frac{z^{n-1}\,\tilde{F}(z)}{(1+\epsilon z^2)^{n+1}}, \tag{32}$$

where

$$\tilde{F}(z) = \int_z^1 dy\, \frac{F(y)}{y^2}. \tag{33}$$

From Eq. (32), one finds

$$\begin{aligned} U_n(\sigma) &= U_n(\infty) + \frac{1}{2M^2\, n(n+2)}\Big[4\sigma^4\Phi_n^{(3)}(\sigma) \\ &+ 12(2-n)\sigma^3\Phi_n^{(2)}(\sigma) + (13n^2-34n+24)\sigma^2\Phi_n^{(1)}(\sigma) \\ &- 2n(n-1)(3n-2)\sigma\Phi_n(\sigma) - n^2(n+2)(n-1)\int_\sigma^\infty ds\,\Phi_n(s)\Big], \end{aligned} \tag{34}$$

where $\Phi_n(\sigma)$ is expressed via the function (33) as follows

$$\Phi_n(\sigma) = \left(\frac{\sigma}{M^2}\right)^{(n-2)/2} \tilde{F}\left(\sqrt{\frac{\sigma}{M^2}}\right). \tag{35}$$

We represent the function $xF_3(x,Q^2)$ in the form

$$xF_3(x,Q^2) \equiv W_3(x,Q^2) = W_3^{(0)}(x,Q^2) + w_3(x,Q^2).$$

Here $W_3^{(0)}(x,Q^2)$ is expressed via the quark distribution function as in Eq. (26) and $w_3(x,Q^2)$ has the form

$$w_3(x,Q^2) = \int_0^1 d\beta\,\theta[f(\beta;x,\epsilon)]\,\theta(z)\,\theta(1-z)\,\Phi(\beta;x,\epsilon), \tag{36}$$

where

$$\begin{aligned} \Phi(\beta;x,\epsilon) &= \frac{1}{8}\Bigg[\left(-1+\frac{\beta^2}{z^2}\right) z^3 \left(\frac{F(z)}{z^2}\right)^{(2)} \\ &+ 3\left(-1+3\frac{\beta^2}{z^2}\right) z^2 \left(\frac{F(z)}{z^2}\right)^{(1)} + 3\left(1+5\frac{\beta^2}{z}\right) z \left(\frac{F(z)}{z^2}\right)\Bigg]. \end{aligned} \tag{37}$$

The quantities β and z connected by the relation

$$\beta = \eta\,(1+\epsilon z^2), \qquad \eta = \frac{x}{\sqrt{1+4\epsilon x^2}}. \tag{38}$$

Finally, we get

$$
\begin{aligned}
w_3(x,Q^2) &= 2\epsilon\eta^3 \int_{z_-}^{1} dz\, \frac{F(z)}{z^2} \qquad (39)\\
&- \frac{\epsilon\eta}{4}\Big[\left(\eta^2\epsilon^2 z_-^4 + (-1+2\eta^2\epsilon)z_-^2 + \eta^2\right)F^{(1)}(z_-)\\
&+ \left(\eta^2\epsilon^2 z_-^4 + 3(1+2\eta^2\epsilon)z_-^2 + 5\eta^2\right)\frac{F(z_-)}{z_-}\Big],
\end{aligned}
$$

where $z_- \equiv z(\beta_-)$ and β_- is defined in (25). Similar expressions can be found for the structure functions F_1 and F_2.

Conclusions. The Jost-Lehmann-Dyson representation reflecting the general principles of the local quantum field theory (covariance, Hermiticity, spectrality, and causality) has been applied for studying the inelastic lepton-hadron process. We have concentrated on the well-known trouble that is a characteristic feature of the so-called "ξ"-scaling approach. We have argued that the approach based on the Jost-Lehmann-Dyson representation gives the self-consistent method of incorporating the target mass dependence into the structure function and does not lead to the conflict with the spectral condition.

Acknowledgments. The authors would like to express sincere thanks to S.V. Mikhailov, K.A. Milton, D.I. Kazakov, V.I. Kuvshinov, D.V. Shirkov, A.N. Sissakian, O.P. Solovtsova, and N.M. Shumeyko for interest in the work and helpful discussions. The work of I.L.S. was supported by the RFBR, grants 99-01-00091, 99-02-17727, 00-15-96691, and the US National Science Foundation, grant PHY-9600421.

REFERENCES

[1] N.N. Bogoliubov and D.V. Shirkov, *Introduction to the Theory of Quantum Fields* [*in Russian*], Nauka, Moscow (1973, 1976, 1986), *English transl.*: Wiley, New York (1959, 1980).

[2] I.L. Solovtsov, Part. Nucl. Lett. 4[101] (2000) 10.

[3] D.V Shirkov and I.L. Solovtsov, JINR Rapid Comm. 2[76]-96 (1996) 5, hep-ph/9604363.

[4] D.V Shirkov and I.L. Solovtsov, Phys. Rev. Lett. **79** (1997) 1209.

[5] R. Jost and H. Lehmann, Nuovo Cim. **5** (1957) 1598.

[6] F.J. Dyson, Phys. Rev. **110** (1958) 1460.

[7] H. Georgi and H.D. Politzer, Phys. Rev. D **14** (1976) 1829.

[8] O. Nachtmann, Nucl. Phys. B **63** (1973) 237.
[9] D.J. Gross, S.B. Treiman and Wilczek, Phys. Rev. D **15** (1977).2486.
[10] A. De Rújula, H. Georgi and H.D. Politzer, Phys. Rev. D **15** (1977) 2495.
[11] J.L. Miramontes and J.S. Guillén, Z. Phys. C **41** (1988) 247.
[12] R.G. Roberts, *The Structure of the Proton. Deep Inelastic Scattering*, Cambridge University Press, 1990.
[13] K.A. Milton and I.L. Solovtsov, Phys. Rev. D **55** (1997) 5295; Phys. Rev. D **59** (1999) 107701.
[14] O.P. Solovtsova, Pis'ma v ZhETF, **64** (1996) 664 [JETP Lett. **64** (1996) 714]; Yad. Fiz. **63** (2000) 738 [Phys. At. Nucl. **63** (2000) 672].
[15] D.V. Shirkov, Nucl. Phys. B (Proc. Suppl.) **64** (1998) 106.
[16] K.A. Milton, I.L. Solovtsov and O.P. Solovtsova, Phys. Lett. B **415** (1997) 104; Proceedings the XXIX Int. Conference on High Energy Physics, Vancouver, B.C., Canada, July 23-29, 1998, v.II, p.1608.
[17] K.A. Milton and O.P. Solovtsova, Phys. Rev. D **57** (1998) 5402.
[18] I.L. Solovtsov and D.V. Shirkov, Phys. Lett. B **442** (1998) 344.
[19] K.A. Milton, I.L. Solovtsov and O.P. Solovtsova, Phys. Lett. B **439** (1998) 421; Phys. Rev. D **60** (1999) 016001.
[20] I.L. Solovtsov and D.V. Shirkov, Teor. Mat. Fiz. **120** (1999) 482 [Theor. Math. Phys. **120** (1999) 1220].
[21] D.V. Shirkov, Lett. Math. Phys. **48** (1999) 135; Teor. Mat. Fiz. **119** (1999) 55; Teor. Mat. Fiz. **127** (2001) 3.
[22] K.A. Milton, I.L. Solovtsov, O.P. Solovtsova, and V.I. Yasnov, Eur. Phys. J. C **14** (2000) 495.
[23] K.A. Milton, I.L. Solovtsov and O.P. Solovtsova, Phys. Rev. D **64** (2001) 016005.
[24] N.N. Bogoliubov, V.S. Vladimirov and A.N. Tavkhelidze, Theor. Math. Phys. **12** (1972) 305.
[25] B. Geyer, D. Robaschik and E. Wieczorek, Fortschr. Phys. **27** (1979) 75; Phys. Part. Nucl. **11** (1980) 132.
[26] W. Wetzel, Nucl. Phys. B **139** (1978) 170.
[27] S. Deser, W. Gilbert and E.C.S. Sudarshan, Phys. Rev. **117** (1960) 266.
[28] B.V. Geshkenbein and A.I. Komech, Sov. J. Nucl. Phys. **26** (1977) 446.
[29] R. Barbieri, J. Ellis, M.K. Gailard, and G.G. Ross, Phys. Lett. **64**B (1976) 171; Nucl. Phys. B **117** (1976) 50.