# Variational perturbation theory and analytic properties of the running expansion parameter of QCD 

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#### Abstract

Analytic properties of the nonperturbative running expansion parameter in quantum chromodynamics which is appeared within variational perturbation theory are investigated. We argue that a requirement of the Källén-Lehmann analyticity allows one to define a variational parameter the value of which is agreed well with nonperturbative data coming from the meson spectroscopy.


Perturbation theory is the basic method of quantum field theory for performing calculations involving only Lagrangian parameters. Its use along with the renormalization procedure allows important results to be obtained in quantum electrodynamics, in the theory of electroweak interactions, and in the description of the perturbative region of QCD. However, specific features of quantum field theory are such that a study of the structure of a quantum field model is not completed within the framework of perturbation theory, even in theories with a small coupling constant. In particular, as it is well known, a lot of problems of QCD require nonperturbative approaches. The development of nonperturbative methods has received a great deal of attention. In this paper we, according to ideas of the analytic approach to QCD [1-4], investigate analytic properties of the nonperturbative running expansion parameter which is appear in QCD [5,6] by using the method of variational perturbation theory (VPT) [7-10]. Within the VPT method a quantity under consideration is represented in the form of variational or so-called "floating" expansion which determines the algorithm of calculating corrections up to any order. Moreover, the existence of free parameters allows one to control the convergence properties of the VPT series.

The method described in Refs. [5, 6] starts with the standard action of QCD, written as

$$
\begin{equation*}
S(A, q, \varphi)=S_{2}(A)+S_{2}(q)+S_{2}(\varphi)+g S_{3}(A, q, \varphi)+g^{2} S_{4}(A) \tag{1}
\end{equation*}
$$

where $S_{2}(A)$, including the gauge-fixing term, $S_{2}(q)$ and $S_{2}(\varphi)$ are the standard free actions of the gluon, quark and ghost fields. The term $S_{3}(A, q, \varphi)$ describes the Yukawa interaction of gluons, gluons with quarks, and gluon with ghosts

$$
\begin{equation*}
S_{3}(A, q, \varphi)=S_{3}(A)+S_{3}(A, q)+S_{3}(A, \varphi) \tag{2}
\end{equation*}
$$

and the term $S_{4}(A)$ in Eq.(1) generates the four-gluon vertex. By use of an auxiliary field $\chi_{\mu \nu}$ the latter action can be rewritten as a trilinear action between $A$ and $\chi$, so that $S$ becomes

$$
\begin{equation*}
S(A, q, \varphi, \chi)=S_{2}(q)+S_{2}(\varphi)+S_{2}(\chi)+S(A, \chi)+g S_{3}(A, q, \varphi) \tag{3}
\end{equation*}
$$

Following the ideas of the VPT method, we introduce auxiliary parameters $\xi$ and $\zeta$ and rewrite the action in the form

$$
\begin{equation*}
S(A, q, \varphi, \chi)=S_{0}^{\prime}(A, q, \varphi, \chi)+S_{I}^{\prime}(A, q, \varphi, \chi) \tag{4}
\end{equation*}
$$

where

$$
\begin{equation*}
S_{0}^{\prime}=\zeta^{-1}\left[S(A, \chi)+S_{2}(q)+S_{2}(\varphi)\right]+\xi^{-1} S_{2}(\chi) \tag{5}
\end{equation*}
$$

and

$$
\begin{align*}
S_{I}^{\prime} & =g S_{3}(A, q, \varphi)-\left(\zeta^{-1}-1\right)[S(A, \chi) \\
& \left.+S_{2}(q)+S_{2}(\varphi)\right]-\left(\xi^{-1}-1\right) S_{2}(\chi) \tag{6}
\end{align*}
$$

The exact value of the quantity under consideration, for instance, the Green function does not depend on the parameters $\xi$ and $\zeta$. However, the approximation of that quantity with a finite number of terms of the VPT series, which results from the expansion in powers of the action $S_{I}^{\prime}(A, q, \varphi, \chi)$, does depend on those parameters. We can employ the freedom in the choice of the parameters $\xi$ and $\zeta$ for construction of a new small parameter of the expansion. Analysis of the structure of the VPT series shows $[5,6]$ that the two parameters must be related by $\xi=\zeta^{3}$ in order to preserve gauge invariance. After a rescaling of the fields we obtain the following expression for a general Green function:

$$
\begin{equation*}
G(\cdots)=\int D_{Q C D}(\cdots) V(A, q, \varphi) \exp \left(i S_{0}\right) \tag{7}
\end{equation*}
$$

with

$$
\begin{array}{r}
V(A, q, \varphi)=\sum_{n} \sum_{k=0}^{n} \frac{1}{(n-k)!}\left(-\frac{\partial}{\partial \kappa}\right)^{n-k}  \tag{8}\\
\times \frac{i^{k}}{k!} \frac{1}{\left[1+\kappa\left(\zeta^{-1}-1\right)\right]^{\nu / 2}}\left[g_{3} S_{3}(A, q, \varphi)\right]^{k} \exp \left\{i\left[g_{4}^{2} S_{4}(A)\right]\right\},
\end{array}
$$

where

$$
\begin{equation*}
g_{3}=\frac{g}{\left[1+\kappa\left(\zeta^{-1}-1\right)\right]^{3 / 2}}, \quad g_{4}=\frac{g}{\left[1+\kappa\left(\xi^{-1}-1\right)\right]^{1 / 2}} . \tag{9}
\end{equation*}
$$

Here $S_{0}$ is the standard free action of QCD and $\kappa$ is a parameter introduced for convenience which is set equal to 1 at the end of the calculation.

Analysis of the structure of this variational perturbation series shows that it can be organized in powers of the new small parameter $a \equiv 1-\zeta$ if the standard coupling constant $g$ is related to $a$ by

$$
\begin{equation*}
\lambda=\frac{1}{C} \frac{a^{2}}{(1-a)^{3}}, \tag{10}
\end{equation*}
$$

where $\lambda=g^{2} /(4 \pi)^{2}=\alpha_{s} / 4 \pi$ and $C$ is a positive constant. As follows from Eq.(10), at any values of the coupling constant $g$, the new expansion parameter $a$ obeys the inequality $0 \leq a<1$. The parameter $C$ is a positive constant which plays the role of a variational parameter. The original quantity which is approximated by this expansion does not depend on the auxiliary parameters $C$; however, any finite approximation depends on it on account of the truncation of the series.

One can define the parameter $C$ using hadronic spectroscopy data [6] from the condition that the renormalization group $\beta$-function at large enough values of the coupling constant behaves as $\beta(\lambda) \simeq-\lambda$. Such a behaviour corresponds to the singular infrared behavior of the running coupling constant $\lambda\left(Q^{2}\right) \sim Q^{-2}$ and leads to the linear growth of the nonrelativistic static quark-antiquark potential at large distances. To obtain the nonperturbative $\beta$-function here we use the renormalization constants including terms up to $O\left(a^{9}\right)$ and get

$$
\begin{equation*}
\beta^{(k)}(\lambda)=-\frac{1}{C^{2}} \frac{a^{2}}{(2+a)(1-a)^{2}} \varphi^{(k)}(a) \tag{11}
\end{equation*}
$$

with

$$
\begin{align*}
& \varphi^{(9)}(a)=2 \beta_{0} a^{2}+9 \beta_{0} a^{3}+4\left(6 \beta_{0}+\frac{\beta_{1}}{2 C}\right) a^{4} \\
+ & 5\left(10 \beta_{0}+3 \frac{\beta_{1}}{C}\right) a^{5}+6\left(15 \beta_{0}+21 \frac{\beta_{1}}{2 C}+\frac{\beta_{2}}{3 C^{2}}\right) a^{6}  \tag{12}\\
+ & 7\left(21 \beta_{0}+28 \frac{\beta_{1}}{C}+3 \frac{\beta_{2}}{C^{2}}\right) a^{7}+8\left(28 \beta_{0}+63 \frac{\beta_{1}}{C}+15 \frac{\beta_{2}}{C^{2}}+\frac{\beta_{3}}{4 C^{3}}\right) a^{8} \\
+ & 9\left(36 \beta_{0}+126 \frac{\beta_{1}}{C}+55 \frac{\beta_{2}}{C^{2}}+3 \frac{\beta_{3}}{C^{3}}\right) a^{9},
\end{align*}
$$

and the perturbative coefficients of the $\beta$-function up to the four-loop level in the $\overline{\mathrm{MS}}$ renormalization scheme taken from [11]

$$
\begin{align*}
\beta_{0} & =11-\frac{2}{3} n_{f} \\
\beta_{1} & =102-\frac{38}{3} n_{f}  \tag{13}\\
\beta_{2} & =\frac{2857}{2}-\frac{5033}{18} n_{f}+\frac{325}{54} n_{f}^{2}, \\
\beta_{3} & =\left(\frac{149753}{6}+3564 \zeta_{3}\right)-\left(\frac{1078361}{162}+\frac{6508}{27} \zeta_{3}\right) n_{f} \\
& +\left(\frac{50065}{162}+\frac{6472}{81} \zeta_{3}\right) n_{f}^{2}+\frac{1093}{729} n_{f}^{3} .
\end{align*}
$$

Here $n_{f}$ is the number of quark flavors and $\zeta_{3}=1.2020569$.


Fig. 1: The behaviour of the functions $-\beta^{k}(\lambda) / \lambda$ for $k=3,5,7,9$.

In Fig. 1 we have shown the values of $-\beta^{(k)}(\lambda) / \lambda$ as functions of the coupling constant for parameters $C_{3}=4.1, C_{5}=21.3, C_{7}=61.1$ and $C_{9}=132.7$. One can see that at large values of the coupling constant $\lambda$ the function $-\beta^{(k)}(\lambda) / \lambda \simeq 1$ that corresponds to $\lambda\left(Q^{2}\right) \sim Q^{-2}$ at a small $Q^{2}$.

In oder to fix our parameters completely, we will use the information from the meson spectroscopy. The value of the coefficient $\sigma$ in the linear part of the quark-antiquark static potential $V_{\text {lin }}(r)=\sigma r$ is $\sigma \simeq 0.15-$ $\div 0.20 \mathrm{GeV}^{2}[12-15]$. The corresponding behaviour of $\alpha_{s}\left(Q^{2}\right)$ is

$$
\begin{equation*}
\bar{\alpha}_{s}\left(Q^{2}\right) \simeq \frac{3}{2} \frac{\sigma}{Q^{2}} \tag{14}
\end{equation*}
$$

at a small value of $Q^{2}$. Here we will use the value $\sigma=0.1768 \mathrm{GeV}^{2}$ [13]. By using the renormalization group method we get

$$
\begin{equation*}
\ln \frac{Q^{2}}{Q_{0}^{2}}=\phi\left(a, n_{f}\right)-\phi\left(a_{0}, n_{f}\right) \tag{15}
\end{equation*}
$$

where

$$
\begin{equation*}
\phi\left(a, n_{f}\right)=\frac{1}{2} \int^{\lambda} \frac{d \lambda}{\beta(\lambda)} \tag{16}
\end{equation*}
$$

and the $\beta$-function is defined by Eq.(11). The behavior of $\sigma\left(Q^{2}\right)$ as a function of $Q^{-1}$ is shown in Fig. 2 for normalization point $Q_{0}=100 \mathrm{MeV}$.

The running expansion parameter $a\left(Q^{2}\right)$ as a function of $Q^{2}$ is determined from the renormalization group equation

$$
\begin{equation*}
Q^{2}=Q_{0}^{2} \exp \left[\frac{C_{k}}{2 \beta_{0}}\left(f_{k}(a)-f_{k}\left(a_{0}\right)\right)\right] \tag{17}
\end{equation*}
$$

where

$$
\begin{equation*}
f_{k}(a)=\frac{2 \beta_{0}}{C_{k}} \int^{\lambda} \frac{d \lambda}{\beta_{k}(\lambda)} \tag{18}
\end{equation*}
$$

with

$$
\begin{aligned}
f_{2}(a) & =\frac{2}{a^{2}}+\frac{12}{a}-\frac{9}{1-a}+21 \ln (1-a)-21 \ln (a) \\
f_{3}(a) & =\frac{2}{a^{2}}-\frac{6}{a}-\frac{18}{11} \frac{1}{1-a}-48 \ln (a) \\
& +\frac{624}{121} \ln (1-a)+\frac{5184}{121} \ln \left(1+\frac{9}{2} a\right)
\end{aligned}
$$



Fig. 2: The tension $\sigma\left(Q^{2}\right)$ for normalization point $Q_{0}=100 \mathrm{MeV}$ and $k=$ $3,5,7,9$.

$$
\begin{aligned}
f_{4}(a) & =\frac{2}{a^{2}}-\frac{6}{a}-\frac{0.493}{1-a}+3.096 \ln (a)+1.959 \ln (1-a) \\
& -2.527 \ln \left[(x+0.176)^{2}+0.047\right]-38.441 \arctan (4.6 x+0.81) \\
f_{5}(a) & =\frac{2}{a^{2}}-\frac{6}{a}-\frac{0.197}{1-a}+1.489 \ln (a)+0.942 \ln (1-a) \\
& +13.13 \ln (x+0.303)-7.783 \ln \left[(x+0.071)^{2}+0.114\right] \\
& +11.505 \arctan (-2.965 x-0.211) \\
f_{6}(a) & =\frac{2}{a^{2}}-\frac{6}{a}-\frac{0.094}{1-a}+0.833 \ln (a)+0.527 \ln (1-a) \\
& -5.339 \ln \left[(x-0.038)^{2}+0.161\right]+1.866 \arctan (-2.489 x+0.095) \\
& +4.658 \ln \left[(x+0.296)^{2}+0.032\right]+14.19 \arctan (-5.629 x-1.664) \\
f_{7}(a) & =\frac{2}{a^{2}}-\frac{6}{a}-\frac{0.051}{1-a}+0.465 \ln (a)+0.326 \ln (1-a)+6.379 \ln (x+0.371) \\
& -0.323 \ln \left[(x+0.238)^{2}+0.095\right]-11.333 \arctan (3.235 x+0.769)
\end{aligned}
$$

$$
-3.263 \ln \left[(x-0.137)^{2}+0.189\right]+0.778 \arctan (2.299 x-0.316)
$$

Thus the running parameter $a\left(Q^{2}\right)$ is defined as an implicit function of $Q^{2}(a)$ via the equation (17). Consider the following function (with an accuracy of $O\left(a^{2}\right)$ )

$$
\begin{equation*}
z=\exp \left[\frac{C_{2}}{2 \beta_{0}}\left(\frac{2}{a^{2}}+\frac{12}{a}-\frac{9}{1-a}+21 \ln \frac{1-a}{a}\right)\right] \tag{19}
\end{equation*}
$$

with

$$
z=\frac{Q^{2}}{Q_{0}^{2}} \exp \left(\frac{C_{2}}{2 \beta_{0}} f_{2}\left(a_{0}\right)\right)
$$

The existence and regularity of the inverse function to the regular function $z=z(a)$ in a vicinity of every point $z_{0}=z\left(a_{0}\right)$ can be guaranteed if only the derivative $z^{\prime}\left(a_{0}\right) \neq 0$. The function (19) is regular in any point $a$ except $a=0$ and $a=1$ and $z^{\prime}(a)=0$ for $a=-2$. Consequently, an inverse function $a=a(z)$ may exist in the neighborhood of any point $z$, except singular points of the function and branch point $a=-2$. One cannot make any conclusions about the existence of the inverse function to $z=z(a)$ determined alongside the whole set $z$. However, if $z^{\prime}(a) \neq 0$ in the region and we know that the inverse function exists, then it means that this inverse function is regular.


Fig. 3: The complex a-plane.


Fig. 4: The complex $z$-plane.

Let us set the region in the complex $a$-plane as it is shown in Fig. 3 and examine which lines will be the images of the cuts along negative part of real axes on the complex $z$-plane (see Fig. 4) and what is the role of the parameter $C$ in this case. Let $a=\rho \exp (\imath \varphi)$, then

$$
\begin{equation*}
z=R(\rho, \varphi) \exp (\imath(\rho, \varphi)) \tag{20}
\end{equation*}
$$

where

$$
\begin{aligned}
R(\rho, \varphi)= & \exp \left[\frac { C _ { 2 } } { 2 \beta _ { 0 } } \left(\frac{2 \cos 2 \varphi}{\rho^{2}}+\frac{12 \cos \varphi}{\rho}\right.\right. \\
& \left.\left.-\frac{9(1-\rho \cos \varphi)}{1+\rho^{2}-2 \rho \cos \varphi}+21 \ln \frac{\sqrt{1+\rho^{2}-2 \rho \cos \varphi}}{\rho}\right)\right] \\
I(\rho, \varphi)= & \frac{C_{2}}{2 \beta_{0}}\left[\frac{-2 \sin 2 \varphi}{\rho^{2}}-\frac{12 \sin \varphi}{\rho}\right. \\
- & \left.\frac{9 \rho \sin \varphi}{1+\rho^{2}-2 \rho \cos \varphi}+21(\arg (1-\rho \cos \varphi-i \rho \sin \varphi)-\varphi)\right]
\end{aligned}
$$

with

$$
\arg (1-\rho \cos \varphi-i \rho \sin \varphi)= \begin{cases}\arctan \frac{-\rho \sin \varphi}{1-\rho \cos \varphi}, & \text { if } 1-\rho \cos \varphi \geq 0 \\ \pi+\arctan \frac{-\rho \sin \varphi}{1-\rho \cos \varphi}, & \text { if } 1-\rho \cos \varphi<0 \\ & -\rho \sin \varphi \geq 0 \\ -\pi+\arctan \frac{-\rho \sin \varphi}{1-\rho \cos \varphi}, & \text { if } 1-\rho \cos \varphi<0 \\ & -\rho \sin \varphi<0\end{cases}
$$

In order that the cuts on the a-plane are the images of the negative real $z$-axes, we have to put certain conditions on the parameter $C$. We have to choose parameter $C_{2}$ as following:

$$
C_{2}=\frac{2 \beta_{0}}{21}, \quad C_{2}\left(n_{f}=3\right)=0.857
$$

In this case we have only one meaning of $z_{0}$ in the $z$-plane for given $a_{0}$ from the $a$-plane.

However, the question about the existence of the inverse function $a(z)$ is still unsolved. To resolve this it is necessary to have, for the equation (19), only one solution of $a$ in the region $a$ for any $z$ from the $z$-plane. Even from a simple example of the function $z=e^{a}$ one can see that the inverse function may be a many-valued analytic function. To define it one has to pick out the principal branch on the $a$-plane which is the image of the $z$-plane $(-\pi<\arg z \leq \pi)$. For simplicity, we will change $a \rightarrow 1 / \zeta$, then

$$
\begin{equation*}
z=\exp \left[\frac{C_{2}}{2 \beta_{0}}\left(2 \zeta^{2}+12 \zeta-\frac{9 \zeta}{\zeta-1}+21 \ln (\zeta-1)\right)\right] . \tag{21}
\end{equation*}
$$

We put $\zeta=W(z)$. To extend $\zeta$ to the complex plane, we must define all of the branches of $W$. We specify the boundary curves that maximally partition the $\zeta$-plane and find the curves which separate the principal branch $W_{0}$ from the branches $W_{1}$ and $W_{-1}$. The $\zeta$-plane corresponding to (21) is shown in Fig. 5. All of the solid boundary lines in Fig. 5 reflect into the line running along the negative real axis on the $z$-plane, and all of the dash near-boundary lines in Fig. 5 reflect into the line lying just below the negative real axis on the $z$-plane. The negative real axis in the $z$-plane is called the branch cut for the logarithm, and the limiting value $z=0$ is called the branch point. The curves which separate the principal branch $W_{0}$, from the branches $W_{1}$ and $W_{-1}$ are:

$$
\frac{1}{21}\left(4 x y+12 y+\frac{9 y}{(x-1)^{2}+y^{2}}+21 \arg (x-1+i y)\right)=\mp \pi
$$



Fig. 5: The complex $\zeta$-plane.
where

$$
\zeta=x+i y
$$

Branch points are $\zeta=0$ and $\zeta=-1 / 2$ (here $z^{\prime}(\zeta)=0$ ) and, consequently, $\zeta=W(z)$ will be a many-valued analytic function.

Similarly we determine the values of parameters $C_{k}$ in higher orders. With an accuracy $O\left(a^{3}\right)$ we get

$$
\begin{equation*}
z=R(\rho, \varphi) \exp (\imath(\rho, \varphi)) \tag{22}
\end{equation*}
$$

where

$$
\begin{aligned}
R(\rho, \varphi) & =\exp \left[\frac { C _ { 3 } } { 2 \beta _ { 0 } } \left(\frac{2 \cos 2 \varphi}{\rho^{2}}-\frac{6 \cos \varphi}{\rho}-48 \ln \rho-\frac{18}{11} \frac{1-\rho \cos \varphi}{1+\rho^{2}-2 \rho \cos \varphi}\right.\right. \\
& \left.\left.+\frac{624}{121} \ln \sqrt{1+\rho^{2}-2 \rho \cos \varphi}+\frac{5184}{121} \ln \sqrt{\left(\frac{2}{9}\right)^{2}+\rho^{2}+\frac{4}{9} \rho \cos \varphi}\right)\right]
\end{aligned}
$$

$$
\begin{array}{r}
I(\rho, \varphi)=\frac{C_{3}}{2 \beta_{0}}\left[\frac{-2 \sin 2 \varphi}{\rho^{2}}+\frac{6 \sin \varphi}{\rho}-48 \varphi-\frac{18}{11} \frac{\rho \sin \varphi}{1+\rho^{2}-2 \rho \cos \varphi}\right. \\
\left.+\frac{624}{121} \arg (1-\rho \cos \varphi-i \rho \sin \varphi)+\frac{5184}{121} \arg (2 / 9+\rho \cos \varphi+i \rho \sin \varphi)\right]
\end{array}
$$

with

$$
\arg (1-\rho \cos \varphi-i \rho \sin \varphi)= \begin{cases}\arctan \frac{-\rho \sin \varphi}{1-\rho \cos \varphi}, & \text { if } 1-\rho \cos \varphi \geq 0 \\ \pi+\arctan \frac{\rho \sin \varphi}{1-\rho \cos \varphi}, & \text { if } 1-\rho \cos \varphi<0 \\ & -\rho \sin \varphi \geq 0 \\ -\pi+\arctan \frac{-\rho \sin \varphi}{1-\rho \cos \varphi}, & \text { if } 1-\rho \cos \varphi<0 \\ & -\rho \sin \varphi<0\end{cases}
$$

and

$$
\begin{aligned}
& \arg (2 / 9+\rho \cos \varphi+i \rho \sin \varphi)= \\
& \begin{cases}\arctan \frac{\rho \sin \varphi}{2 / 9+\rho \cos \varphi}, & \text { if } 2 / 9+\rho \cos \varphi \geq 0 ; \\
\pi+\arctan \frac{\rho \sin \varphi}{2 / 9+\rho \cos \varphi}, & \text { if } 2 / 9+\rho \cos \varphi<0, \\
& \rho \sin \varphi \geq 0 ; \\
-\pi+\arctan \frac{\rho \sin \varphi}{2 / 9+\rho \cos \varphi}, & \text { if } 2 / 9+\rho \cos \varphi<0, \\
& \rho \sin \varphi<0 .\end{cases}
\end{aligned}
$$

We fix parameters $C_{i}: C_{3}=3.5, C_{4}=9.2, C_{5}=19.1, C_{6}=34.1$, and $C_{7}=55.6$. One can see the values of variational parameters $C_{k}$ obtained do not too differ from that obtained earlier on basis of the meson spectroscopy data.

Thus within the variational perturbation theory we have examined a complex plane of running expansion parameter $a=1 / \zeta$ and defined the branches of the many-valued function $\zeta=W(z)$. The requirement of certain analytic properties of the running expansion parameter allowed us to define the values of the variational parameters $C_{k}$ which turned out to be close to the values coming from the nonperturbative information of the meson spectroscopy.

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