

Electromagnetic Decays of Light Vector Mesons

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Abstract

This work dedicated to describing two-body compounded system of quarks (meson) in point form of Poincare invariant quantum mechanics with potential, which was offered by so-called Mock-meson model. Authors shows process of calculation the basis parameters of the model by the variational method and followed applications for radiative decay processes. For such calculation authors use the simplest radiative decay scheme. Comparing results with experimental data was shown how to calculate the anomalous part of quark magnetic moment.

1 Introduction

The radiative processes, in particular the decays of vector mesons has been a convenient tool for studying the structure of hadrons. There are quite a number of approaches for the model to describe radiative transitions mesons(see,[1, 2, 3, 4]). In our work, the calculation of the form-factor of the radiative decay conducted within the constituent relativistic quark model based on the point form of Poincare invariant quantum mechanics (about Poincare invariant quantum mechanics, see, eg [5, 6]).

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2 Elements of quark model, based on Poincare invariant quantum mechanics

For the our relativistic quark model, based on the Poincare invariant quantum mechanics, using the potential proposed by the so-called Mock-meson model [7] using a model parameterization running coupling constant of the strong interaction, modified in [8]:

$$\alpha_S(Q^2) = \sum_{k=1}^7 \alpha_k \exp(-Q^2/4\gamma_k^2) . \quad (1)$$

The interquark potential in coordinate representation from [7] is used, which is considered a sum of Coulomb, linear confinement, and spin-spin parts for pseudoscalar and vector mesons:

$$\hat{V}(\mathbf{r}) = \hat{V}_{\text{Coul}}(\mathbf{r}) + \hat{V}_{\text{conf}}(\mathbf{r}) + \hat{V}_{\text{SS}}(\mathbf{r}) \quad (2)$$

where

$$\hat{V}_{\text{Coul}}(r) = -\frac{4}{3} \frac{\alpha_S(r)}{r} = -\frac{4}{3r} \sum_{k=1}^7 \alpha_k \text{erf}(\tau_k r) , \quad (3)$$

$$\hat{V}_{\text{conf}}(\mathbf{r}) = w_0 + \sigma r \left(\frac{\exp(-b^2 r^2)}{\sqrt{\pi} b r} + \left(1 + \frac{1}{2b^2 r^2} \right) \text{erf}(br) \right) \quad (4)$$

and

$$\hat{V}_{\text{SS}}(\mathbf{r}) = -\frac{32 (\mathbf{S}_q \mathbf{S}_{\bar{Q}})}{9\sqrt{\pi} m_q m_{\bar{Q}}} \sum_{k=1}^7 \alpha_k \tau_k^4 \exp(-\tau_k^2 r^2) . \quad (5)$$

Potential has following free parameters: gluon string tension parameter σ , parameter of perturbative part w_0 and masses of quarks $m_{q,\bar{Q}}$. Parameter τ_k determined from relation

$$\tau_k^2 = \frac{\gamma_k^2}{\gamma_k^2 + b} b , \quad (6)$$

where b is smearing parameter [7].

Parameter of linear part of the potential in most number of models lies in limits $\sigma = (0.18 \div 0.20) \text{ GeV}^2$, that's why we assume, that

$$\sigma = \bar{\sigma} \pm \Delta\sigma = (0.19 \pm 0.01) \text{ GeV}^2 . \quad (7)$$

The parameters α_k, γ_k are fixed on the basis of requirements, consistent with experimental data for the difference in the first moments of the proton and neutron spin structure functions (QCD-modified Bjorken sum rule [9])[8].

In work [10] were calculated integral representation of decay constant for pseudoscalar and vector meson in framework of point form of Poincare invariant quantum mechanics:

$$f_P(m_q, m_{\bar{Q}}, \beta) = \frac{3}{\sqrt{2\pi}} \int_0^\infty dk k^2 \Phi(k, \beta) \sqrt{\frac{M_0^2 - (m_q - m_Q)^2}{\omega_{m_q}(k) \omega_{m_Q}(k)} \frac{(m_q + m_Q)}{M_0^{3/2}}}, \quad (8)$$

$$\begin{aligned} f_V(m_q, m_Q) &= \\ &= \frac{3}{\sqrt{2\pi}} \int_0^\infty dk k^2 \Phi(k, \beta) \frac{\sqrt{(\omega_{m_q}(k) + m_q)(\omega_{m_Q}(k) + m_Q)}}{\sqrt{\omega_{m_q}(k) + \omega_{m_Q}(k)} \sqrt{\omega_{m_q}(k) \omega_{m_Q}(k)}} \times \\ &\times \left(1 + \frac{k^2}{3(\omega_{m_q}(k) + m_q)(\omega_{m_Q}(k) + m_Q)} \right), \end{aligned} \quad (9)$$

where $M_0 = \omega_{m_q}(k) + \omega_{m_{\bar{Q}}}(k)$ and $\omega_m(k) = \sqrt{k^2 + m^2}$.

Using equations (2)-(5) and (8),(9) we get the following system of equations [8]:

$$M_P(m_q, m_{\bar{Q}}, w_0, \beta) = M_P^{\text{exp}} \pm \Delta M_P, \quad (10)$$

$$M_V(m_q, m_{\bar{Q}}, w_0, \beta) = M_V^{\text{exp}} \pm \Delta M_V, \quad (11)$$

$$M_V(m_q, m_{\bar{Q}}, w_0, \beta) - M_P(m_q, m_{\bar{Q}}, w_0, \beta) = M_V^{\text{exp}} - M_P^{\text{exp}} \pm \delta M_{V,P}^{\text{exp}}, \quad (12)$$

$$f_P(m_q, m_{\bar{Q}}, \beta) = f_P^{\text{exp}} \pm \Delta f_P^{\text{exp}}, \quad (13)$$

$$f_V(m_q, m_{\bar{Q}}, \beta) = f_V^{\text{exp}} \pm \Delta f_V^{\text{exp}}, \quad (14)$$

where $M_P^{\text{exp}}, M_V^{\text{exp}}$ are experimental value of pseudoscalar and vector mesons, respectively. The last two equations (13)-(14) express condition of equality lepton coupling constants for the pseudoscalar and vector mesons, calculated in the framework of Poincare covariant model, with experimental values of decay constants. It's should be notice, that during the calculation the wave function was taken in form

$$\Phi(k, \beta) = \frac{2}{\sqrt{3}\pi^{1/4}\beta^{3/2}} \exp\left(-\frac{k^2}{2\beta^2}\right). \quad (15)$$

After solving the system of equations (10)-(14) for light mesons we have following values of quark masses and β -parameters of wave function:

$$\begin{aligned} m_u &= (239.9 \pm 2.3) \text{ MeV}, \quad m_d = (243.8 \pm 2.3) \text{ MeV}, \\ m_s &= (466.6 \pm 28) \text{ MeV}, \\ \beta_{uu} \simeq \beta_{dd} \simeq \beta_{ud} &= (328.78 \pm 2.1) \text{ MeV}, \quad \beta_{us} \simeq (360.3 \pm 12.1) \text{ MeV}. \end{aligned} \quad (16)$$

Thus, we have fixed all basic parameters of the model by equation (16).

3 Radiative decay of vector mesons

Matrix element of the radiative decay process $V \rightarrow P\gamma^*$ could be parameterized using 4-velocity of the vector and pseudoscalar mesons by expression:

$${}_P\langle Q', M_P | \hat{J}^\alpha | Q, M_V \rangle_V = \frac{e}{(2\pi)^3} g_{VP\gamma^*}(t) K^\alpha(\mu) \frac{\sqrt{M_V M_P}}{\sqrt{4V_0 V'_0}}, \quad (17)$$

where $K^\alpha(\mu) = i\epsilon^{\alpha\nu\rho\sigma} \varepsilon_\nu(\mu) V_\rho V'_\sigma$ and $e = \sqrt{4\pi\alpha_{\text{QED}}}$.

In framework of Poincare invariant quantum mechanics we consider mesons P and V as relativistic constituent quark-antiquark system. In such approach decay caused by the emission of quark a γ^* -quantum. In generalized Breit system it's easy to show, that

$$\begin{aligned} g_{VP\gamma^*}(t) &= \frac{1}{4\pi\sqrt{M_V M_P}} \int d\mathbf{k} \ k^2 \Phi(\mathbf{k}, \beta) \sqrt{\frac{1}{\omega_{m_q}(\mathbf{k}) \omega_{m_{\bar{q}}}(\mathbf{k})}} \times \\ &\times \sqrt{\frac{3 + 4\nu_1(\lambda - \nu_1)}{2}} \nu'_1 \times \left[\Phi(\mathbf{k}_2, \beta) \sqrt{\frac{\omega_{m_{\bar{q}}}(\mathbf{k}_2)}{\omega_{m_q}(\mathbf{k}_2)}} \bar{u}_{\nu'_1}(\mathbf{k}_2, m_q) B^{-1}(\mathbf{v}_{\mathbf{Q}}) \times \right. \\ &\times \frac{(\Gamma_2 \cdot K^*)}{(K \cdot K^*)} u_{\nu_1}(\mathbf{k}, m_q) D_{-\nu'_1, \lambda - \nu_1}^{1/2}(\mathbf{n}_{W_2}(\mathbf{k}, \mathbf{v}_{\mathbf{Q}})) + \Phi(\mathbf{k}_2, \beta) \sqrt{\frac{\omega_{m_q}(\mathbf{k}_2)}{\omega_{m_{\bar{q}}}(\mathbf{k}_2)}} \times \\ &\left. \times \bar{v}_{\lambda - \nu_1}(\mathbf{k}, m_{\bar{q}}) \frac{(\Gamma_1 \cdot K^*)}{(K \cdot K^*)} B(\mathbf{v}_{\mathbf{Q}}) v_{-\nu'_1}(\mathbf{k}_2, m_{\bar{q}}) D_{\nu'_1, \nu_1}^{1/2}(\mathbf{n}_{W_1}(\mathbf{k}, \mathbf{v}_{\mathbf{Q}})) \right], \end{aligned} \quad (18)$$

where \mathbf{k} is relative momentum and

$$\mathbf{n}_{W_{2,1}} = -\frac{[\mathbf{k}, \mathbf{V}]}{\omega_{m_q, \bar{q}} + m_{q, \bar{q}} - (\mathbf{k} \mathbf{V})}, \quad (19)$$

$$\Gamma_{1,2}^\mu = F_1(t)\gamma^\mu + iF_2(t)\frac{\sigma^{\mu\nu}(k_{1,2} - k)}{2m_{q,\bar{q}}}. \quad (20)$$

In relation (20) form-factors $F_1(t)$ and $F_2(t)$ normalized in the natural units magnetic μ_q and anomalous magnetic moments of quarks κ_q :

$$F_1(t=0) + F_2(t=0) = \mu_q, \quad F_2(t=0) = \kappa_q. \quad (21)$$

It's also should be notice, than in (18) relation for \mathbf{k}_2 and $\omega_{m_{q,\bar{q}}}(\mathbf{k}_2)$ given by:

$$\mathbf{k}_2 = \mathbf{k} + \mathbf{v}_Q \left((\varpi + 1)\omega_{m_{q,\bar{q}}} + \sqrt{\varpi^2 - 1} k \cos \theta_k \right), \quad (22)$$

$$\omega_{m_{q,\bar{q}}}(\mathbf{k}_2) = \omega_{m_{q,\bar{q}}}(\mathbf{k})\varpi - \sqrt{\varpi^2 - 1} k \cos \theta_k \quad (23)$$

and

$$\varpi = \frac{M_0^2 + M_0'^2 - t}{2M_0M_0'}. \quad (24)$$

Using experimental data for radiative decay of light vector mesons ρ^+ , K^\pm and K^* [11] and carrying numerical integration from (18) and (21) we obtain following values of magnetic moments of u, d and s quarks in units μ_N (nuclear magneton)(Table (1)):

Table 1: Quarks magnetic moments, μ_N

Magnetic moment	This work	[12]	[13]
μ_u	2.080 ± 0.082	2.066	2.08 ± 0.07
μ_d	-1.261 ± 0.015	-1.110	-1.31 ± 0.06
μ_s	-0.621 ± 0.011	-0.633	-0.77 ± 0.06

4 Conclusion and remarks

The work conducted within the framework of the relativistic quark model based on the point form of the Poincare-invariant quantum mechanics to obtain an integral representation for the form factor of the $V \rightarrow P\gamma$ transition. From the condition of compliance with model calculations decay width of experimental values found values of the magnetic moments of quarks, which are correlated with the data obtained using the experimental values of the magnetic moments of baryons.

5 Acknowledgments

We are grateful to the organizational and material support to the organizers of the Conference DSPIN'15. This work was supported by the Belarusian Republican Foundation for Basic Research.

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