Low Energy Hadron Interaction of Tau Lepton

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Abstract

Hadronic decays of τ-lepton have been investigated in the framework of Quark Confinement Model.Branching ratios of τ decays with one pseudoscalar, vector or axial vector meson and with two pions in the final state have been calculated. The numerical results are in a satisfactory agreement with experimental data.

1 Introduction

Since opening in 1975 [1] τ-lepton is an essential tool for testing the fundamental aspects of the electroweak interaction. In particular, due to the large mass of τ -lepton hadronic decays are cinematically . This makes it possible further study as a phenomenon related to the strong interaction, as well as phenomena associated with the weak interaction. Unlike a well-known process of hadrons, which gives an indication only of the electromagnetic vector current, semi-leptonic decays lepton provide an opportunity to study both vector and axial currents.Unlike a well-known process $e^+e^- \to \gamma$ and hadrons, which gives an indication only of the electromagnetic vector current, semi-leptonic decays of τ-lepton provide an opportunity to study both vector and axial currents.This kind of decays were studied in different theoretical approaches [2] Currently hadron decays of heavy lepton are studied by such collaborations as ALEPH [3],BaBar[4],CLEO[5],BELLE[6]. The study of hadron decays requires attraction of additional models of strong interactions at low energies.In the

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present investigation we we study τ - lepton decays in Quark Confinement Model (QCM) [7]. This model based on the certain assumptions about nature of quark confinement and hadronization allows to describe the electromagnetic,strong and weak interactions of light (nonstrange and strange)mesons from a unique point of view.

2 Two particle τ-Decays with Pseudoscalar Mesons in the Final State

The hadron fields in QCM are assumed to arise after integration over gluon and quark variables in the QCM generating function. The transition of hadrons to quarks and vice versa is given by the interaction Lagrangian. In particular necessary interaction Lagrangians for $\pi$and$K$ mesons look like:

$$\mathcal{L}_P = \frac{g_M}{\sqrt{2}} P \bar{q}^a i\gamma_5 \lambda^m q^a \tag{1}$$

λ^m - is a corresponding SU(3)-matrix,q- quark vector

$$q_j^a = \begin{pmatrix} u^a \\ d^a \\ s^a \end{pmatrix}$$

The coupling constants g_M for meson-quark interaction are defined from so-called compositeness condition. It us convenient to use interaction constant in a form:

$$h_M = \frac{3g_M^2}{4\pi^2} = -\frac{1}{\tilde{\prod}'_M(m_M)} \tag{2}$$

instead of g_M in the further calculations.

a)$\tau \to \pi\nu_\tau$ Decay

The matrix element of this decay can be written as

$$M(\tau \to \pi\nu_\tau) = \frac{G_F}{\sqrt{2}} f_\pi \cos\theta_C p^\mu \bar{\nu}(\hat{q})\gamma^\mu(1-\gamma^5)\tau(\hat{k}) \tag{3}$$

where

$$f_\pi = \frac{\sqrt{3}\Lambda F_P(\mu_\pi^2)}{\pi\sqrt{2F_{PP}(\mu_\pi^2)}} \tag{4}$$

with $\mu_\pi^2 = \frac{m_\pi^2}{\Lambda^2}$

$$\Gamma(\tau \to \pi\nu_\tau) = \frac{1}{16\pi} G_F^2 f_\pi^2 \cos^2\theta_C m_\tau^3 \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 \tag{5}$$

Structure integrals $F_P(x), F_{PP}(x)$ has the following form

$$F_P(x) = \int_0^\infty dua(u) + \frac{x}{4}\int_0^1 dua\left(-\frac{ux}{4}\right)\sqrt{1-u} \tag{6}$$

$$F_{PP}(x) = \int_0^\infty dub(u) + \frac{x}{4}\int_0^1 dub\left(-\frac{ux}{4}\right)\frac{1-\frac{u}{2}}{\sqrt{1-u}} \tag{7}$$

Functions $a(u)$ and $b(u)$ are QCM confinment functions:

$$\begin{aligned} a(u) &= a_0 e^{-u^2-a_1u} \\ b(u) &= b_0 e^{-u^2-b_1u} \end{aligned} \tag{8}$$

Decay width for $\tau \to P\nu_\tau$ is written as:

$$\Gamma(\tau \to P\nu_\tau) = \frac{1}{16\pi}G_F^2 {g_{\tau\nu P}}^2 V_{ij}^2 m_\tau^3 \left(1 - \frac{m_P^2}{m_\tau^2}\right)^2 \tag{9}$$

where V_{ij} denotes the ij element of CKM matrix [8]. In case of $\tau \to P\nu_\tau$ $V_{ij} = V_{ud}$ Branching ratio of this decay have been recieved

b) $\tau \to K\nu_\tau$ ***Decay***

To describe the interaction of heavy lepton with kaons it is necessary to take into account the difference in the parameters of the non-strange and strange quark. The matrix element of this decay can be written as

$$M(\tau \to K\nu_\tau) = \frac{G_F}{\sqrt{2}} g_{\tau\nu K} \sin\theta_C p^\mu \bar{\nu}(\hat{q})\gamma^\mu(1-\gamma^5)\tau(\hat{k}) \tag{10}$$

Form factor $g_{\tau\nu K}$ have been recieved as

$$g_{\tau\nu K} = \frac{\Lambda}{\pi}\sqrt{\frac{3h_K}{2}} F_P\left(\mu_K^2, \Lambda_u, \Lambda_s\right) \tag{11}$$

where h_K is defined by (2) Loop integral F_P in this case is:

$$\begin{aligned} F_P\left(\mu_K^2, \Lambda_u, \Lambda_s\right) &= \frac{\delta}{2}(\int_0^\infty dua(u) + s\int_0^{u_\Delta} dua(-us)\sqrt{1-u+\left(\frac{u\Delta}{2}\right)^2}) + \\ &+ \frac{\Delta}{4}\delta s^2 \int_0^{u_\Delta} duua(-us)\sqrt{1-u+\left(\frac{u\Delta}{2}\right)^2} \end{aligned} \tag{12}$$

The following notations in (12) have been introduced:

$$\Lambda^2 = \frac{\Lambda_s^2 + \Lambda_u^2}{2}; \Delta = \frac{\Lambda_s^2 - \Lambda_u^2}{\Lambda_s^2 + \Lambda_u^2}; \delta = \sqrt{1-\Delta} + \sqrt{1+\Delta}. \tag{13}$$

The decay width $\tau \to K\nu_\tau$ can be calculated by (9) with $V_{ij} = V_{sd}$

3 τ Lepton Interaction with Vector mesons

The study of τ- lepton decay in vector particles is very important due to the fact that the ρ - mesons channel is the main channel of heavy lepton decays. Interaction Lagrangians for ρandK^* mesons in QCM is:

$$\mathcal{L}_V = \frac{g_V}{\sqrt{2}} V^\mu \bar{q}^a \gamma_\mu \lambda^m q^a \tag{14}$$

a)$\tau \to \rho \nu_\tau$ **Decay**

The matrix element of this decay can be written in the following way:

$$M^{\mu\nu} = \left[g^{\mu\nu} q^2 - q^\mu q^\nu\right] F_{\tau\rho\nu}\left(q^2\right) \tag{15}$$

where

$$F_{\tau\rho\nu}\left(q^2\right) = \frac{G_F}{\sqrt{2}} V_{ud} \Lambda^2 \frac{\sqrt{3h_\rho}}{2\pi} \Pi_\rho\left(q^2\right) \tag{16}$$

Constant of ρ- quark interactions h_ρ can be calculated by (2). Form factor $\Pi_\rho(x)$ have been received as

$$\Pi_\rho(x) = \frac{1}{3\Lambda^2}\left(\int_0^\infty dub(u) + \frac{x}{4}\int_0^1 dub\left(-\frac{ux}{4}\right)\sqrt{1-u}\right) \tag{17}$$

b)$\tau \to K^* \nu_\tau$ **Decay**

Matrix element of $\tau \to K^* \nu_\tau$ decay can be written in a form similiar to (15) and (16), with V_{ud} changed to V_{sd}.

$$M^{\mu\nu} = \left[g^{\mu\nu} q^2 - q^\mu q^\nu\right] F_{\tau K^*\nu}\left(q^2\right) \tag{18}$$

$$F_{\tau K^*\nu}\left(q^2\right) = \frac{G_F}{\sqrt{2}} V_{sd} \Lambda^2 \frac{\sqrt{3h_{K^*}}}{2\pi} \Pi_{K^*}\left(q^2\right) \tag{19}$$

We have taken into account difference between nonstrange and strange quarks, so form factor $\Pi_K^*(x)$ is written as

$$\begin{aligned}
\Pi_{K^*}(x) = &\left(\sqrt{1-\Delta^2} - 1\right) \times \qquad (20)\\
&\times \left(\int_0^\infty duub(u) - \left(\frac{x}{4}\right)^2 \int_0^1 duub\left(\frac{-ux}{4}\right)\sqrt{1-u+\left(\frac{u\Delta}{2}\right)^2}\right) - \\
&- \left(\frac{x\Delta}{8}\right)^2 \int_0^1 duu^2 b\left(\frac{-ux}{4}\right)\sqrt{1-u+\left(\frac{u\Delta}{2}\right)^2} + \\
&+ \frac{4}{3\Lambda^2}\left(\int_0^\infty dub(u) + \frac{x}{4}\int_0^1 duub\left(\frac{-ux}{4}\right)\sqrt{1-u+\left(\frac{u\Delta}{2}\right)^2}\right)
\end{aligned}$$

The widthes of $\tau \to V\nu_\tau$ can be calculated in standard way and can be written using (15)-(17) and (18)-(20) as

$$\Gamma(\tau \to V\nu_\tau) = \frac{3G_F^2 V_{ij}^2 h_V \Lambda^4}{128\pi^3 m_V^2} m_\tau^3 \left(1 - \frac{m_V^2}{m_\tau^2}\right)^2 \left(1 + \frac{2m_V^2}{m_\tau^2}\right) \Pi_V^2\left(m_V^2\right) \quad (21)$$

4 Interaction of τ-Lepton with Axial a_1 Meson

The study of τ- lepton interactions with axial vector meson is extremely interesting from the point of view of studying its decay into (2n + 1) meson, as well as a testing of model because the calculation of the decay constants can not be linked with the phenomenological constants of the low-energy physics, as is done in most of the approaches in the case of final pseudoscalar and vector states.In the QCM axial vector meson-quark interactions are described by

$$i\gamma_\mu\gamma_5\lambda^m q^a \quad (22)$$

Matrix element for $\tau \to a_1\nu_\tau$ is writtten as

$$M^{\mu\nu}(\tau \to a_1\nu_\tau) = \frac{G_F}{\sqrt{2}} V_{ud}\Lambda^2 \frac{\sqrt{3h_{a_1}}}{2\pi} \left[g^{\mu\nu}q^2 F_1^A\left(q^2\right) - q^\mu q^\nu F_2^A\left(q^2\right)\right] \quad (23)$$

where form factors $F_1^A(x)$ and $F_2^A(x)$ have been obtained in a form

$$F_1^A(x) = \quad (24)$$
$$= -2\int_0^\infty du u b(u) - \frac{x}{3}\left(\int_0^\infty du b(u) + \frac{x}{4}\int_0^1 du b\left(-\frac{ux}{4}\right)(2u-1)\sqrt{1-u}\right)$$

$$F_2^A(x) = \frac{1}{3\Lambda^2}\left(\int_0^\infty du b(u) + \frac{x}{4}\int_0^1 du b\left(-\frac{ux}{4}\right)\sqrt{1-u}\right) \quad (25)$$

The decay width is calculated by the formula:

$$\Gamma(\tau \to a_1\nu_\tau) = \frac{3G_F^2 V_{ud}^2 h_{a_1}\Lambda^4}{128\pi^3 m_{a_1}^2} m_\tau^3 \left(1 - \frac{m_{a_1}^2}{m_\tau^2}\right)^2 \left(1 + \frac{2m_{a_1}^2}{m_\tau^2}\right)\left(F_1^A\left(m_{a_1}^2\right)\right)^2 \quad (26)$$

5 Three Particles Decays

Decay $\tau \to \pi\pi\nu_\tau$ is one of the main modes of heavy lepton decays. The matrix element is defined by the contact graph and graphs with an intermediate vector meson.

Contribution from the contact graph can be written as

$$M^{\mu}_{dir}(\tau \to \pi\pi\nu_\tau) = G_F V_{ud} h_\pi (q_1 - q_2)^\mu F_- \left(s, q_1^2, q_2^2\right) \tag{27}$$

where $s = (p_\tau - p_{\nu_\tau})^2$.

Form factor $F_- \left(s, q_1^2, q_2^2\right)$ have been obtained as

$$F-\left(s, q_1^2, q_2^2\right) = \frac{1}{2}\left(\int_0^\infty dub(u) + \frac{s}{4\Lambda^2}\int_0^1 dub\left(-\frac{us}{4\Lambda^2}\right)\sqrt{1-u}\right) + \tag{28}$$
$$+\frac{1}{2\Lambda^2}\int_0^1 d^3\alpha\cdot\delta\left(1-\sum_{i=1}^3 \alpha_i\right)\left(s\alpha_1\alpha_2 + q_1^2\alpha_1\left(1+\alpha_3\right) + q_2^2\left(1+\alpha_3\right)\right) b\left(-Q\right)$$

where

$$Q = \frac{s\alpha_1\alpha_2 + q_1^2\alpha_1\alpha_3 + q_2^2\alpha_2\alpha_3}{\Lambda^2} \tag{29}$$

Intermediate vector meson contribution to the matrix element in the general case can be written as

$$M^{\mu}_{int}(\tau \to \pi\pi\nu_\tau) = M^{\mu\lambda}_{\tau\to\rho\nu_\tau}(s) D^{\lambda\sigma}_\rho(s)(q_1 - q_2)^\sigma F_- \left(s, q_1^2, q_2^2\right) \tag{30}$$

$M^{\mu\lambda}_{\tau\to\rho\nu_\tau}(s)$ and $F_- \left(s, q_1^2, q_2^2\right)$ are defined by (27) and (28).

$M^{\mu\lambda}_{\tau\to\rho\nu_\tau}(s) D^{\lambda\sigma}_\rho(s)$ contents $h_\rho D^{\mu\nu}_\rho\left(p^2\right)$. Its analytical expression in chain approximation have to be modified due of $\rho-$resonance. The following form have been used

$$h_\rho D^{\mu\nu}_\rho\left(p^2\right) = \frac{1}{\Pi_{1\rho}\left(p^2\right) - \Pi_{1\rho}\left(m_\rho^2\right) + i m_\rho \Gamma_\rho} \times$$
$$\times\left(-g^{\mu\nu} + p^\mu p^\nu \frac{\Pi_{2\rho}\left(p^2\right)}{\Pi_{1\rho}\left(p^2\right) - \Pi_{1\rho}\left(m_\rho^2\right) + p^2\Pi_{2\rho}\left(p^2\right)}\right) \tag{31}$$

where m_ρ and Γ_ρ are mass and full width of ρ resonance. Matrix element of $\tau \to \pi\pi\nu_\tau$ is a sum of mentioned above contributions. So it have been obtained in the form:

$$M^{\mu}(\tau \to \pi\pi\nu_\tau) = G_F V_{ud} h_\pi (q_1 - q_2)^\mu F_- \left(s, q_1^2, q_2^2\right) \times$$
$$\times \left(\frac{\Pi_{1\rho}\left(p^2\right)}{\Pi_{1\rho}\left(p^2\right) - \Pi_{1\rho}\left(m_\rho^2\right) + i m_\rho \Gamma_\rho} - 1 \right) \tag{32}$$

The width of the decay have been received under standard transformations is written in the following way

$$\Gamma(\tau \to \pi\pi\nu_\tau) = \frac{G_F^2 h_\pi^2 V_{ud}^2}{64\pi^3 m_\tau} \times$$
$$\times \int_{4m_\pi^2}^{m_\tau^2} \frac{ds}{s} \lambda\left(s, m_\tau^2, 0\right) \lambda^{\frac{3}{2}}\left(s, m_\pi^2, m_\pi^2\right) \left(1 + \frac{2s}{m_\tau^2}\right) F_-^2\left(s, q_1^2, q_2^2\right) \tag{33}$$

6 Numerical Results

The following QCM parameters were used for calculation of numerical values of matrix elements [9]

$$\begin{aligned} \Lambda_u &= 460\ MeV \\ \Lambda_s &= 506\ MeV \\ b_0 = 2 &\qquad b_1 = 0.2 \\ a_0 = 2 &\qquad a_1 = 0.5 \end{aligned} \tag{34}$$

Branching rations calculated by

$$Br(\tau \to M\nu_\tau) = \frac{\Gamma(\tau \to M\nu_\tau)}{\Gamma_{tot}} \tag{35}$$

are given in Table.

τ-Decays	Br (QCM)	Br (Experiment) [10]
$\tau \to \pi\nu_\tau$	$11,25\%$	$(10,83 \pm 0.06)\,\%$
$\tau \to K\nu_\tau$	$7,6 \cdot 10^{-3}$	$(7,0 \pm 0.1) \cdot 10^{-3}$
$\tau \to \rho\nu_\tau$	$23,5\%$	$(25,52 \pm 0,09)\,\%$
$\tau \to K^*\nu_\tau$	$1,68\%$	$(1,20 \pm 0,07)\,\%$
$\tau \to a_1\nu_\tau$	$10,4\%$	-
$\tau \to \pi\pi\nu_\tau$	$23,7\%$	$(25,24 \pm 0,16)\,\%$

The table shows that our values are in reasonable agreement with the experimental data.

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