On Target Mass Corrections to Deep-inelastic Structure Functions

V. I. Lashkevich, O. P. Solovtsova*

International Center for Advanced Studies, Sukhoi State Technical University of Gomel

Abstract

We investigate target mass corrections to the unpolarized structure functions of the deep-inelastic scattering by using the traditional Georgi–Politzer method and another approaches. The recent methods for solving the 'threshold' problem arisen in the limit as the Bjorken variable x tends to unity are discussed. We present results of a new approach and demonstrate that, in the large-x region, target mass corrections to structure functions calculated by using this method noticeably differ that other approaches give.

1 Introduction

To compare correctly QCD predictions with experimental data of the deep-inelastic scattering at low Q^2 scales, $Q^2 \lesssim 1 - 2$ GeV^2, it is important to take into account in the analysis additional power terms are known as target mass corrections (TMCs) arising from purely kinematic effects associated with finite mass of the nucleon target. In the QCD analysis of the deep-inelastic scattering data the operator product expansion (OPE) method is widely used. However the OPE was derived in the massless limit and if a finite mass of the nucleon target is included, then the TMCs arise. Many years ago, the OPE was used to include TMC effects systematically via the Nachtmann ξ variable [1] by Georgi and Politzer (GP) [2]. The GP method, named also as ξ-scaling method, showed the importance of the

*E-mail: solovtsova@gstu.gomel.by

accounting of TMCs. However within this method there was a problem to describe the structure functions behavior as the Bjorken variable x tends to unity. This problem was widely discussed in the literature ever since its appearance and continues to be discussed until now (see, e.g., [3–5]).

In the present work we extend our previous analysis [5] and analyze several frameworks for the TMCs in order to improve a knowledge of TMC effects for the unpolarized proton structure functions.

2 Target Mass Corrections

The inclusive cross section of the deep-inelastic scattering process can be written as $d\sigma \sim L^{\mu\nu}W_{\mu\nu}$ in terms of leptonic and hadronic tensors, $L^{\mu\nu}$ and $W_{\mu\nu}$. The hadronic tensor $W_{\mu\nu}$ is parameterized by structure functions which is defined via structure functions $F_{i=1,2,3}(x,Q^2)$.[1]

2.1 Operator product expansion: GP approach

According to the GP approach the structure functions are given by [6]

$$F_1(x,Q^2) = \frac{x}{\xi\rho}F_1^0(\xi,Q^2) + \frac{\varepsilon x^2}{\rho^2}h_2(\xi,Q^2) + \frac{2\varepsilon^2x^3}{\rho^3}g_2(\xi,q^2)\,, \tag{1}$$

$$F_2(x,Q^2) = \frac{x^2}{\xi^2\rho^3}F_2^0(\xi,Q^2) + \frac{6\varepsilon x^3}{\rho^4}\,h_2(\xi,Q^2) + \frac{12\varepsilon^2x^4}{\rho^5}\,g_2(\xi,q^2)\,, \tag{2}$$

$$h_2(\xi,Q^2) = \int\limits_\xi^1 \frac{F_2^0(y,Q^2)}{y^2}dy, \quad g_2(\xi,Q^2) = \int\limits_\xi^1 dy \int\limits_y^1 \frac{F_2^0(z,Q^2)}{z^2}dz\,,$$

$$F_3(x,Q^2) = \frac{x}{\xi\rho^2}F_3^0(\xi,Q^2) + \frac{2\varepsilon x^2}{\rho^3}h_3(\xi,q^2)\,,\ h_3(\xi,Q^2) = \int\limits_\xi^1 \frac{F_3^0(y,Q^2)}{y}dy\,. \tag{3}$$

Here $x = Q^2/2\nu = Q^2/2(q\cdot P)$ is the Bjorken scaling variable, ξ is the Nachtmann variable [1]

$$\xi = \frac{2\,x}{1+\sqrt{1+4\varepsilon x^2}} = \frac{2x}{1+\rho}\,, \tag{4}$$

[1] Other structure functions, $i = 4,\ 5,\ 6$, are proportional to the lepton mass and are therefore negligible for the kinematics of the deep-inelastic region.

$\rho = \sqrt{1 + 4\varepsilon x^2}$, $\varepsilon = M^2/Q^2$, M is the target mass, the functions $F_i^0(\xi, Q^2) = \lim_{M\to 0} F_i(x, Q^2)_{x=\xi}$.

The expressions (1)–(3) are known to suffer from the "threshold problem", in which the target mass corrected structure functions do not vanish as $x \to 1$, and are in fact nonzero in the kinematically forbidden $x > 1$ region. A numerous of attempts have been made to ameliorate the threshold problem using various prescriptions.

2.2 Known approximations

Recently, Kulagin and Petti (KP) [7] showed that by expanding the target mass corrected structure functions to leading order in $1/Q^2$, the resulting functions have the correct $x \to 1$ limits (see also Ref. [8]).

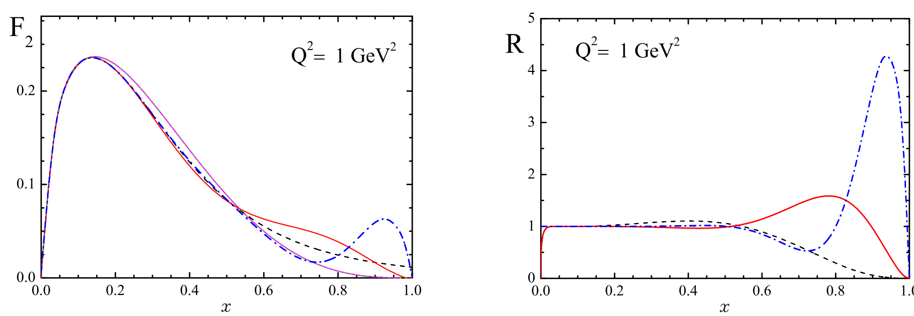

Figure 1: Left panel: The behavior of the structure function F_2 obtained vs the Bjorken variable x. The solid (red) line corresponds to result of $1/Q^2$ KP approximation, the dash-dotted (blue) line – $1/Q^4$ approximation, the dashed (black) line – the GP result, and dotted (green) line – without target mass corrections. Right panel: Ratio of the target mass corrected F_2 structure function by using the $1/Q^2$ (solid, red) and $1/Q^4$ (dash-dotted, blue) the KP approximation, and GP approximation (dashed, black) compared with the structure function without target mass corrections.

While avoiding the threshold problem, this prescription, however, raises the question of whether the $1/Q^2$ approximation is sufficiently accurate for structure functions near $x \approx 1$ at moderate Q^2. To test the convergence of the $1/Q^2$ expansion at large x, we further expand the GP result to include $\mathcal{O}(1/Q^4)$ corrections. Figure 1 illustrates the accuracy of the KP approach. In order to isolate the target mass effect from the specific form of the structure function parametrization we take for simplicity the form $F_2 \sim (1-x)^3$. One can see that both the $1/Q^2$ and $1/Q^4$ approximations are found to reproduce the GP result well up to $x \approx 0.6$, but significant deviations are visible at larger x. The reliability of a low order $1/Q^2$ expansion is therefore questionable at large x values, and hence their efficacy in removing the $x \to 1$ threshold problem.

An alternative approach to TMCs relies on the collinear factorization (CF) formalism [9–11], which makes use of the factorization theorem to relate the hadronic tensor for lepton–hadron scattering to that for scattering from a parton. Here parton distributions are formulated directly in momentum space, avoiding the need to perform an inverse Mellin transform to obtain the PDF from its moments. The first study of TMCs within CF was made by Ellis, Furmanski, and Petronzio (EFP) [9]. Using the same notation as above, the EFP results for the target mass corrected structure functions are given by

$$F_1^{\rm EFP}(x,Q^2) = \frac{2}{1+\rho} F_1^0(\xi,Q^2) + \frac{(\rho^2-1)}{(1+\rho)^2} h_2(\xi,Q^2)\,, \tag{5a}$$

$$F_2^{\rm EFP}(x,Q^2) = \frac{1}{\rho^2} F_2^0(\xi,Q^2) + \frac{3\xi(\rho^2-1)}{\rho^2(1+\rho)} h_2(\xi,Q^2)\,, \tag{5b}$$

$$F_3^{\rm EFP}(x,Q^2) = \frac{1}{\rho} F_3^0(\xi,Q^2) + \frac{2(\rho^2-1)}{\rho(1+\rho)^2} h_3(\xi,Q^2)\,, \tag{5c}$$

where again the F_i^0 refer to the uncorrected structure functions. Because the massless functions F_i^0 are evaluated at ξ, the target mass corrected structure functions will suffer from the same threshold problem as in the OPE result in Eqs. (1)–(3). So, in both the EFP and OPE treatments of TMCs, the resulting structure functions are nonzero for $x > 1$.

Other approach for target mass corrected structure functions is the approach of Steffens and Melnitchouk (SM) [3] which effectively corresponds to use of a new variable

$$\xi_{SM} = x\frac{1+\sqrt{1+4x^2}}{1+\sqrt{1+4\varepsilon x^2}}\,, \tag{6}$$

and the modified moments $A_n^{\rm (SM)} \equiv \int_0^{\xi_0} d\xi\ \xi^n\ F(\xi,\xi_0)$, with $\xi_0 \equiv \xi(x = 1) = 2/(1+\sqrt{1+4\epsilon}) < 1$ (see Refs. [5] for more details).

2.3 JLD-approach

Let us now pass to new approach which based on the Jost-Lehmann-Dyson (JLD) integral representation [12, 13]. As it was shown by Solovtsov [14] that the threshold problem is a similar to the problem that appears for an invariant charge in quantum chromodynamics, when the violation of the general principles of the theory, which are reflected in the Källén–Lehmann representation, leads to unphysical singularities. A solution of this problem was proposed proposed by Shirkov and Solovtsov[2] [15] (see Ref. [16] as review). By using the JLD integral representation it was shown [14] that the natural scaling variable is a new variable ξ_S,

$$\xi_S = x\frac{\sqrt{1+4\varepsilon}}{\sqrt{1+4\varepsilon x^2}}\,, \tag{7}$$

which leads to the moments $\mathcal{M}_n(Q^2)$ that are analytic functions. In this case, the spectral property for the structure functions is satisfied automatically, and no problem arises in the limit as the Bjorken variable x tends to unity (see, e.g. Refs. [5]). Note the proof of the JLD representation is based on the most general principles of the theory, such as the covariance, Hermiticity, spectrality, and causality.

According to JLD-approach, instead of the function $F_i^0(\xi)$ we must use

$$F_i^0(x,Q^2) = \begin{cases} F_i^0(\xi_-) - F_i^0(1)\,, & 0 \leqslant x < \overline{x}\,, \\ F_i^0(\xi_-) - F_i^0(\xi_+)\,, & \overline{x} \leqslant x \leqslant 1\,, \end{cases} \tag{8}$$

where $\overline{x} = 1/\sqrt{1+4\varepsilon^2}$,

$$\xi_\mp(x) = \frac{x\sqrt{1+4\varepsilon x^2}}{1+4\varepsilon x^2+4\varepsilon^2 x^2}\cdot\left[1+2\varepsilon \mp 2\varepsilon\cdot\frac{\sqrt{1-x^2}}{\sqrt{1+4\varepsilon x^2}}\right]. \tag{9}$$

Follow this recipe, we transform Eqs. (1)–(3) and, for example, for the structure function F_3 it turns out:
for $x \leqslant \overline{x}$

$$F_3^S(x,Q^2) = \frac{x\cdot F_3^{(0)}(\xi_-(x),Q^2)}{\xi_-(x)(1+4\varepsilon x^2)} + \frac{2\varepsilon x^2}{\sqrt{(1+4\varepsilon x^2)^3}}h_3(\xi_-(x),Q^2), \tag{10a}$$

$$F_3^S(x,Q^2) = \frac{x}{(1+4\varepsilon x^2)}\left[\frac{F_3^{(0)}(\xi_-(x),Q^2)}{\xi_-(x)} - \frac{F_3^{(0)}(\xi_+(x),Q^2)}{\xi_+(x)}\right] + \tag{10b}$$

[2] This analytic approach called the Analytic Perturbation Theory (APT).

$$+\frac{2\varepsilon x^2}{\sqrt{(1+4\varepsilon x^2)^3}}\left[h_3(\xi_-(x),Q^2)-h_3(\xi_+(x),Q^2)\right]$$

for $\overline{x} \leqslant x \leqslant 1$.

3 Numerical result

In our calculations we take the distributions of light u, d and s quarks and anti-quarks from Ref. [17], where was fixed the next to leading (NLO) value of the parameter $\Lambda_{\rm QCD} = 0.248$ GeV. We have verified that distributions provided in other papers, for example, in Refs. [18] in the region of $x > 0.2$, for which become essential the TMCs, very close to distributions given in Ref. [17].

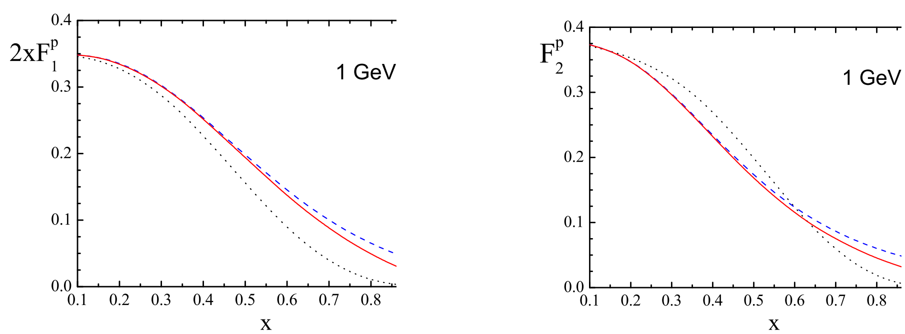

Figure 2: The behavior of the proton structure function F_1 (left panel) and F_2 (right panel) vs the Bjorken variable x at Q^2=1 GeV^2. The solid (red) line corresponds to our result obtained by using the JLD-approach, the dashed (blue) curve reflects the result obtained by standard GP method, and the dotted (green) line is the initial proton distribution [17].

Figure 2 shows the behavior of the proton structure function at Q^2=1 GeV^2 for the structure functions F_1 (left panel) and F_2 (right panel). One can see that target mass corrections to these structure functions calculated by using the JLD-approach are noticeably differ, in the large-x region, that the traditional GP method gives. The same we obtain for the proton structure function F_3 of the neutrino nucleon deep inelastic scattering obtained by using the expressions (3) and (10) (see Ref. [19] for more details).

4 Conclusion

In this report we have sought to discuss the the ‘threshold’ problem in the standard TMC analysis. Historically it has been argued that the problem in the threshold region exists because at low Q^2 the higher twist contributions cannot be neglected. The inclusion of target mass corrections in the fits of deep-inelastic scattering data is important as change the magnitude of the higher twist terms needed to describe the experimental data.

We discussed available to target mass corrections approaches and suggested to use the new JLD-approach. We observed that at low $Q^2 \sim 1 \div 2$ GeV^2 the TMCs to structure functions calculated by using the JLD-approach noticeably differ from the standard GP-method or another approaches results. We believe that the JLD-approach including target mass effects will be useful in extracting the magnitude of the structure functions from the experimental data and to precisely extract the higher twist contribution.

Acknowledgments

We are grateful to A.V. Sidorov and D.V. Shirkov for interest in this work and helpful discussions. This work was partly supported by the RFBR grant 14-01-00647 and the BelRFFR-JINR grant F14D-007.

References

[1] O. Nachtmann, Nucl. Phys. B **63**, 237 (1973).

[2] H. Georgi and H.D. Politser, Phys. Rev. D **14**, 1829 (1976).

[3] W. Melnitchouk, R. Ent, and C. Keppel, Phys. Rep. **406**, 127 (2005); F.M. Steffens, W. Melnitchouk, Phys. Rev. C **73**, 055202 (2006); I. Schienbein *et al.*, J. Phys. G. **35**, 053101 (2008).

[4] A. Accardi and J.W. Qiu, JHEP **07**, 90 (2008); A. Accardi *et al.*, Phys. Rev. D **81**, 034016 (2010).

[5] V.I. Lashkevich and O.P. Solovtsova, Theor. Math. Phys. **160**, 1260 (2009); Nonlin. Phenom. Complex Syst. **13**, 286 (2010); Nonlin. Phenom. Complex Syst. **14**, 184 (2011).

[6] A. De Rújula, H. Georgi, and H.D. Politzer, Annals of Phys. **103**, 315 (1977).

[7] S. A. Kulagin and R. Petti, Nucl. Phys. A **765**, 126 (2006).

[8] F.J. Yndurain, *The Theory of Quark and Gluon Interactions*, Springer, 2006, 476 pp.

[9] R.K. Ellis, W. Furmfnski, R.Petronsio, Nucl. Phys. B **212** 29 (1983).

[10] S. Kretzer and M. H. Reno, Phys. Rev. D **66**, 113007 (2002).

[11] A. Accardi and J. W. Qiu, JHEP **07**, 090 (2008).

[12] R. Jost and H. Lehmann, Nuov. Cim. **5**, 1598 (1957).

[13] F.J. Dyson, Phys. Rev. **110**, 1460 (1958).

[14] I.L. Solovtsov, Part. Nucl. Lett. **4**[101], 10 (2000).

[15] D.V. Shirkov, I.L. Solovtsov, Phys. Rev. Lett. **79**, 1209 (1997).

[16] D.V. Shirkov and I.L. Solovtsov, Theor. Math. Phys. **150**, 132 (2007).

[17] M. Glück, E. Reya, and A. Vong, Z. Phys. C **7**, 433 (1995); Eur. Phys. J. C **5**, 461 (1998).

[18] A.D. Martin, R.G. Roberts, W.J. Stirling, and R.S. Thorne, Eur. Phys. J. C **16**, 117 (2000);
A.D. Martin, W.J. Stirling, R.S. Thorne, and G. Watt, Eur. Phys. J. C **63**, 189 (2009).

[19] V.I. Lashkevich and O.P. Solovtsova, Proc. of Annual Seminar NPCS'2012, Vol. **19**, 127 (2012).