Low Energy Hadronic Interactions of Scalar Mesons.

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Abstract

Low energy interactions of scalar mesons are investigated in the Quark Confinement Model. The good description of these decays in two-quark scheme was achieved by means of the addition term in the scalar-quarks Lagrangian.

1 Introduction

The scalar O^{++} mesons are very important in the low energy physics. The linear realization of the chiral symmetry required introduction of σparticles, which turned out to be convenient for construction the phenomenological chiral Lagrangians reproducing the low- energy relations of the current algebra. The phenomenological analysis and model investigations of $\pi\pi, \pi N, NN$ scattering, the pion polarizabilities,the $K \to 2\pi$ and $K \to 2\gamma$ decays indicate the importance of taking into account an intermediate light scalar particles.But the question had been asked by P.G. Estabrooks almost thirty years ago :"What and where is scalar mesons ?"[1] is still actual . The recent KLOE [2], E97 Collaboration [3]-[5],BES [6]-[8] experiments answer the second part of this question, while the first one remains unanswered.
There are several points of view on the structure of scalar mesons. There are many models [9]-[11] where this particles are treated as two quark systems.But mass spectra calculated in this schemes is not agree with experimental one. The estimations of the $f_0 \to \pi\pi$ decay width in the simple two quark models are in the contradiction with experimental data.
In some approaches scalar particles are treated as four-quark systems [12],[13].In this scheme light scalar mesons have the following contents:

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$$f_0(980) \Rightarrow s\bar{s}\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\cos\alpha + u\bar{u}d\bar{d}\sin\alpha;$$
$$f_0(1300) \Rightarrow s\bar{s}\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\sin\beta + u\bar{u}d\bar{d}\cos\beta;$$
$$a_0(980) \Rightarrow \frac{u\bar{u}-d\bar{d}}{\sqrt{2}}s\bar{s}$$

The existence of $\varepsilon(650)$ is predicted in this approach. The development of this approach have led to the models where the ground state of $qq\bar{q}\bar{q}$ system is considered as the $K\bar{K}$ molecule [14].
In some approaches scalar mesons are associated with scalar glueballs, predicted by QCD. The lattice calculations [15] give the opportunity to estimate the scalar mesons masses. In [16]the descriptions of scalar particles in the framework of instanton liquid model of the QCD vacuum is proposed. There are a number of articles where scalar mesons are treated as a mixture of quark and gluonic states.

There is a problem in the evaluation of the hadronic matrix elements. We perform the calculations in the Quark Confinement Model (QCM)[17]. This model based on the certain assumptions about nature of quark confinement and hadronization allows to describe the electromagnetic,strong and weak interactions of light (nonstrange and strange)mesons from a unique point of view. In the QCM we treat scalar mesons as two-quark states.But in the case of O^{++} mesons we strike with the problem of description of triangle diagram corresponding to $S \to PP$ decay by the simplest quark-meson Lagrangian. So one have to modify it by additional term with derivative. This problem seems to testify of a more complicated (then simple two-quark) structure of scalars. A four-quark component may be essential in scalar mesons.

In this article we consider the parameter of additional interaction and a mixing angle of scalar mesons as free parameters.They are fitting by the Adler condition and by the experimental value of the $f_0(975) \to \pi\pi$ width. The received numerical values for hadronic and electromagnetic decays are in a good agreement with experimental data.

2 Quark Confinement Model

The hadronic interactions will be described in the QCM [17].This model is based on the following assumptions:

The hadron fields are assumed to arise after integration over gluon and quark variables in the QCM generating function. The transition of hadrons to quarks and vice versa is given by the interaction Lagrangian. In particular necessary interaction Lagrangians for $\pi^{\pm}$,$K^{\pm}$ and η mesons look like:

$$\mathcal{L}_{\pi^{\pm}} = \frac{g_{\pi^{\pm}}}{\sqrt{2}}\pi^{\pm}\bar{q}(a)i\gamma_5\frac{\lambda^1 \pm i\lambda^2}{\sqrt{2}}q^a \tag{1}$$

$$\mathcal{L}_{K^{\pm}} = \frac{g_{K^{\pm}}}{\sqrt{2}}K^{\pm}\bar{q}(a)i\gamma_5\frac{\lambda^4 \pm i\lambda^5}{\sqrt{2}}q^a \tag{2}$$

$$\mathcal{L}_{\eta} = \frac{g_{\eta^{\pm}}}{\sqrt{2}}\eta\bar{q}(a)i\gamma_5\frac{\lambda^8 sin\theta + \lambda^0 cos\theta}{\sqrt{2}}q^a. \tag{3}$$

Angle θ in (3) , is a mixing angle for η - mesons. The coupling constants g_M for meson-quark interaction are defined from so-called compositeness condition It us convenient to use interaction constant in a form:

$$h_M = \frac{3g_M^2}{4\pi^2} = -\frac{1}{\tilde{\prod}'_M(m_M)} \tag{4}$$

instead of g_M in the further calculations. All hadron-quark interactions are described by quark diagrams induced by S matrix averaged over vacuum backgrounds.

The second QCM assumption is that the quark confinement is provided by nontrivial gluon vacuum background. The averaging of quark diagrams generated by S-matrix over vacuum gluon fields $\hat{B}_{VAC}$ is suggested to provide quark confinement and to make the ultraviolet finite theory. The confinement ansatz in the case of one-loop quark diagrams consists in following replacement:

$$\int d\sigma_{VAC} Tr|M(x_1)S(x_1,x_2|B_{VAC})...M(x_n)S(x_n,x_1|B_{VAC})| \longrightarrow$$
$$\int d\sigma_v Tr|M(x_1)S_v(x_1-x_2)...M(x_n)S_v(x_n-x_1)|, \tag{5}$$

where

$$S_v(x_1-x_2) = \int \frac{d^4p}{i(2\pi)^4}e^{-ip(x_1-x_2)}\frac{1}{v\Lambda_q - \hat{p}} \tag{6}$$

The parameter Λ_q characterizes the confinement rang of quark with flavor number $q = u, d, s$. The measure $d\sigma_v$ is defined as:

$$\int \frac{d\sigma_v}{v - \hat{z}} = G(z) = a(-z^2) + \hat{z}b(-z^2) \tag{7}$$

The function $G(z)$ is called the confinement function. $G(z)$ is independent on flavor or color of quark. $G(z)$ is an entire analytical function on the z-plane.$G(z)$ decreases faster then any degree of z in Eucidean region.The choice of $G(z)$,or as the same of $a(-z^2)$ $b(-z^2)$, is one of model assumptions.In the notes $a(-z^2)$ and $b(-z^2)$ are chosen as:

$$\begin{aligned} a(u) &= a_0 e^{-u^2-a_1 u} \\ b(u) &= b_0 e^{-u^2-b_1 u} \end{aligned} \tag{8}$$

The request of satisfaction of Ward anomaly identity in QCM gives the additional correlation between $a(0)$ and $b(0)$: $b(0) = -a'(0)$, $a(0) = 2$. Using $a(u)$ and $b(u)$ as (8), one can receive: $a_0 = 2$, $a_1 = \frac{b_0}{4}$. So, the free parameters of the model are Λ_q, b_0, b_1. The model parameters were fixed in the by fitting the well-established constants of low-energy physics.(f_π, f_K, $g_{\rho\gamma}$, $g_{\pi\gamma\gamma}$, $g_{\omega\pi\gamma}$, $g_{\rho\pi\pi}$, $g_{K^*\pi\gamma}$)

$$\begin{aligned} \Lambda_u &= 460\ MeV \\ \Lambda_s &= 506\ MeV \\ b_0 = 2 \qquad & b_1 = 0.2 \\ a_0 = 2 \qquad & a_1 = 0.5 \end{aligned} \tag{9}$$

We put $\Lambda_u = \Lambda_d$ in the most of decays.

3 $S \to PP$ form factor.

Let us calculate form factor for $S \to PP$ decay which is described by the triangle diagram (Fig.1) with Γ_S,$i\gamma^5$, $i\gamma^5$ vertexes.

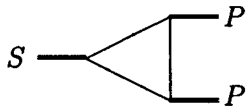

Fig.1

In the case of simple Lagrangian analogous to (1)with $\Gamma_S = I$ the corresponding structure integral calculated for zero masses of final states and

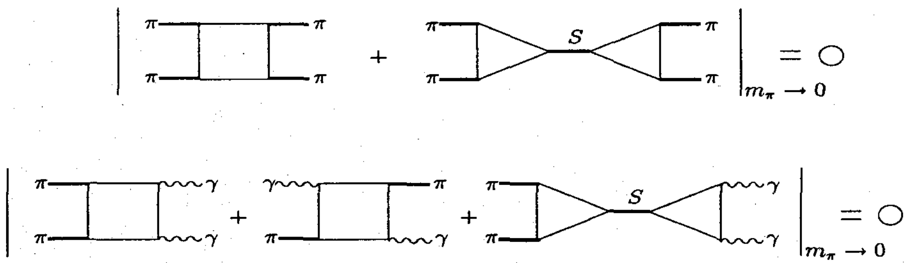

Fig.2

normalized to unity at $m_S = 0$ is:

$$I_0(s) = \int_0^{\infty} dua(u) - s\int_0^1 dua(-u\cdot s)\left[\frac{1}{2}\ln\left(\frac{1+\sqrt{1-u}}{1-\sqrt{1-u}}\right) - \sqrt{1-u}\right] \quad (10)$$

It turns out, that the value of $I_0(s)$ decreases with m_S and becomes equal zero at $m_S \approx 1000 \div 1100 MeV$. This leads to a theoretical value of $f_0(975) \to \pi\pi$ decay width to the underestimated ($\Gamma \sim 1 MeV$) in comparison with the experimental one $\Gamma_{exp} = (26 \pm 5) MeV$. We propose the solution of this problem by introducing additional interaction with first derivative. So instead of simple "minimal" scalar vertex $\Gamma_S = 1$ one have to consider the following one:

$$\Gamma_S = I - i\frac{H}{\Lambda}\hat{\partial} \quad (11)$$

where $\hat{\partial} \equiv \overrightarrow{\hat{\partial}} - \overleftarrow{\hat{\partial}}$, H-the additional free parameter.

4 Additional Scalar Meson Parameters.

As a basic for fitting the parameters H and δ_S-mixing angle for scalars we take,first (Fig.2), the Adler condition wich means that amplitudes of $\pi\pi \to \pi\pi$ and $\gamma\pi \to \gamma\pi$ processes are equal to zero at $m_\pi \to 0$ limit, and the second , the experimental value of $f_0(975) \to \pi\pi$ decay width.

The Adler condition in the QCM can be represented by following relations:

$$5b_0 = 2\Lambda^2 cos\delta_S a_0 \left[\int_0^\infty a(u)du - 4H\int_0^\infty b(u)udu\right] \times$$
$$\times(5cos\delta_S - \sqrt{2}sin\delta_S)h_\varepsilon D_\varepsilon(0) \quad (12)$$

$$\int_0^\infty b(u)du = 2\Lambda^2 cos^2\delta_S \left[\int_0^\infty a(u)du - 4H\int_0^\infty b(u)udu\right]^2 h_\varepsilon D_\varepsilon(0) \quad (13)$$

For fitting the parameters H and δ_S it is convenient to eliminate form factor $h_\varepsilon D_\varepsilon(0)$ connected with "unknown" ε meson and use the following ratio

$$R = -\frac{5b_0 cos\delta_S\left[\int_0^\infty a(u)du - 4H\int_0^\infty b(u)udu\right]}{\int_0^\infty b(u)du a_0(5cos\delta_S - \sqrt{2}sin\delta_S)} = 1 \quad (14)$$

which is independent of m_ε.

The matrix element of $f_0(975) \to \pi\pi$ can be written in the form:

$$G_{f_0\pi\pi} = 4sin\delta_S\Lambda h_\pi\sqrt{\frac{1}{6}h_S(H)}[I_0(s) - 4HI_1(s)] \quad (15)$$

where $s = \frac{m_{f_0}^2}{4\Lambda^2}$, $I_0(s)$ and$I_1(s)$are defined by (10) and

$$I_1(s) = \int_0^\infty duu\cdot b(u) - \frac{s^2}{2}\int_0^1 duu\cdot b(-u\cdot s)\left[\frac{1}{2}\ln\left(\frac{1+\sqrt{1-u}}{1-\sqrt{1-u}}\right) - \sqrt{1-u}\right] \quad (16)$$

The decay width for $f_0(975) \to \pi\pi$ was calculated in standard way and is written as:

$$\Gamma(f_0(975) \to \pi\pi) = \frac{3}{16\pi}\sqrt{1 - \frac{4m_\pi^2}{m_{f_0}}}G_{f_0\pi\pi}\frac{1}{m_{f_0}} \quad (17)$$

We have found the following numerical values for scalar parameters

$$H = 0,55, sin\delta_s = 0,25 \quad (18)$$

5 Hadronic and Electromagnetic Decays of Scalar Mesons.

The calculation of hadronic and electromagnetic decay widths is crucial for any scheme for description 0^{++} mesons.For description of meson - quark interaction the Lagrangian (1)- (3) will be used. The Lagrangian of scalar - quark interaction is written as:

$$\mathcal{L}_S = \frac{g_S}{\sqrt{2}} S \bar{q}^a \Gamma_S \frac{\lambda^1 \lambda^2}{\sqrt{2}} q^a \tag{19}$$

with Γ_S defined by (11). Matrix element for $S \to PP$ decay is described by diagram Fig.1. Analytically it is expressed by (15).We will ignore π-meson mass, so for $f_0(975) \to \pi\pi$ the structure integrals in (15) are defined by (10) and (16).

In the case of $a_0(980) \to \pi\eta$ the structure integrals are calculated with nonzero η-meson mass:

$$I_{SPP}(m_a, m_\eta, o) = I_o\left(\frac{m_a^2}{4\Lambda^2}, \frac{m_\eta^2}{4\Lambda^2}\right) - 4HI_1\left(\frac{m_a^2}{4\Lambda^2}, \frac{m_\eta^2}{4\Lambda^2}\right), \tag{20}$$

where

$$\begin{aligned} I_0(x,y) = & \int_0^\infty du a(u) - x\int_0^1 du a(-ux)\left[\frac{1}{2}\ln\left(\frac{1+\sqrt{1-u}}{1-\sqrt{1-u}}\right)\right. \\ & \left. -\sqrt{1-u}\right] - y\int_0^1 du a(-uy)\left[\frac{1}{2}\ln\left(\frac{1+\sqrt{1-u}}{1-\sqrt{1-u}}\right)\right. \\ & \left. -\sqrt{1-u}\right] \end{aligned} \tag{21}$$

$$\begin{aligned} I_1(x,y) = & \int_0^\infty du u b(u) - \frac{1}{2}x(x-y)\int_0^1 du u b(-ux)\sqrt{1-u} + \\ & +\frac{1}{4}\int_0^1 du u\left[x^2 b(-ux) - y^2 b(-uy)\right]\ln\left(\frac{1+\sqrt{1-u}}{1-\sqrt{1-u}}\right) \end{aligned} \tag{22}$$

The decay widths can be calculated in standard way:

$$\Gamma\left(S \to PP'\right) = \frac{1}{16\pi}\frac{p^* g_{SPP'}(m_S^2)}{m_S^2} \tag{23}$$

$$\Gamma\left(S \to PP'\right) = \frac{1}{8\pi}\frac{p^* g_{SPP'}(m_a^2, m_\eta^2)}{m_a^2} \tag{24}$$

(23) for $S \to \pi\pi$ and (24) for $S \to \pi\eta$ decays.

The electromagnetic decays amplitude is defined by diagram Fig.3. It can be written as:

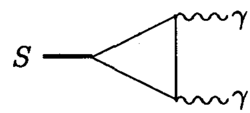

Fig.3

$$M\left(S\rightarrow\gamma\gamma\right)=\left[g^{\mu\nu}q_1q_2-q_1^{\mu}q_2^{\nu}\right]g_{s\gamma\gamma}\left(m_s^2\right) \tag{25}$$

where

$$g_{s\gamma\gamma}\left(m_s^2\right)=\alpha\sqrt{6h_s(H)}\frac{1}{\Lambda}Tr\left\{Q^2\lambda_s\right\}\left[F_{SVV}^1\left(m_s^2\right)+HF_{SVV}^2\left(m_s^2\right)\right] \tag{26}$$

Where $\alpha=\frac{e^2}{4\pi}$, $Q=\frac{1}{3}diag(2,-1,-1)$ and $F_{SVV}^{1,2}\left(m_s^2\right)$ are defined in following way:

$$F_{SVV}^1\left(m_s^2\right)=\frac{x}{4}\int_0^1 dua\left(-u\frac{x}{4}\right)(1+u)\ln\left(\frac{1+\sqrt{1-u}}{1-\sqrt{1-u}}\right) \tag{27}$$

$$F_{SVV}^2\left(m_s^2\right)=\frac{x}{4}\int_0^1 dub\left(-u\frac{x}{4}\right)u\ln\left(\frac{1+\sqrt{1-u}}{1-\sqrt{1-u}}\right) \tag{28}$$

The decay widths for scalar electromagnetic decays $S\rightarrow\gamma\gamma$ can be calculated by:

$$\Gamma\left(S\rightarrow\gamma\gamma\right)=\frac{m_s^2}{64\pi}g_{s\gamma\gamma}^2(m_s^2) \tag{29}$$

The numerical results for strong and electromagnetic decays of scalar mesons are represented in Table.

DECAY MODE	QCM	EXPERIMENTAL DATA
$f_0(975) \to \pi\pi$	$22 MeV$	$26 \pm 5 MeV$[18]
$a_0(980) \to \pi\eta$	$56 MeV$	$54 \pm 7 MeV$[18]
$\varepsilon(660) \to \pi\pi$	$357 MeV$	–
$f_0(975) \to \gamma\gamma$	$0,368 KeV$	$0,39^{+0,08}_{-0,11} KeV$[19]
$a_0(980) \to \gamma\gamma$	$0,43 KeV$	$0,30 \pm 0,10 KeV$[20]
$\varepsilon(660) \to \gamma\gamma$	$0,33 KeV$	–

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