Mixing Angles and Radiative Decays of η, η'- Mesons

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Abstract

The η, η' system was under investigation.Mixing angles for octet-singlet mixing and for quark-flavor bases were fixed. The decay constants of radiative $V \to P\gamma$ and $P \to V\gamma$ ($P \equiv \eta, \eta'$; $V \equiv \rho, \omega, \phi$) decays were calculated. The weak decay constants for η, η' mesons were concidered.We have obtained the numerical relations between constants in different bases.

1 Introduction

The world of light quarks and hadrons is very reach with interesting phenomena. At short distances there are only free quarks and gluons. They are governed by Quantum Chromodynamics.At large distances there are only hadrons. This point-like particles are described by the standard quantum field equations.And intermediate distances colour confinement and hadronization take place.From the physical point of view, this is a low energy region of hadronic physics where physical processes with the liberated energy $1 \div 2$ GeV proceed.The investigation of simple quark-antiquark systems such as pseudoscalar mesons π^0, η, η'... is of extraordinary interest as a source of information about structure of hadrons. It's well known that SU(3)-symmetry predicts the existence of massless pseudoscalar octet $\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - s\bar{s})$ and massive singlet $\eta_0 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$.Physical states η, η' are the mixture of η_8 and η_0. The study of $\eta - \eta'$ mixing is very important for understanding of basic properties of quark - hadron matter. This phenomena was considered in different approaches [1]. Recent CLEO [2] and L3[3]experiments call the additional interest to this phenomena.

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Theoretical analysis is model dependent and mixing angle θ prediction varies from $-12°$[4] to $-20°$ [5].

Apart of mixing angles two parameters f_8 and f_0-η, η' decay constants are usually introduce.Numerical estimations for this parameters are also model dependent and varies strongly. For example,f_8 is estimated from $0.71 f_\pi$ in[6]to $1.28 f_\pi$ [7]. Numerical value forf_0 varies in the limits:from $0.94 f_\pi$ in[6]to $1.25 f_\pi$ [7].

Recently another scheme with two mixing angles was proposed by [8].

Also there exists approach connected with quark basis $q\bar{q} = \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}, s\bar{s}$[4]. In this case decay constant mixing is considers to be the same as for meson states. The analysis of mentioned schemes can be performed by study two-photon decays of π^0, η, η' mesons.

Another problem is the evaluation of the hadronic matrix elements.We perform the calculations in the Quark Confinement Model (QCM) [9]. This model based on the certain assumptions about nature of quark confinement and hadronization allows to describe the electromagnetic,strong and weak interactions of light (nonstrange and strange)mesons from a unique point of view.

2 Quark Confinement Model

The hadronic interactions will be described in the QCM.This model is based on the following assumptions [9]:

The hadron fields are assumed to arise after integration over gluon and quark variables in the QCM generating function. The transition of hadrons to quarks and vice versa is given by the interaction Lagrangian. In particular necessary interaction Lagrangians for $\pi^\pm$,η and η' mesons look like:

$$\mathcal{L}_M = \frac{g_M}{\sqrt{2}} M \bar{q}^a \Gamma \lambda^m q^a \tag{1}$$

where Γ- Dirak matrix,λ^m - is a corresponding SU(3)-matrix,q- quark vector

$$q_j^a = \begin{pmatrix} u^a \\ d^a \\ s^a \end{pmatrix}$$

The coupling constants g_M for meson-quark interaction are defined from so-called compositeness condition [10] It us convenient to use interaction constant in a form:

$$h_M = \frac{3g_M^2}{4\pi^2} = -\frac{1}{\tilde{\Pi}'_M(m_M)} \qquad (2)$$

instead of g_M in the further calculations. All hadron-quark interactions are described by quark diagrams induced by S matrix averaged over vacuum backgrounds.

The second QCM assumption is that the quark confinement is provided by nontrivial gluon vacuum background. The averaging of quark diagrams generated by S-matrix over vacuum gluon fields $\hat{B}_{VAC}$ is suggested to provide quark confinement and to make the ultraviolet finite theory. The confinement ansatz in the case of one-loop quark diagrams consists in following replacement:

$$\int d\sigma_{VAC} Tr|M(x_1)S(x_1,x_2|B_{VAC})...M(x_n)S(x_n,x_1|B_{VAC})| \longrightarrow$$

$$\int d\sigma_v Tr|M(x_1)S_v(x_1-x_2)...M(x_n)S_v(x_n-x_1)|, \qquad (3)$$

where

$$S_v(x_1-x_2) = \int \frac{d^4p}{i(2\pi)^4} e^{-ip(x_1-x_2)} \frac{1}{v\Lambda_q - \hat{p}} \qquad (4)$$

The parameter Λ_q characterizes the confinement rang of quark with flavour number $q = u, d, s$. The measure $d\sigma_v$ is defined as:

$$\int \frac{d\sigma_v}{v - \hat{z}} = G(z) = a(-z^2) + \hat{z}b(-z^2) \qquad (5)$$

The function $G(z)$ is called the confinement function. $G(z)$ is independent on flavor or color of quark. $G(z)$ is an entire analytical function on the z-plane.$G(z)$ decreases faster then any degree of z in Eucidean region.The choice of $G(z)$,or as the same of $a(-z^2)$ $b(-z^2)$, is one of model assumptions.In the notes [9], [11] $a(-z^2)$ and $b(-z^2)$ are chosen as:

$$a(u) = a_0 e^{-u^2 - a_1 u}$$
$$b(u) = b_0 e^{-u^2 - b_1 u} \qquad (6)$$

The request of satisfaction of Ward anomaly identity in QCM gives the additional correlation between $a(0)$ and $b(0)$: $b(0) = -a'(0)$, $a(0) = 2$. Using $a(u)$ and $b(u)$ as (6), one can receive: $a_0 = 2$, $a_1 = \frac{b_0}{4}$. So, the free parameters of the model are Λ_q, b_0, b_1. The model parameters were fixed in the [11] by fitting the well-established constants of low-energy physics.(f_π, f_K, $g_{\rho\gamma}$, $g_{\pi\gamma\gamma}$, $g_{\omega\pi\gamma}$, $g_{\rho\pi\pi}$, $g_{K^*\pi\gamma}$)

$$\begin{aligned} \Lambda_u &= 460\ MeV \\ \Lambda_s &= 506\ MeV \\ b_0 = 2 \qquad & b_1 = 0.2 \\ a_0 = 2 \qquad & a_1 = 0.5 \end{aligned} \tag{7}$$

We put $\Lambda_u = \Lambda_d$ in the most of decays.

3 The description of η, η' mixing

In order to quantify the mixing in the η, η' system, one have to define appropriate mixing parameters, which can be related to physical observables.

The octet-singlet mixing (OSM).This approach is based on chiral perturbation theory which traditionally leads to description of η, η' mixing in terms of singlet-octet parameters [1]. In this case the physical states η and η' are the mixture of massive singlet $\eta_0 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$ and massless octet $\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - s\bar{s})$:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta - \sin\theta \\ \sin\theta \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix} \tag{8}$$

Last time there mass dependence of mixing angle θ was proposed [12]:

$$\begin{aligned} \eta &= \eta_8 \cos\theta_\eta - \eta_0 \sin\theta_\eta \\ \eta' &= \eta_8 \sin\theta_{\eta'} + \eta_0 \cos\theta_{\eta'} \end{aligned} \tag{9}$$

The theoretical prediction for numerical values of mixing angles $\theta_{\eta,\eta'}$ is model dependent and varies from $-12°$ in [12] to $-53°$ in [13].

Mixing in the quark basis(QBM). The parametrization of the decay constants look much simpler in another basis, where the two independent axial-vector currents are taken as [4]

$$\begin{aligned} J^q_{\mu 5} &= \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d) \\ J^s_{\mu 5} &= \bar{s}\gamma_\mu\gamma_5 s \end{aligned} \tag{10}$$

In this scheme η, η' mixing is defined as

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\varphi - \sin\varphi \\ \sin\varphi \cos\varphi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} \tag{11}$$

where $\eta_q = \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}, \eta_s = s\bar{s}$.

The numerical value for mixing angle φ varies as $\varphi = 30° \div 45°$ in different approaches [1].

4 Two-photon decays and η, η' mixing parameters.

The constant of two-photon decay $P \to \gamma\gamma$ is a very important source of information about two-photon η, η' mixing. We can use the experimental data about η, η' decays to fix numerical values of mixing angles $\theta_{\eta,\eta'}$ and φ.

The experimental values of widths of this decays are [14]

$$\begin{aligned} W(\eta \to \gamma\gamma) &= (0.46 \pm 0.04) KeV \\ W(\eta' \to \gamma\gamma) &= (4.27 \pm 0.19) KeV \end{aligned} \tag{12}$$

and the corresponding decay constants are

$$g^{exp}_{\eta\gamma\gamma} = 0.259 GeV^{-1}, g^{exp}_{\eta'\gamma\gamma} = 0.341 GeV^{-1} \tag{13}$$

Analytical expressions for radiative η, η' decay constants was obtained in QCM both in OSM and QBM, mentioned above.

In the QBM approach the decay constants were received as

$$\begin{aligned} g_{\eta\gamma\gamma}(\varphi) = \frac{\sqrt{3h_\eta(\varphi)}}{\pi}(\frac{1}{\sqrt{2}} \cdot \frac{5}{9} \cdot F_{PVV}(m^2_\eta, 0, 0, \Lambda_n) \cdot \cos\varphi - \\ - \frac{1}{9} \cdot F_{PVV}(m^2_\eta, 0, 0, \Lambda_s) \cdot \sin\varphi) \end{aligned} \tag{14}$$

$$\begin{aligned} g_{\eta'\gamma\gamma}(\varphi) = \frac{\sqrt{3h_{\eta'}(\varphi)}}{\pi}(\frac{1}{\sqrt{2}} \cdot \frac{5}{9} \cdot F_{PVV}(m^2_{\eta'}, 0, 0, \Lambda_n) \cdot \sin\varphi + \\ + \frac{1}{9} \cdot F_{PVV}(m^2_{\eta'}, 0, 0, \Lambda_s) \cdot \cos\varphi) \end{aligned} \tag{15}$$

The structure integral $F_{PVV}(m^2, 0, 0, \Lambda)$ in (14),(15) is written as

$$F_{PVV}(p^2, q_1^2, q_2^2, \Lambda) = \frac{1}{\Lambda} \int_0^1 \{d^3\alpha\} a(-Q) \tag{16}$$

where $a(-Q)$ is confinement function defined by (6)with

$$Q = \frac{p^2\alpha_1\alpha_3 + q_1^2\alpha_2\alpha_3 + q_2^2\alpha_1\alpha_2}{\Lambda^2}.$$

The best agreement with experimental data (12)can be achieved with the

$$\varphi = 39.3^\circ \tag{17}$$

In the OSM scheme the decay constants were received as

$$g_{\eta\gamma\gamma}(\theta_\eta) = \frac{\sqrt{3}}{\pi}(g_{\eta 8\gamma\gamma}(m_\eta^2)\cos\theta_\eta - g_{\eta 0\gamma\gamma}(m_\eta^2)\sin\theta_\eta) \tag{18}$$

$$g_{\eta'\gamma\gamma}(\theta_{\eta'}) = \frac{\sqrt{3}}{\pi}(g_{\eta 8\gamma\gamma}(m_{\eta'}^2)\sin\theta_{\eta'} + g_{\eta 0\gamma\gamma}(m_{\eta'}^2)\cos\theta_{\eta'}) \tag{19}$$

where

$$g_{\eta 8\gamma\gamma}(x) = \sqrt{h_{\eta 8}(x)}\frac{1}{\sqrt{6}} \cdot (\frac{5}{9} \cdot F_{PVV}(x, 0, 0, \Lambda_n) - \frac{2}{9} \cdot F_{PVV}(x, 0, 0, \Lambda_s)) \tag{20}$$

$$g_{\eta 0\gamma\gamma}(x) = \sqrt{h_{\eta 0}(x)}\frac{1}{\sqrt{3}} \cdot (\frac{5}{9} \cdot F_{PVV}(x, 0, 0, \Lambda_n) + \frac{2}{9} \cdot F_{PVV}(x, 0, 0, \Lambda_s)) \tag{21}$$

One can find the numerical values for mixing angles $\theta_\eta, \theta_{\eta'}$ by solving the equations (18) and (19).

The following resolutions have been received

$$\theta_\eta = -15.4^\circ \tag{22}$$

$$\theta_{\eta'} = 58^\circ \quad or \quad \theta_{\eta'} = -17.9^\circ \tag{23}$$

5 Radiative decays of η, η' mesons.

Let us calculate the constants of $V \to P\gamma$ and $P \to V\gamma$ ($P \equiv \eta, \eta'$; $V \equiv \rho, \omega, \phi$)decays using the mixing parameters received in previous section.

The decay amplitudes are defined as

$$A(V \to P\gamma) = eg_{VP\gamma}\varepsilon^{\mu\nu\alpha\beta}\epsilon^{\mu}(p_V)\epsilon^{\nu}(q_\gamma)q_\gamma^\alpha p_V^\beta \tag{24}$$

$$A(P \to V\gamma) = eg_{PV\gamma}\varepsilon^{\mu\nu\alpha\beta}\epsilon^{\mu}(q_\gamma)\epsilon^{\nu}(p_V)q_\gamma^\alpha p_V^\beta \tag{25}$$

and the corresponding decay widths are

$$W(P \to V\gamma) = m_V^3 \alpha g_{PV\gamma}^2 \tag{26}$$

$$W(V \to P\gamma) = \frac{1}{3} m_p^3 \alpha g_{VP\gamma}^2 \tag{27}$$

The following expressions for decay constants were received in QCM:
In the QBM approach

$$g_{\eta\rho\gamma} = \sqrt{h_\rho h_\eta(\varphi)} \cos\varphi F_{PVV}(m_\eta^2, m_\rho^2, 0, \Lambda_n) \tag{28}$$

$$g_{\eta'\rho\gamma} = \sqrt{h_\rho h_{\eta'}(\varphi)} \sin\varphi F_{PVV}(m_{\eta'}^2, m_\rho^2, 0, \Lambda_n) \tag{29}$$

$$g_{\eta\omega\gamma} = \sqrt{h_\omega h_\eta(\varphi)} \cos\varphi \frac{1}{3} F_{PVV}(m_\eta^2, m_\omega^2, 0, \Lambda_n) \tag{30}$$

$$g_{\eta'\omega\gamma} = \sqrt{h_\omega h_{\eta'}(\varphi)} \sin\varphi \frac{1}{3} F_{PVV}(m_{\eta'}^2, m_\omega^2, 0, \Lambda_n) \tag{31}$$

$$g_{\eta\phi\gamma} = \sqrt{h_\phi h_\eta(\varphi)} \sin\varphi \frac{2}{3} F_{PVV}(m_\eta^2, m_\phi^2, 0, \Lambda_s) \tag{32}$$

$$g_{\eta'\phi\gamma} = \sqrt{h_\phi h_{\eta'}(\varphi)} \cos\varphi \frac{2}{3} F_{PVV}(m_{\eta'}^2, m_\phi^2, 0, \Lambda_s) \tag{33}$$

In the OSM scheme the singlet η_0 and octet η_8 decay constants have been received as

$$g_{\eta,\eta' 8\rho\gamma} = \sqrt{h_\rho h_{\eta 8}(m_{\eta,\eta'}^2)} \frac{1}{\sqrt{3}} F_{PVV}(m_{\eta,\eta'}^2, m_\rho^2, 0, \Lambda_n) \tag{34}$$

$$g_{\eta,\eta' 0\rho\gamma} = \sqrt{h_\rho h_{\eta 0}(m_{\eta,\eta'}^2)} \frac{\sqrt{2}}{\sqrt{3}} F_{PVV}(m_{\eta,\eta'}^2, m_\rho^2, 0, \Lambda_n) \tag{35}$$

$$g_{\eta,\eta' 8\omega\gamma} = \sqrt{h_\omega h_{\eta 8}(m_{\eta,\eta'}^2)} \frac{2}{3\sqrt{6}} F_{PVV}(m_{\eta,\eta'}^2, m_\omega^2, 0, \Lambda_n) \tag{36}$$

$$g_{\eta,\eta' 0\omega\gamma} = \sqrt{h_\omega h_{\eta 0}(m_{\eta,\eta'}^2)} \frac{2}{3\sqrt{3}} F_{PVV}(m_{\eta,\eta'}^2, m_\omega^2, 0, \Lambda_n) \tag{37}$$

$$g_{\eta,\eta' 8\phi\gamma} = \sqrt{h_\phi h_{\eta 8}(m^2_{\eta,\eta'})}\frac{4}{3\sqrt{6}}F_{PVV}(m^2_{\eta,\eta'}, m^2_\phi, 0, \Lambda_s) \quad (38)$$

$$g_{\eta,\eta' 0\phi\gamma} = \sqrt{h_\phi h_{\eta 0}(m^2_{\eta,\eta'})}\frac{2}{3\sqrt{3}}F_{PVV}(m^2_{\eta,\eta'}, m^2_\phi, 0, \Lambda_s) \quad (39)$$

Corresponding decay constants of physical η, η' states to vector particles ($V \equiv \rho, \omega, \phi$) and γ quanta are defined as

$$g_{\eta V\gamma} = g_{\eta 8 V\gamma}\cos\theta_\eta - g_{\eta 0 V\gamma}\sin\theta_\eta \quad (40)$$

$$g_{\eta' V\gamma} = g_{\eta' 8 V\gamma}\sin\theta_{\eta'} + g_{\eta' 0 V\gamma}\cos\theta_{\eta'} \quad (41)$$

The numerical values for $V \to P\gamma$ and $P \to V\gamma$ decays are given in tables 1 and 2.

Table 1

$g_{V\eta\gamma}(GeV^{-1})$	Experiment [14]	QBM $\varphi = 39.3°$	OSM $\theta_\eta = -15.4°$
$g_{\rho\eta\gamma}$	$1.47 + 0.25 - 0.28$	1.49	1.495
$g_{\omega\eta\gamma}$	0.5 ± 0.04	0.496	0.497
$g_{\phi\eta\gamma}$	0.69 ± 0.02	0.715	0.72

Table 2

$g_{V\eta'\gamma}(GeV^{-1})$	Experiment [14]	QBM $\varphi = 39.3°$	OSM $\theta_{\eta'} = 57.97°$	OSM $\theta_{\eta'} = -17.88°$
$g_{\rho\eta'\gamma}$	1.31 ± 0.06	1.065	1.55	1.05
$g_{\omega\eta'\gamma}$	0.45 ± 0.03	0.353	0.514	0.33
$g_{\phi\eta'\gamma}$	1.00 ± 0.28	0.761	0.381	0.786

One can see that numerical value $\theta_{\eta'} = -17.88°$ provides better agreement with experimental data.

6 Weak decay constants in $\eta - \eta'$ system.

The decay constants in the $\eta - \eta'$ system are defined as matrix elements of axial-vector currents [1]:

$$\langle 0|J_{\mu 5}|M(p)\rangle = i f_M p_\mu \quad (42)$$

In the QBM scheme axial-vector currents are defined by(10).

One receives in QCM

$$f_\eta = f_\eta^q \cos\varphi - f_\eta^s \sin\varphi \quad (43)$$

$$f_{\eta'} = f_{\eta'}^q \sin\varphi + f_{\eta'}^s \cos\varphi \quad (44)$$

where

$$f_P^{q,s} = \frac{\sqrt{3h_P(\varphi)}}{2\pi}\Lambda_{n,s}F_{PW}(m_P^2, \Lambda_{n,s}), \quad P = \eta, \eta' \quad (45)$$

and structure integral is written as

$$F_{PW}(x, \Lambda) = \frac{1}{2}(\int_0^\infty a(u)du + \frac{x}{4\Lambda^2}\int_0^1 dua(-u\frac{x}{4\Lambda^2}\sqrt{1-u}) \quad (46)$$

The following numerical relations have been received

$$\frac{f_\eta^q}{f_\pi} = 1.263, \ \frac{f_\eta^s}{f_\pi} = 1.304, \ \frac{f_{\eta'}^q}{f_\pi} = 1.97, \ \frac{f_{\eta'}^s}{f_\pi} = 1.95. \quad (47)$$

In the OSM scheme one can write

$$\langle 0|J_{\mu 5}|\eta\rangle = \langle 0|J_{\mu 5}|\eta_8\rangle \cos\theta_\eta - \langle 0|J_{\mu 5}|\eta_0\rangle \sin\theta_\eta \quad (48)$$

$$\langle 0|J_{\mu 5}|\eta'\rangle = \langle 0|J_{\mu 5}|\eta_8\rangle \sin\theta_{\eta'} + \langle 0|J_{\mu 5}|\eta_0\rangle \cos\theta_{\eta'} \quad (49)$$

Corresponding constants have been received as

$$f_\eta^a = f_8^a \cos\theta_\eta - f_0^a \sin\theta_\eta \quad (50)$$

$$f_{\eta'}^a = f_8^a \sin\theta_{\eta'} + f_0^a \cos\theta_{\eta'} \quad (51)$$

where

$$f_8^8(m_P) = \frac{\sqrt{3h_8(m_P^2)}}{6\pi\sqrt{2}}(\Lambda_n F_{PW}(m_P^2, \Lambda_n) + 2\Lambda_s F_{PW}(m_P^2, \Lambda_s)) \quad (52)$$

$$f_0^0(m_P) = \frac{\sqrt{3h_0(m_P^2)}}{6\pi}(2\Lambda_n F_{PW}(m_P^2, \Lambda_n) + \Lambda_s F_{PW}(m_P^2, \Lambda_s)) \quad (53)$$

$$f_8^0(m_P) = \frac{\sqrt{3h_8(m_P^2)}}{6\pi}(\Lambda_n F_{PW}(m_P^2, \Lambda_n) - \Lambda_s F_{PW}(m_P^2, \Lambda_s)) \quad (54)$$

$$f_0^8(m_P) = \frac{\sqrt{3h_0(m_P^2)}}{3\pi\sqrt{2}}(\Lambda_n F_{PW}(m_P^2, \Lambda_n) - \Lambda_s F_{PW}(m_P^2, \Lambda_s)) \quad (55)$$

The relations $\frac{f_0^8(m_P^2)}{f_8^8(m_P^2)}$ and $\frac{f_8^0(m_P^2)}{f_0^0(m_P^2)}$ $(P=\eta,\eta')$ turn out to be very small, so one can neglect corresponding parts in(50),(51).

Therefore we obtain η,η' decay constants as

$$f_\eta^8 = f_8^8\cos\theta_\eta,\ f_\eta^0 = -f_0^0\sin\theta_\eta \tag{56}$$

$$f_{\eta'}^8 = f_8^a\sin\theta_{\eta'},\ f_{\eta'}^0 = f_0^0\cos\theta_{\eta'} \tag{57}$$

There also exists another mixing scheme with mixture angles θ_8 and θ_0 [15]

$$\begin{pmatrix} f_\eta^8 f_\eta^0 \\ f_{\eta'}^8 f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} f_8\cos\theta_8 - f_0\sin\theta_0 \\ f_8\sin\theta_8 \quad f_0\cos\theta_0 \end{pmatrix} \tag{58}$$

The relation of θ_8 and θ_0 with mixing angle φ is [4]

$$\theta_8 = \varphi - \arctan\frac{\sqrt{2}f_s}{f_q},\qquad \theta_0 = \varphi - \arctan\frac{\sqrt{2}f_q}{f_s} \tag{59}$$

So in the case of $f_{q,s} = f_\eta^{q,s}$ from (59) we obtain

$$\theta_8 = -16.3°,\qquad \theta_0 = -14.5° \tag{60}$$

If we use $f_{q,s} = f_{\eta'}^{q,s}$

$$\theta_8 = -12.29°,\qquad \theta_0 = -18.45° \tag{61}$$

The relation of θ_8 and θ_0 with mixing angles $\theta_{\eta,\eta'}$ is [1]

$$\tan\theta_8 = \frac{\sin\theta_{\eta'}}{\cos\theta_\eta},\qquad \tan\theta_0 = \frac{\sin\theta_\eta}{\cos\theta_{\eta'}} \tag{62}$$

So for $\theta_\eta = -15.4°$ from (22) and the first value $\theta_{\eta'} = 58°$ from (23) θ_8 and θ_0 are

$$\theta_8 = 41.3°,\qquad \theta_0 = -26.6° \tag{63}$$

In the case of $\theta_\eta = -15.4°$ from (22) and the second value $\theta_{\eta'} = -17.88°$ from (23) we have

$$\theta_8 = -17.8°,\qquad \theta_0 = -15.59° \tag{64}$$

Comparison of numerical results (63),(64)with numbers from (61) shows the value $\theta_{\eta'} = -17.88°$ to be chosen for description the η' state. This statement is in a good agreement with results of previous section.

7 Conclusion

The η, η' system was studied in different mixing schemes.The numerical values for mixing angles were obtained by study two-photon decays of π^0, η, η' mesons. In the case of octet-singlet mixing

$$\theta_\eta = -15.4^\circ$$

$$\theta_{\eta'} = 58^\circ \quad or \quad \theta_{\eta'} = -17.9^\circ$$

And

$$\varphi = 39.3^\circ$$

for quark-basis scheme.

The constants of radiative $V \to P\gamma$ and $P \to V\gamma$ ($P \equiv \eta, \eta'$; $V \equiv \rho, \omega, \phi$) decays were calculated. The comparison of obtained results with experimental data allow us to choose one from two obtained values for $\theta_{\eta'} = -17.9^\circ$. The η, η' decay constants also were calculated. The numerical results for relations between constants and different sets of mixing parameters confirms our choice of $\theta_{\eta'}$.

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