Phenomenology of Gauge-Higgs Unification Models

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Abstract

The class of models in which the Higgs field is a part of gauge fields in higher dimensions, the Higgs sector has been controlled by the gauge principle is discussed. The difference between the Higgs particle and gauge bosons originates from the structure of the extra-dimensional space. This scenario is called as the Gauge-Higgs Unification.

1 Introduction

After the discovery of a Higgs-like boson at LHC [1]- [2] many fundamental questions remains unresolved. Indeed, The Standard Model seems economical, but it hides a lot of secrets. We believe that physics ought to be based on simple principles. But is there a principle governing the Higgs field? What is the origin of the Higgs particle? After all, what is the mechanism of the electroweak gauge symmetry breaking? As a result the SM is afflicted with many arbitrary parameters.

There have been many proposals (technicolor, supersymmetry and so on). There are the class of models in which the Higgs field is a part of gauge fields in higher dimensions, the Higgs sector has been controlled by the gauge principle. The difference between the Higgs particle and gauge bosons originates from the structure of the extra-dimensional space. This scenario is called as the Gauge-Higgs Unification.

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2 Old gauge-Higgs unification

The idea of the gauge-Higgs unification is very old. In the Kaluza-Klein theory the gravity in five dimensional spacetime of topology $M^4 \times S^1$ unifies the four-dimensional gravity with the electromagnetism. The part of the metric, a namely, $g_{\mu 5}$ ($\mu = 0, 1, 2, 3$) , contains the 4D vector potential A_μ in the electromagnetism. In the Gauge-Higgs Unification one instead of gravity in higher dimensional spacetime considers gauge theory. Extra-dimensional components, A_{y_j}, of gauge potentials transform as 4D scalars under 4D Lorentz transformations. The 4D Higgs field is identified with a low energy mode of A_{y_j}. The Higgs field becomes a part of gauge fields.

This scenario was proposed by Fairlie and by Forgacs and Manton in 1979 [3]- [6]. They tried to achieve unification by restricting configurations of gauge fields in extra dimensions with symmetry ansatz. Manton considered gauge theory with gauge group $\mathcal{G}$ defined on $M^4 \times S^2$. It is assumed that only spherically symmetric configurations are allowed and gauge fields have non-vanishing flux (field strengths) on S^2. Further it is demanded that the gauge group $\mathcal{G}$ breaks down to $SU(2)_L \times U(1)_Y$ by non-vanishing flux. There appears a Higgs doublet as a low energy mode of A_{y_j}. Quite amazingly the Higgs doublet turns out to have a negative mass squared so that the symmetry further breaks down to $U(1)_{EM}$.

There are two parameters: the radius R of S^2 and the gauge coupling g_6 in the six-dimensional spacetime. These two parameters are fixed by the Fermi constant and the four-dimensional $SU(2)_L$ gauge coupling g. The masses m_W, m_Z, and m_H are determined as functions of g_6 and R. The Weinberg angle θ_W is determined by the gauge group only. There are three gauge groups which satisfy the above requirements. The result is summarized in Table 1.

$\mathcal{G}$	$\sin^2\theta_W$	m_W	m_Z	m_H
$SU(3)$	3/4	44 GeV	88 GeV	88 GeV
$O(5)$	1/2	54 GeV	76 GeV	76 GeV
G_2	1/4	76 GeV	88 GeV	88 GeV

Table 1: Spectrum in the Gauge-Higgs Unification model by Manton.

The unification is achieved and the Higgs mass is predicted, but though numerical values are not realistic. There are generic problems in this scheme. First, the mass m_Z is $\sim 1/R$. In other words, it necessarily predicts a too small Kaluza-Klein scale $1/R$. Secondly, and more importantly, there is no justification for the ansatz of non-vanishing flux. The restriction to spherically symmetric configurations is not justified either.

3 New gauge-Higgs unification

New attempts of construction of realistic models are based on realisation of several key ideas, such as orbifolds, warped spacetime, Hosotani mechanism of dynamical breaking gauge symmetry and some other.

Further we will shortly characterize them.

3.1 Larger gauge group $\mathcal{G}$

In the EW symmetry breaking $SU(2)_L \times U(1)_Y \to U(1)_{\text{EM}}$ the Higgs field is an $SU(2)_L$ doublet in the fundamental representation. In the Gauge-Higgs Unification the Higgs field is a part of gauge fields which are in the adjoint representation of the gauge group $\mathcal{G}$. This implies that one needs to start with a larger gauge group $\mathcal{G}$ which contains $SU(2)_L \times U(1)_Y$ as a subgroup. Examples are $SU(3)$, $SU(3) \times U(1) \times U(1)$, and $SO(5) \times U(1)$. The most promising models [7]- [9] include namely last group as the gauge group: $SO(5) \times U(1)_X$ breaks down to $SO(4) \times U(1)_X$ by the orbifold boundary conditions, to $SU(2)'_L \times U(1)'_Y$ by brane dynamics, and to $U(1)_{\text{EM}}$ by the Hosotani mechanism.

3.2 Orbifolds

An extra-dimensional space has orbifold structure [10]. Shortly, orbifold is the manifold with the some identified points. The simplest example is S^1/Z_2 in which the points y, $y + 2\pi R$, and $-y$ are identified. Physics must be the same at those points, but gauge potentials need not. Gauge potentials obey, around two fixed points $y_0 = 0$ and $y_1 = \pi R$,

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix} (x, y_j - y) = P_j \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix} (x, y_j + y) P_j^\dagger \,, \tag{1}$$

where $P_j = P_j^{-1} \in \mathcal{G}$. It follows that $A_M(x, y + 2\pi R) = U A_M(x, y) U^\dagger$ where $U = P_1 P_0$. The Lagrangian density remains invariant under the parity transformations. The set $\{P_0, P_1\}$ defines the orbifold boundary conditions (BC).

3.3 Four-dimensional (4D) Higgs

4D Higgs fields reside in the A_y components which are even under P_0 and P_1. Take $\mathcal{G} = \mathcal{SO}(\triangledown)$ and $P_0 = P_1 = \text{diag}\,(-1,-1,-1,-1,1)$. With this orbifold BC $SO(5)$ breaks down to $SO(4)$. A_μ's have zero modes (4D gauge fields) in the diagonal $SO(4) \simeq SU(2)_L \times SU(2)_R$. A_y, on the other hand, has zero modes in the off-diagonal parts:

$$SO(5):\ A_y = \begin{pmatrix} & & & & \phi_1 \\ & & & & \phi_2 \\ & & & & \phi_3 \\ & & & & \phi_4 \\ -\phi_1 & -\phi_2 & -\phi_3 & -\phi_4 & \end{pmatrix} \quad , \quad \Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_4 - i\phi_3 \end{pmatrix} \ . \tag{2}$$

The zero mode multiplet is an $SO(4)$ vector, or a $(\mathbf{2},\mathbf{2})$ representation of $SU(2)_L \times SU(2)_R$. It can be identified with the EW Higgs field Φ.

3.4 Chiral fermions

In Nature, as known, fermion content is chiral. This is highly nontrivial in higher dimensional gauge theory, as a spinor in higher dimensions always contains both right- and left-handed components in four dimensions, in other words, no exist 2D Weyl spinors, exist only 4D Dirac bispinor. The left-right asymmetry in fermion modes at low energies can be induced from nontrivial topology of extra-dimensional space and non-vanishing flux of gauge fields in extra dimensions. However, there is another, simpler and more powerful, way to have chiral fermions. If the extra-dimensional space is an orbifold, appropriate boundary conditions naturally give rise to chiral fermion content.

Take a vector (4D) fermion multiplet Ψ in the $SO(5)$ model. The orbifold BC for Ψ is given by

$$\Psi(x, y_j - y) = \pm P_j \gamma^5 \Psi(x, y_j + y) \ . \tag{3}$$

The 4D matrix $\gamma^5 = diag(1, 1, , -1, -1)$ is necessary to assure the invariance of $\overline{\Psi} i(\gamma^\mu D_\mu + \gamma^5 D_5)\Psi$. With $+$ sign in (3), the first four components of Ψ have zero modes only for $(\gamma^5)_{ij} = -1$ (left-handed components), whereas the fifth component has a zero mode only for $(\gamma^5)_{ij} = 1$ (a right-handed component). All the massive Kaluza-Klein excited states appear vector-like, but the lowest, light modes appear chiral.

3.5 Wilson line phase θ_H

When the space is not simply connected, a configuration of vanishing field strengths does not necessarily mean trivial. The phenomenon is called the Aharonov-Bohm (AB) effect in quantum mechanics. Consider $SU(N)$ gauge theory on $M^4 \times S^1$ with coordinates (x^μ, y), and impose periodic boundary conditions $A_M(x, y + 2\pi R) = A_M(x, y)$. A configuration $A_y(x, y) =$constant gives $F_{MN} = 0$, but gives

$$W \equiv P\exp\left\{ig\int_0^{2\pi R} dy\, A_y\right\} = U \begin{pmatrix} e^{i\theta_1} & & \\ & \ddots & \\ & & e^{i\theta_N} \end{pmatrix} U^\dagger \tag{4}$$

where $U^\dagger = U^{-1}$ and $\sum_{j=1}^N \theta_J = 0$ $(mod\ 2\pi)$. θ_j's are Yang-Mills AB phases or Wilson line phases in the theory. This set we will be denoted as θ_H. They cannot be eliminated by gauge transformations preserving the boundary conditions.

Classical vacua are degenerate. Wilson line phases θ_H label flat directions of the classical potential. The degeneracy is lifted at the quantum level. The mass spectrum $\{m_n\}$ of various fields depends on θ_H. The effective potential $V_{\rm eff}(\theta_H)$ is given at the one loop level by

$$V_{\rm eff}(\theta_H) = \sum \mp\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\sum_n \ln\left\{-p^2 + m_n^2(\theta_H)\right\} \ . \tag{5}$$

The value of θ_H is determined by the location of the global minimum of $V_{\rm eff}(\theta_H)$.

3.6 Dynamical gauge symmetry breaking

Once the matter content is specified, the effective potential is determined and so is the value of θ_H in the true vacuum. In general the global minimum is located at $\theta_H \neq 0$. Suppose that all fields are periodic so that the boundary conditions are $SU(N)$ symmetric. If $\theta_H \neq 0$, the symmetry breaks down to a subgroup of $SU(N)$ in general. In other words we have dynamical gauge symmetry breaking.

Instead of periodic boundary conditions, one might impose more general twisted boundary conditions. For instance, one can impose $A_M(x, y+2\pi R) = \Omega A_M(x,y)\Omega^\dagger$ $(\Omega \in SU(N))$. It can be shown that on $M^4 \times S^1$ physics does not depend on the choice of Ω, thanks to dynamics of phases θ_H.

3.7 Flat v.s. warped

The gauge-Higgs unification scenario in flat spacetime is afflicted with a few intrinsic difficulties. The electroweak symmetry is spontaneously broken by θ_H. Non-vanishing θ_H gives rise to non-vanishing masses for W and Z bosons. m_W, for instance, is typically given by

$$m_W \sim \frac{\theta_H}{2\pi} \times \frac{1}{R} \sim \frac{\theta_H}{2\pi} \times m_{KK} \quad . \tag{6}$$

Here R is the size of the extra-dimensions. Secondly, the effective potential $V_{\rm eff}(\theta_H)$ is generated at the one-loop level, and therefore is $O(\alpha_W)$ where $\alpha_W = g_W^2/4\pi$ is the $SU(2)_L$ coupling. The Higgs mass m_H^2 becomes $O(\alpha_W)$ as well. Evaluation of $V_{\rm eff}$ shows that

$$m_H \sim \sqrt{\alpha_W} \times \frac{1}{R} \sim \sqrt{\alpha_W}\, \frac{2\pi}{\theta_H}\, m_W \quad . \tag{7}$$

The relations (6) and (7) are generic predictions from the gauge-Higgs unification in flat spacetime. Once the value of θ_H is given, m_{KK} and m_H are predicted. The value of θ_H is determined from the location of the global minimum of $V_{\rm eff}(\theta_H)$. It depends on the matter content in the theory. Given standard matter content of quarks and leptons with a minimal set of additional matter, the global minimum of $V_{\rm eff}(\theta_H)$ is typically located either at $\theta_H = 0$ or at $\theta_H = (.2 \sim .8)\pi$, as confirmed in various models. In the former case the electroweak symmetry remains unbroken. What we want is the latter. In this case $m_{KK} \sim 10 m_W$ and $m_H \sim 10\,\text{GeV}$. One has too low m_{KK} and too small m_H.

There are two approaches to solve these problems. One way is to stay in flat space and tune the matter content such that $V_{\rm eff}(\theta_H)$ is minimized at a small value for θ_H. For instance, one can introduce many matter multiplets, or even supersymmetry, to have cancellation among dominant parts of the contributions to V_{eff}. Or, one can incorporate quarks in several representations of the gauge group to have small θ_H.

An alternative way is to consider models in the curved space, particularly in the Randall-Sundrum (RS) warped space [11]- [14]. It is remarkable that all the problems mentioned above are naturally solved in the RS space.

4 SO(5) × U(1) unification in warped spacetime

Recently, the first realistic $SO(5) \times U(1)$ Gauge-Higgs Unification model in the Rundull-Sundrum (RS) warped space with the Higgs boson mass $m_H = 126$ GeV is constructed [15].

Metric is given by

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \tag{8}$$

where $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$, $\sigma(y) = \sigma(y + 2L) = \sigma(-y)$, and $\sigma(y) = k|y|$ for $|y| \leq L$. The RS space is viewed as bulk AdS space $(0 < y < L)$ with AdS curvature $-6k^2$ sandwiched by the Planck brane at $y = 0$ and the TeV brane at $y = L$.

The 5D Lagrangian density consists of

$$\begin{aligned} \mathcal{L} = {} & \mathcal{L}^{\text{gauge}}_{\text{bulk}}(A, B) + \mathcal{L}^{\text{fermion}}_{\text{bulk}}(\Psi_a, \Psi_F, A, B) \\ & + \mathcal{L}^{\text{fermion}}_{\text{brane}}(\hat{\chi}_\alpha, A, B) + \mathcal{L}^{\text{scalar}}_{\text{brane}}(\hat{\Phi}, A, B) + \mathcal{L}^{\text{int}}_{\text{brane}}(\Psi_a, \hat{\chi}_\alpha, \hat{\Phi}) \ . \end{aligned} \tag{9}$$

$SO(5)$ and $U(1)_X$ gauge fields are denoted by A_M and B_M, respectively. The two associated gauge coupling constants are g_A and g_B. Two quark multiplets and two lepton multiplets Ψ_a are introduced in the vector representation of $SO(5)$ in each generation, whereas n_F extra fermion multiplets Ψ_F are introduced in the spinor representation. These bulk fields obey the orbifold boundary conditions at $y_0 = 0$ and $y_1 = L$ given by

$$\begin{aligned} &\begin{pmatrix} A_\mu \\ A_y \end{pmatrix} (x, y_j - y) = P_j \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix} (x, y_j + y) P_j^{-1}, \\ &\begin{pmatrix} B_\mu \\ B_y \end{pmatrix} (x, y_j - y) = \begin{pmatrix} B_\mu \\ -B_y \end{pmatrix} (x, y_j + y) \ , \\ &\Psi_a(x, y_j - y) = P_j \Gamma^5 \Psi_a(x, y_j + y) \ , \end{aligned}$$

$$\Psi_F(x, y_j - y) = (-1)^j P_j^{\rm sp} \Gamma^5 \Psi_F(x, y_j + y) ~,$$

$$P_j = {\rm diag}\,(-1,-1,-1,-1,1) ~, \quad P_j^{\rm sp} = {\rm diag}\,(1,1,-1,-1) ~. \qquad (10)$$

The orbifold boundary conditions break $SO(5) \times U(1)_X$ to $SO(4) \times U(1)_X \simeq SU(2)_L \times SU(2)_R \times U(1)_X$.

The brane interactions are invariant under $SO(4) \times U(1)_X$. The brane scalar $\hat\Phi$ is in the $(\mathbf{1},\mathbf{2})_{-1/2}$ representation of $[SU(2)_L, SU(2)_R]_{U(1)_X}$. It spontaneously breaks $SU(2)_R \times U(1)_X$ to $U(1)_Y$ by non-vanishing $\langle \hat\Phi \rangle$ whose magnitude is supposed to be much larger than the KK scale $m_{\rm KK}$. At this stage the residual gauge symmetry is $SU(2)_L \times U(1)_Y$. Brane fermions $\hat\chi_\alpha$ are introduced in the $(\mathbf{2},\mathbf{1})$ representation. The quark-lepton vector multiplets Ψ_a are decomposed into $(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{1})$. The $(\mathbf{2},\mathbf{2})$ part of Ψ_a, $\hat\chi_\alpha$ in $(\mathbf{2},\mathbf{1})$ and $\hat\Phi$ in $(\mathbf{1},\mathbf{2})$ form $SO(4) \times U(1)_X$ invariant brane interactions. With $\langle \hat\Phi \rangle \neq 0$ they yield mass terms. The resultant spectrum of massless fermions is the same as in the SM. All exotic fermions become heavy, acquiring masses of $O(m_{\rm KK})$. Further with brane fermions all anomalies associated with gauge fields of $SO(4) \times U(1)_X$ are cancelled.

With the orbifold boundary conditions (10) there appear four zero modes of A_y in the components $(A_y)_{a5} = -(A_y)_{5a}$ $(a = 1, \cdots, 4)$. They form an $SO(4)$ vector, or an $SU(2)_L$ doublet, corresponding to the Higgs doublet in the SM. The Wilson line phase is defined with these zero modes by

$$e^{i\Theta_H/2} \sim P \exp \left\{ i g_A \int_0^L dy\, A_y \right\} ~. \qquad (11)$$

At the tree level the value of the Θ_H is not determined, as it gives vanishing field strengths. At the quantum level its effective potential $V_{\rm eff}$ becomes non-trivial. The value of Θ_H is determined by the location of the minimum of $V_{\rm eff}$. This is the Hosotani mechanism and induces dynamical gauge symmetry breaking. It leads to gauge-Higgs unification, resolving the gauge-hierarchy problem. Without loss of generality one can assume that $(A_y)_{45}$ component develops a non-vanishing expectation value. Let us denote the corresponding component of Θ_H by θ_H. If θ_H takes a non-vanishing value, the electroweak symmetry breaking takes place.

5 $V_{\rm eff}(\theta_H)$ and m_H

Given the matter content one can evaluate $V_{\rm eff}(\theta_H)$ at the one loop level unambiguously. The θ_H dependent part of $V_{\rm eff}(\theta_H)$ is finite, being free from divergence. $V_{\rm eff}(\theta_H)$ depends on several parameters of the theory; $V_{\rm eff} = V_{\rm eff}(\theta_H; \xi, c_t, c_F, n_F, k, z_L)$ where ξ is the gauge parameter in the generalized R_ξ gauge, c_t and c_F are the bulk mass parameters of the top and extra fermion multiplets, n_F is the number of the extra fermion multiplets, and k, z_L are parameters specifying the RS metric (8). Given these parameters, $V_{\rm eff}$ is fixed, and the location of the global minimum of $V_{\rm eff}(\theta_H)$, $\theta_H^{\rm min}$, is determined.

With $\theta_H^{\rm min}$ determined, m_Z, g_w, $\sin^2\theta_W$ are determined from g_A, g_B, k, z_L and $\theta_H^{\rm min}$. The top mass m_t is determined from c_t, k, $z_L, \theta_H^{\rm min}$, whereas the Higgs boson mass m_H is given by

$$m_H^2 = \frac{1}{f_H^2} \frac{d^2 V_{\rm eff}}{d\theta_H^2}\bigg|_{\rm min} , \qquad f_H = \frac{2}{g_w} \sqrt{\frac{k}{L(z_L^2 - 1)}} ~. \tag{12}$$

Let us take $\xi = 1$. Then the theory has seven parameters

$$\{g_A, g_B, k, z_L, c_t, c_F, n_F\}$$

. Adjusting these parameters, we reproduce the values of five observed quantities $\{m_Z, g_w, \sin^2\theta_W, m_t, m_H\}$. This leaves two parameters, say z_L and n_F, free. Put differently, the value of $\theta_H^{\rm min}$ is determined as a function of z_L and n_F; $\theta_H^{\rm min} = \theta_H(z_L, n_F)$. We comment that contributions from other light quark/lepton multiplets to $V_{\rm eff}$ are negligible.

$V_{\rm eff}(\theta_H)$ in the absence of the extra fermions ($n_F = 0$) was evaluated. It was found there that the global minima naturally appear at $\theta_H = \pm\frac{1}{2}\pi$ at which the Higgs boson becomes absolutely stable. It is due to the emergence of the H parity invariance. In particular the Higgs trilinear couplings to W, Z, quarks and leptons are all proportional to $\cos\theta_H$ and vanish at $\theta_H = \pm\frac{1}{2}\pi$.

This, however, conflicts with the observation of an unstable Higgs boson at LHC. To have an unstable Higgs boson the H parity invariance must be broken, which is most easily achieved by introducing extra fermion multiplets Ψ_F in the spinor representation of $SO(5)$ in the bulk.

Let us take $n_F = 3$, $z_L = e^{kL} = 10^7$ as an example. $\{g_w, \sin^2\theta_W\}$ are

z_L	θ_H	$m_{\rm KK}$	$m_{Z^{(1)}}$	$m_{F^{(1)}}$
10^8	0.360	3.05 TeV	2.41 TeV	0.668 TeV
10^7	0.258	3.95	3.15	0.993
10^6	0.177	5.30	4.25	1.54
10^5	0.117	7.29	5.91	2.53

Table 2: Values of the various quantities with given z_L for $n_F = 3$. $m_{Z^{(1)}}$ and $m_{F^{(1)}}$ are masses of the first KK Z boson and the lowest mode of the extra fermion multiplets. Relations among θ_H, $m_{\rm KK}$ and $m_{Z^{(1)}}$ are universal, independent of n_F.

related to $\{g_A, g_B\}$ by

$$g_w = \frac{g_A}{\sqrt{L}} \ , \quad \tan\theta_W = \frac{g_B}{\sqrt{g_A^2 + g_B^2}} \ , \tag{13}$$

where $z_L = e^{kL}$. The observed values of $\{m_Z, g_w, \sin^2\theta_W, m_t, m_H\}$ are reproduced with $k = 1.26 \times 10^{10}\,$GeV, $c_t = 0.330$, $c_F = 0.353$ for which the minima of $V_{\rm eff}$ are found at $\theta_H = \pm 0.258$. The KK mass scale is $m_{\rm KK} = \pi k z_L^{-1} = 3.95\,$TeV.

5.1 Phenomenological consequences

The Gauge-Higgs Unification gives many definitive predictions to be tested by experiments. The values of various quantities determined from $m_H = 126\,$GeV with given z_L for $n_F = 3$ represents in Table 2. The relation between θ_H and $m_{\rm KK}$ is well summarized with

$$m_{\rm KK} \sim \frac{1350\,{\rm GeV}}{(\sin\theta_H)^{0.787}} \ . \tag{14}$$

In considered model all Higgs couplings $HWW, HZZ, Hc\bar{c}, Hb\bar{b}, H\tau\bar{\tau}$ are suppressed by a factor $\cos\theta_H$ at the tree level. The corrections to $\Gamma[H \to \gamma\gamma]$ and $\Gamma[H \to gg]$ due to KK states amount only to 0.2% (2%) for $\theta_H = 0.117(0.360)$. Hence may conclude that

$$\text{branching fraction:} \qquad B(H \to j) \sim B^{\rm SM}(H \to j)$$

$$j = WW, ZZ, \gamma\gamma, gg, b\bar{b}, c\bar{c}, \tau\bar{\tau}, \cdots$$

$$\gamma\gamma \text{ production rate:} \qquad \sigma^{\rm prod}(H) \cdot B(H \to \gamma\gamma) \sim (\text{SM}) \times \cos^2\theta_H \ . \tag{15}$$

The signal strength in the $\gamma\gamma$ production relative to the SM is about $\cos^2\theta_H$. It is about 0.99 (0.91) for $\theta_H = 0.1$ (0.3). This contrasts to the prediction in the UED models in which the contributions of KK states can add up in the same sign to sizable amount.

Couplings of quarks and leptons to W and Z also suffer from modification, but the amount of deviation from the standard model turns out tiny. The μ-e universality in weak interactions played an important role in the development of the theory. In the modern language it says that all left-handed leptons and quarks have the same coupling to the W boson. It is dictated by the $SU(2)_L$ gauge invariance in four dimensions. In the gauge-Higgs unification, however, the universality is not guaranteed at $\theta_H \neq 0$. As explained earlier, non-vanishing θ_H mixes various components in the gauge group and various levels in the Kaluza-Klein tower. This mixing for fermions depends on, say, the kink mass parameter c, and therefore is not universal.

For $c > 0.6$ wave functions are mostly localized near the Planck brane at $y = 0$ so that the 4D gauge coupling to W becomes almost universal for any values of θ_H. Define $r_\mu(\theta_H) = g_\mu^W(\theta_H)/g_e^W(\theta_H) - 1$ where g_e^W and g_μ^W are the gauge (W) couplings of e and μ, respectively. One finds typically that $r_\mu \sim -10^{-8}$ for $\theta_H = 0.5\pi$. For τ, $r_\tau \sim -2\times10^{-6}$. These numbers are well within the experimental limit, being very hard to test in the near future. For top quarks, the deviation becomes bigger ($r_t(0.5\pi) \sim -2 \times 10^{-2}$), but is difficult to measure accurately.

6 Conclusion and remarks

There are several constraints to be imposed on the gauge-Higgs unification:

- For the consistency with the S parameter, it is need $\sin\theta_H < 0.3$.
- The tree-level unitarity requires $\theta_H < 0.5$.
- Z' search at Tevatron and LHC. The first KK Z corresponds to Z'. No signal has been found so far, which implies that $m_{Z^{(1)}} > 2\,\text{TeV}$. It requires $\theta_H < 0.4$.

- The consistency with other precision measurements such as the Z boson decay and the forward-backward asymmetry on the Z resonance give the constraints: $m_{\mathrm{KK}} > 1.5\,\mathrm{TeV}$.

All of those constraints above point $\theta_H < 0.4$. When θ_H is very small, the KK mass scale m_{KK} becomes very large and it becomes very difficult to distinguish the gauge-Higgs unification from the SM. The range of interest is $0.1 < \theta_H < 0.35$, which can be explored at LHC with an increased energy 13 or 14 TeV.

The Gauge-Higgs Unification predicts the following signals:

1. The first KK Z should be found at $m_{\mathrm{KK}} = 2.5 \sim 6\,\mathrm{TeV}$ for $\theta_H = 0.35 \sim 0.1$.

2. The Higgs self-couplings should be smaller than those in the SM. λ_3 (λ_4) should be $10 \sim 20\%$ ($30 \sim 60\%$) smaller for $\theta_H = 0.1 \sim 0.35$, according to the universality relations. This should be explored at ILC.

3. The lowest mode ($F^{(1)}$) of the KK tower of the extra fermion Ψ_F should be discovered at LHC. Its mass depends on both θ_H and n_F. For $n_F = 3$, the mass is predicted to be $m_{F^{(1)}} = 0.7 \sim 2.5\,\mathrm{TeV}$ for $\theta_H = 0.35 \sim 0.1$.

Some theoretical tasks on future: 1) flavor mixing has to be incorporated to explore flavor physics; 2) the orbifold boundary conditions (P_0, P_1) in (10) have been given by hand so far; 3) not only electroweak interactions but also strong interactions should be integrated in the form of grand gauge-Higgs unification.

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