Fermion Current Calculation by the Method of Basis Spinors

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Abstract

We develop MBS-techniques for more efficient calculating of scattering amplitudes involving both massive fermions of an arbitrary polarization and massless fermions in quantum field theories. The purpose of this work is to calculation matrix elements of fermions with spin 1/2.

1 Introduction

When evaluating a Feynman amplitude involving fermions, the amplitude is expressed as sum of terms which have the form

$$\begin{aligned} \mathcal{M}_{\lambda_p,\lambda_k}\left(p, s_p,\ k, s_k\ ; Q\right) &= \mathcal{M}_{\lambda_p,\lambda_k}\left([p]\,, [k]\,; Q\right) = \\ &= \bar{w}^{A}_{\lambda_p}\left(p, s_p\right) Q\ w^{B}_{\lambda_k}\left(k, s_k\right)\ , \end{aligned} \tag{1}$$

where λ_p and λ_k are spin indices of the external fermions with four-momenta p, k and arbitrary polarization vectors s_p, s_k. The operator Q is a sum of products of Dirac γ-matrices. The notation $w^{A}_{\lambda_p}\left(p, s_p\right)$ stands for either $u_{\lambda_p}\left(p, s_p\right)$ (bispinor of fermion; $A = +1$) or $v_{\lambda_p}\left(p, s_p\right)$ (bispinor of antifermion; $A = -1$).

The main aim of calculation is to transform (1) to explicitly scalar form (scalar products of four-vectors, Lorentz tensors and so on). The main approach, which has gained popularity in the past decades, is to calculate Feynman amplitudes directly. Many different methods of calculating reaction amplitudes with fermions have been developed [1, 2, 3, 4] et.al. In

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the paper we describe an approach to Feynman diagrams which is based on the using of an isotropic tetrad in Minkowski space and massless basis spinors connected with it and we will call it as Method of Basis Spinors (MBS) [5, 6]) Let us briefly to describe the main relationships of **MBS**.

2 Isotropic tetrad and massless basis spinors

Let us introduce the orthonormal four-vector basis in Minkowski space which satisfies the relations:

$$l_0^\mu \bullet l_0^\nu - \sum_{j=1}^{3} l_j^\mu \bullet l_j^\nu = g^{\mu\nu}, \quad (l_A \cdot l_B) = g_{AB} , \tag{2}$$

where g is the Lorentz metric tensor.

With the help of vectors l_A we can define lightlike vectors, which form the isotropic tetrad in Minkowski space

$$b_\rho = (l_0 + \rho l_3)/2, \; n_\lambda = (\lambda l_1 + \mathrm{i} l_2)/2 , \quad (\lambda, \rho = \pm 1) . \tag{3}$$

From Eqs. (2), (3) it follows that

$$(b_\rho \cdot b_{-\lambda}) = (n_\rho \cdot n_{-\lambda}) = \frac{\delta_{\lambda,\rho}}{2} , \; (b_\rho \cdot n_\lambda) = 0 , \tag{4}$$

$$g^{\mu\nu} = \sum_{\lambda=-1}^{1} \left[\tilde{b}_\lambda^\mu \cdot b_{-\lambda}^\nu + \tilde{n}_\lambda^\mu \cdot n_{-\lambda}^\nu \right] , \tag{5}$$
$$\tilde{b}_\lambda^\mu = 2\, b_\lambda^\mu , \; \tilde{n}_\lambda^\mu = 2\, n_\lambda^\mu .$$

It is always possible to construct the basis of an isotropic tetrad (3) as numerical four-vectors

$$(b_{\pm 1})_\mu = (1/2)\,\{1, 0, 0, \pm 1\} \; , \; (n_{\pm 1})_\mu = (1/2)\,\{0, \pm 1, \mathrm{i}, 0\} \tag{6}$$

or by means of physical vectors for reaction.

By means of the isotropic tetrad (3) we define ***basis spinors*** $u_\lambda(b_{-1})$ and $u_\lambda(b_1)$:

$$\not{b}_{-1} u_\lambda (b_{-1}) = 0 , \qquad u_\lambda (b_1) \equiv \not{b}_1 u_{-\lambda} (b_{-1}) , \tag{7}$$

$$\omega_\lambda u_\lambda (b_A) = u_\lambda (b_A) \; , \; (A = \pm 1) \tag{8}$$

with matrix $\omega_\lambda = 1/2\ (1 + \lambda\gamma_5)$ and normalization condition

$$u_\lambda\,(b_A)\,\bar{u}_\lambda\,(b_A) = \omega_\lambda\ \not{b}_A\ . \tag{9}$$

The relative phase between basis spinors with different helicity is given by

$$\not{n}_\lambda u_{-\rho}\,(b_{-1}) = \delta_{\lambda,\rho} u_\lambda\,(b_{-1})\ . \tag{10}$$

The important property of basis spinors (7) is the **completeness relation**:

$$\sum_{\lambda,A=-1}^{1} u_\lambda\,(b_A)\,\bar{u}_{-\lambda}\,(b_{-A}) = \mathrm{I}\ , \tag{11}$$

which follows from Eqs.(7)–(10). Thus, the arbitrary bispinor can be decomposed in terms of basis spinors $u_\lambda\,(b_A)$.

3 Main equations of MBS and Dirac spinors

Arbitrary Dirac spinor can be determined through the basis spinor (7) with the help of projection operators $\tau_\lambda\,(p, s_p) = u_{\lambda_p}\,(p, s_p)\,\bar{u}_{\lambda_p}\,(p, s_p)$. The Dirac spinors $w_\lambda^A\,(p, s_p)$ for massive fermion and antifermion with four-momentum p $(p^2 = m_p^2)$, arbitrary polarization vector s_p and spin number $\lambda = \pm 1$ can be obtained with the help of basis spinors by means of equation:

$$w_\lambda^A\,(p, s_p) = (A\lambda)\,\frac{(\not{p} + Am_p)\,(1 + \lambda\gamma_5\ \not{s}_p)}{2\sqrt{(b_1\cdot(p + m_p s_p))}}\ u_{-A\times\lambda}\,(b_1) \tag{12}$$

Spinor products of basis spinors are simple and similar to scalar products of isotropic tetrad vectors

$$\bar{u}_\lambda\,(b_C)\,u_\rho\,(b_A) = \delta_{\lambda,-\rho}\delta_{C,-A}\ . \tag{13}$$

With the help of Eq.(5) Dirac matrix γ^μ can be rewritten as

$$\gamma^\mu = \sum_{\lambda=-1}^{1}\left[\not{b}_{-\lambda}\tilde{b}_\lambda^\mu + \not{n}_{-\lambda}\tilde{n}_\lambda^\mu\right] \tag{14}$$

and using Eqs.(8),(10) and (14) we can obtain that

$$\gamma^\mu\ u_\lambda\,(b_A) = \tilde{b}_A^\mu u_{-\lambda}\,(b_{-A}) - A\ \tilde{n}_{-A\times\lambda}^\mu u_{-\lambda}\,(b_A)\ , \tag{15}$$

which allow to transform Dirac matrix to some combination of isotropic tetrad vectors on basis spinor space and

$$\gamma_5\ u_\rho\,(b_A) = \rho\ u_\rho\,(b_A)\ . \tag{16}$$

Eqs. (13), (15) and (16) underlies the method of basis spinors (MBS).

4 MBS and technique of "building" blocks

The **basic idea of Method of Basis Spinors** is to replace Dirac spinors in Eq.(1) by massless basis spinors $u_\lambda\,(b_{\pm1})$ (Eq.(12)), and to use only three Eqs. (13), (15) and (16) to calculate matrix element (1) in terms of scalar functions.

Let us consider an important type of matrix element (1), when $p = b_{-C}$ and $k = b_A$, i.e.

$$\mathcal{M}_{\sigma,-\rho}\,(b_C\ ,\ b_{-A}\ ;Q) \equiv \Gamma^{C,\,A}_{\sigma,\rho}\,[Q] = \bar{u}_\sigma\,(b_C)\,Q\,u_{-\rho}\,(b_{-A})\ . \qquad (17)$$

We call this type of matrix element as **basic matrix element**. By means of MBS relations (13), (15) and (16) it is easy to calculate $\Gamma^{C,A}_{\sigma,\rho}$ in terms of the isotropic tetrad vectors.

With the help of completeness relation (11) the amplitude (1) with is expressed as combinations of the lower-order matrix elements ("building" blocks)

$$\begin{aligned}\mathcal{M}_{\lambda_p,\lambda_k}\,(p,s_p\ k,s_k;Q) = & \sum_{A,\,C,\sigma,\,\rho=-1}^{1} \left\{\bar{w}^{D}_{\lambda_p}\,(p,s_p)\,u_{-\sigma}\,(b_{-C})\right\} \times \\ & \times \left\{\bar{u}_\sigma\,(b_C)\,Qu_{-\rho}\,(b_{-A})\right\}\left\{\bar{u}_\rho\,(b_A)\,w^{F}_{\lambda_k}\,(k,s_k)\right\} = \\ = & \sum_{\sigma,\,\rho=-1}^{1}\ \sum_{A,\,C=-1}^{1} \bar{s}^{(C,\,D)}_{\sigma,\lambda_p}\,(p,s_p)\,\Gamma^{C,A}_{\sigma,\rho}\,[Q]\,s^{(A,\,F)}_{\rho,\lambda_k}\,(k,s_k)\ . \qquad (18)\end{aligned}$$

Decomposition coefficients for helicity states of fermions can be easily calculated:

$$s^{(A,D)}_{\rho,\lambda}(p,s_{\rm hel}) = D\lambda\,W_m(-\lambda\rho D\mathrm{p})f(\rho\lambda,D)D^{*1/2}_{A\rho/2,-D\lambda/2}(\phi,\theta,-\phi) \qquad (19)$$

where

$$\begin{aligned}&W_m(\pm\mathrm{p}) = \sqrt{\omega_m(\mathrm{p})\pm\mathrm{p}}\ ,\quad \omega_m(\mathrm{p}) = \sqrt{\mathrm{p}^2+m^2}\ ,\quad \mathrm{p} = |\mathbf{p}|\ ,\\ &f(A,D) = \delta_{A,-1} + D\delta_{A,1} \qquad (20)\end{aligned}$$

and $D^{1/2}_{\sigma_1,\sigma_2}(\phi,\theta,-\varphi) = \exp(-i\phi)\,d^{1/2}_{\sigma_1,\sigma_2}(\theta)\exp(-i\varphi)$ is Wigner function [7].

5 Vector boson decays

We will now apply the methods described above by calculating the Born amplitude for the decay of the vector boson with the mass m_V and helicity

σ into fermions

$$V(p,\sigma) \to f_i(k_1,\lambda_{k_1}) + \bar{f}_j(k_2,\lambda_{k_2}) \,. \tag{21}$$

where f is a fermion with helicity λ.

Using Feynman rules we can been written the expression for the amplitude decay (21) in general form

$$M^{\sigma}_{\lambda_{k_1},\lambda_{k_2}}\left(V \to f_i\bar{f}_j\right) = R^V_{ij}\,\bar{u}_{\lambda_{k_1}}(k_1,m_1)\,(\varepsilon^{\mu}_{\sigma}\gamma_{\mu})\left[\sum_{\tau=-1}^{1} g^V_{\tau}\,\omega_{\tau}\right] v_{\lambda_{k_2}}(k_2,m_2), \tag{22}$$

where $g^V_{\pm1}$ denote the generic left- and right-handed fermion-fermion-vector couplings and R^V_{ij} is some function of fermion charges and elements of CKM matrix.

We specify the kinematics of decay (21) in the rest frame and helicity states of quarks and boson

$$\begin{aligned} &p^{\mu} = (m_V,0,0,0)\;,\, k^{\mu}_1 = (\omega_{m_1}(\mathrm{k}), \mathrm{k}\sin\theta, 0, \mathrm{k}\cos\theta)\;, \\ &k^{\mu}_2 = (\omega_{m_2}(\mathrm{k}), -\mathrm{k}\sin\theta, 0, -\mathrm{k}\cos\theta)\;, \end{aligned} \tag{23}$$

$$\begin{aligned} &\mathrm{k} = |\mathbf{k}| = \frac{\sqrt{m_V^4 + (m_1^2-m_2^2)^2 - 2m_V^2(m_1^2+m_2^2)}}{2m_V} = \frac{\lambda^{1/2}\left(m_V^2, m_1^2, m_2^2\right)}{2m_V}, \\ &\omega_{m_1}(\mathrm{k}) = \frac{m_V^2 - m_2^2 + m_1^2}{2m_V}\,, \quad \omega_{m_2}(\mathrm{k}) = \frac{m_V^2 + m_2^2 - m_1^2}{2m_V}\,. \end{aligned} \tag{24}$$

The polarization vector $\varepsilon^{\mu}_{\sigma}$ of boson is

$$\varepsilon^{\mu}_{\sigma=0} = (0,0,0,1), \quad \varepsilon^{\mu}_{\sigma=\pm1} = (0, 1/\sqrt{2},\; \mathrm{i}\sigma/\sqrt{2}, 0) \tag{25}$$

for longitudinal polarization ($\sigma = 0$) and transverse ($\sigma = \pm1$) polarization respectively.

Relations (13),(15) and (18)–(20) allow to calculate of matrix element (22) in terms of scalar products:

$$\begin{aligned} &M^{\sigma}_{\lambda_{k_1},\lambda_{k_2}}\left(V \to f_i\bar{f}_j\right) = R^V_{ij} \sum_{A,\rho=-1}^{1} g^V_{\rho}\, D^{*1/2}_{A\rho/2,-\lambda_{k_2}/2}(\phi,\theta,-\phi)\,\times \\ &\times\Big[A\; D^{*1/2}_{-A\rho/2,\lambda_{k_1}/2}(\phi,\theta,-\phi)(\varepsilon_{\sigma}\cdot\tilde{b}_{-A}) - \\ &-D^{*1/2}_{A\rho/2,\lambda_{k_1}/2}(\phi,\theta,-\phi)(\varepsilon_{\sigma}\cdot\tilde{n}_{-A\rho})\Big]\, W_{m_1}(-\rho\lambda_{k_1}\mathrm{k})W_{m_2}(\rho\lambda_{k_2}\mathrm{k})\,. \end{aligned} \tag{26}$$

Using Eqs. (6), (23), the Clebsh-Gordan decomposition of D-matrix

$$
\begin{aligned}
&D^{1/2}_{\lambda_1,\lambda_2}(\phi,\theta,-\phi)D^{1/2}_{\sigma_1,\sigma_2}(\phi,\theta,-\phi) = \\
&= \frac{1}{4}\sqrt{(3+4\lambda_1\sigma_1)}\sqrt{(3+4\lambda_2\sigma_2)}D^1_{\lambda_1+\sigma_1,\lambda_2+\sigma_2}(\phi,\theta,-\phi) + \\
&+2\lambda_1\lambda_2\delta_{\lambda_1,-\sigma_1}\delta_{\lambda_2,-\sigma_2}\ , \quad \text{here} \ \ (\lambda_{1,2},\sigma_{1,2} = \pm 1/2)\ , \qquad (27)
\end{aligned}
$$

Eqs. (6), (23) and

$$
(\varepsilon_\sigma\cdot\tilde{b}_A) = \delta_{\sigma,0}A\ , \quad (\varepsilon_\sigma\cdot\tilde{n}_\rho) = -\sqrt{2}\,\delta_{\sigma^2,1}\,\sigma\delta_{\sigma,-\rho}\ , \qquad (28)
$$

we get

$$
\begin{aligned}
&M^{\sigma}_{\lambda_{k_1},\lambda_{k_2}}\left(V\to f_i\bar{f}_j\right) = R^V_{ij}\left(\sigma\delta_{\sigma^2,1}-\delta_{\sigma,0}\right)\sqrt{\frac{3-\lambda_{k_1}\lambda_{k_2}}{2}}\, d^1_{\sigma,(\lambda_{k_1}-\lambda_{k_2})/2}(\theta)\ \times \\
&\times \sum_{\rho=-1}^{1} g^V_\rho W_{m_1}(-\rho\lambda_{k_1}\mathrm{k})W_{m_2}(\rho\lambda_{k_2}\mathrm{k})\ . \qquad (29)
\end{aligned}
$$

Let us consider the useful relations for $W_m(\rho\mathrm{k})$ in rest frame. After simple calculations we obtain that

$$
\begin{aligned}
&W_{m_1}(\tau\mathrm{k})W_{m_2}(\rho\mathrm{k}) = \\
&= \frac{1}{\sqrt{2}}\left[\delta_{\tau,\rho}\sqrt{m_V^2 - m_{12}^2 + \tau\lambda^{1/2}\left(m_V^2,m_1^2,m_2^2\right)}\right. \\
&\left.+\delta_{\tau,-\rho}\sqrt{m_{12}^2 - \Delta m^2/m_V^2\left(\Delta m^2 + \tau\lambda^{1/2}\left(m_V^2,m_1^2,m_2^2\right)\right)}\right]\ . \qquad (30)
\end{aligned}
$$

When if $m_1 = m, m_2 = 0$ we obtain that

$$
\begin{aligned}
&W_{m_1=m}(\tau\mathrm{k})W_{m_2=0}(\rho\mathrm{k}) = \\
&= m_V\sqrt{1-\beta_V^2}\delta_{\rho,1}\left[\delta_{\tau,1}+\delta_{\tau,-1}\beta_V\right]\ , \quad \beta_V = \frac{m}{m_V}\ . \qquad (31)
\end{aligned}
$$

If $m_1 = m, m_2 = m$ we get that

$$
\begin{aligned}
&W_m(\lambda\mathrm{k})W_m(\rho\mathrm{k}) = m_V\left[\delta_{\lambda,-\rho}\beta_V + \right. \\
&\left.+\delta_{\lambda,\rho}\,\frac{1}{2}\sqrt{2(1+\rho\sqrt{1-4\beta_V^2})-4\beta_V^2}\right]\ , \quad \beta_V = \frac{m}{m_V}\ . \qquad (32)
\end{aligned}
$$

Evaluating the $\left|M^{\sigma}_{\lambda_{k_1},\lambda_{k_2}}\right|$ with the help of Eq. (29) and (30) we arrive at

$$\left|M^{\sigma}_{\lambda_{k_1},\lambda_{k_2}}\left(V \to f_i\bar{f}_j\right)\right|^2 = \left(\delta_{\sigma^2,1} + \delta_{\sigma,0}\right)\left|R^V_{ij}\right|^2 \left|d^1_{\sigma,(\lambda_{k_1}-\lambda_{k_2})/2}(\theta)\right|^2 \times$$
$$\times\left(\left|\sum_{\rho=-1}^{1} g^V_\rho S^{(I)}_{\rho,\lambda_{k_1}}\right|^2 \delta_{\lambda_{k_1},\lambda_{k_2}} + 2\left|\sum_{\rho=-1}^{1} g^V_\rho S^{(II)}_{\rho,\lambda_{k_1}}\right|^2 \delta_{\lambda_{k_1},-\lambda_{k_2}}\right), \tag{33}$$

where

$$S^{(I)}_{\rho,\lambda_1} = \frac{1}{\sqrt{2}}\sqrt{m^2_{12} - \Delta m^2/m^2_V\left(\Delta m^2 - \lambda_1\rho\lambda^{1/2}\left(m^2_V, m^2_1, m^2_2\right)\right)}\,,$$
$$S^{(II)}_{\rho,\lambda_1} = \frac{1}{\sqrt{2}}\sqrt{m^2_V - m^2_{12} - \lambda_1\rho\lambda^{1/2}\left(m^2_V, m^2_1, m^2_2\right)}\,. \tag{34}$$

The partial decay rate (partial width) of unpolarized vector boson into unpolarized pair of fermions in its rest frame is given

$$\frac{d\Gamma}{d\Omega} = \frac{1}{3}\frac{\mathrm{k}}{32\pi^2 m^2_V}\sum_{\sigma=-1}^{1}\sum_{\lambda_{k_1},\lambda_{k_2}=-1}^{1}\left|M^{\sigma}_{\lambda_{k_1},\lambda_{k_2}}\left(V \to f_i\bar{f}_j\right)\right|^2. \tag{35}$$

Decay width of $V \to f_1\bar{f}_2$

$$\Gamma = \int\frac{d\Gamma}{d\Omega}d\Omega = \int_0^{\pi}\int_0^{2\pi}\sin\theta d\theta d\phi\frac{d\Gamma}{d\Omega} \tag{36}$$

with the help of relation

$$\frac{(2J+1)}{2}\int_0^{\pi} d\theta\sin\theta\left|d^J_{\sigma,\rho}(\theta)\right|^2 = 1 \tag{37}$$

and Eq. (33) reduced to

$$\Gamma\left(V \to f_i\bar{f}_j\right) = \frac{\mathrm{k}}{24\pi m^2_V}\left|R^V_{ij}\right|^2 \times$$
$$\times\sum_{\lambda_{k_1},\lambda_{k_2}=-1}^{1}\left(\left|\sum_{\rho=-1}^{1} g^V_\rho S^{(I)}_{\rho,\lambda_{k_1}}\right|^2 \delta_{\lambda_{k_1},\lambda_{k_2}} + 2\left|\sum_{\rho=-1}^{1} g^V_\rho S^{(II)}_{\rho,\lambda_{k_1}}\right|^2 \delta_{\lambda_{k_1},-\lambda_{k_2}}\right) \tag{38}$$

6 Examples of vector boson decays

6.1 $W \to \ell \bar{\nu}_\ell$

The left- and right-handed couplings of the fermions to the W-boson are defined

$$g_\lambda^W = \delta_{\lambda,-1}\frac{1}{\sqrt{2}s_W}\ , R_W = (-1)\sqrt{4\pi\alpha}\,. \tag{39}$$

In SM the contribution of Born amplitude is given by

$$M^\sigma_{\lambda_{k_1},\lambda_{k_2}}(W \to \ell\bar{\nu}_\ell) = (-1)\sqrt{4\pi\alpha}\,(\sigma\delta_{\sigma^2,1} - \delta_{\sigma,0})\, d^1_{\sigma,(\lambda_{k_1}-\lambda_{k_2})/2}(\theta) \times$$
$$\times \delta_{\lambda_{k_2},-1}\frac{m_W}{\sqrt{2}s_W}\sqrt{1-\beta_W^2}\left(\sqrt{2}\delta_{\lambda_{k_1},-\lambda_{k_2}} + \delta_{\lambda_{k_1},\lambda_{k_2}}\beta_W\right)\,. \tag{40}$$

Using of Eqs.(38),(39) and further simplification gives

$$\Gamma\,(W \to \ell\bar{\nu}_\ell) = \frac{\alpha M_W}{24 s_W^2}\left(1-\beta_W^2\right)^2\left(2+\beta_W^2\right)\,. \tag{41}$$

Neglecting the fermion masses (41) leads to the standard expression of decay width [8]

$$\Gamma\,(W \to \ell\bar{\nu}_\ell) = \frac{\alpha M_W}{12 s_W^2}\,. \tag{42}$$

6.2 $Z^0 \to \ell\bar{\ell}$

The left- and right-handed couplings of the fermions ($\ell = e, \mu, \tau$) to the Z-boson are defined

$$g_{-1}^Z = \frac{(s_W^2 - 1/2)}{c_W s_W},\quad g_1^Z = \frac{s_W}{c_W}\ , R_Z = (-1)\sqrt{4\pi\alpha}\,. \tag{43}$$

Similarly we obtain for the Born amplitude of process $Z^0 \to \ell\bar{\ell}$

$$M^\sigma_{\lambda_{k_1},\lambda_{k_2}}\left(Z^0 \to \ell\bar{\ell}\right) = (-1)\sqrt{4\pi\alpha}\ (\sigma\delta_{\sigma^2,1} - \delta_{\sigma,0})\ d^1_{\sigma,(\lambda_{k_1}-\lambda_{k_2})/2}(\theta) m_Z \times$$
$$\times \sum_{\rho=-1}^{1} g_\rho^Z\left(\frac{\delta_{\lambda_{k_1},-\lambda_{k_2}}}{\sqrt{2}}\sqrt{1-\rho\lambda_{k_1}\sqrt{1-4\beta_Z^2} - 2\beta_Z^2} + \delta_{\lambda_{k_1},\lambda_{k_2}}\,\beta_Z\right)\,. \tag{44}$$

Decay width of $Z^0 \to \ell\bar{\ell}$ is given by

$$\Gamma\left(Z^0 \to \ell\bar{\ell}\right) = \frac{\alpha M_Z}{12}\sqrt{1-4\beta_Z^2}\left(\left(g_{-1}^Z\right)^2 + \left(g_1^Z\right)^2 + 8 g_1^Z g_{-1}^Z \beta_Z^2\right)\,. \tag{45}$$

Neglecting the fermion masses (45) leads to

$$\Gamma\left(Z^0 \to \ell\bar{\ell}\right) = \frac{\alpha M_Z}{12}\left(\left(g^Z_{-1}\right)^2 + \left(g^Z_1\right)^2\right) . \qquad (46)$$

7 Conclusion

We have formulated a effective method to calculate Feynman amplitudes for various processes with fermions. In our approach (MBS):

1. We don't use an explicit form of Dirac spinors and γ–matrices (as well as basis spinors)
2. We don't use calculation of traces

The MBS enables us to calculate blocks of Feynman diagrams (current-like constructions and even more complicated structures) and then use them as universal functions during the process of calculation.

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