Electromagnetic Interactions of Kaons

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Abstract

Analytical expressions for the electromagnetic form factors of the charged and neutral K mesons have been obtained in the Quark Confinement Model.The contribution from the direct diagrams, as well as that one from the intermediate vector states in the form factors are examined.It is shown that for the description of the neutral K^0 meson intermediate vector contribution is crucial. Electromagnetic radius of the charged kaon was calculated. fm. The received value $\langle r_{K^+} \rangle = 0.501$ is in good agreement with the experimental data.The numerical values of the square of the electromagnetic radius of the neutral kaon at different values of the mixing angle of vector mesons was obtained.It was found that the best agreement with the experimental data is achieved when $\delta = 0$.The calculated numerical value $\langle r_{K^0} \rangle = -0.084$.

1 Introduction

To date, it is assumed that the hadrons have a finite size , which in the electromagnetic interaction manifests itself in the form of electromagnetic structure of hadrons , which is phenomenologically described by one or more (depending on the spin of the particles under study) functions of one variable (the momentum transfer) , called electromagnetic form factors . The form factors describing the vertexes containing one real photon and two identical strongly interacting particles are called elastic electromagnetic form factors of the hadron. In the case with a single virtual

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photon and two different particles or hadron and real photons corresponding form factors are called transition elektromagntnetic form factors of hadrons [1]. The most experimentally studied is the elastic pion form factor . The experimental data on elastic electromagnetic form factors of kaons significantly poorer [2] , [3] . These form factors have been studied in various theoretical approaches [4] , [5],[6] . In this work, all calculations were carried out in the Quark Confinement Model(QCM) [7].

2 Lagrangians

Lagrangian describing the electromagnetic and strong interactions of the system consisting of charged pseudoscalar, neutral vector mesons, photons, quarks can be written as:

$$L_I(x) = L_{qqP}(x) + L_{qqV}(x) + L_{q^{em}}(x) + L_{P^{em}}(x). \tag{1}$$

where

$$\begin{aligned} L_{qqP} &= \frac{g_P}{\sqrt{2}} P_Q(x)\overline{q}(x) i\gamma^5 \lambda_Q q(x), \\ L_{qqV} &= \frac{g_V}{\sqrt{2}} V_Q(x)\overline{q}(x)\gamma^\mu \lambda_Q q(x), \\ L_P^{em} &= -ie(P^+\partial^\mu P^- - P^-\partial^\mu P^+)A_\mu,) \\ L_q^{em} &= eA_\mu \overline{q} Q\gamma^\mu q. \end{aligned} \tag{2}$$

the notation is adopted

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \tag{3}$$

$$Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix} \tag{4}$$

λ_Q-combination of Gell-Mann matrices, providing the appropriate quantum numbers of mesons and for ω and ϕ meson matrix λ_Q are:

$$\lambda_\omega = \begin{pmatrix} cos\delta & 0 & 0 \\ 0 & cos\delta & 0 \\ 0 & 0 & -\sqrt{2}\sin\delta \end{pmatrix} \tag{5}$$

$$\lambda_\phi = \begin{pmatrix} -sin\delta & 0 & 0 \\ 0 & -sin\delta & 0 \\ 0 & 0 & -\sqrt{2}\cos\delta \end{pmatrix} \tag{6}$$

The coupling constants g_P, g_V for meson-quark interaction are defined from so-called compositeness condition which means that the renormalization constant of the corresponding meson M is equal to zero:

$$Z_M = 1 + g_M^2 \tilde{\Pi}'_(M)(m_M^2) = 0 \tag{7}$$

Here $\tilde{\Pi}_M(p^2)$- mass operator of meson$M$. It us convenient to use interaction constant in a form:

$$h_M = \frac{3g_M^2}{4\pi^2} = -\frac{1}{\tilde{\Pi}'_M(m_M)} \tag{8}$$

instead of g_M in the further calculations.

3 Electromagnetic Kaon Form Factors

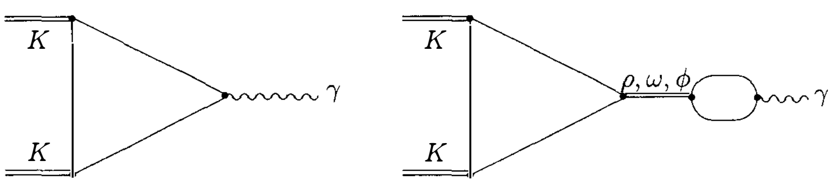

Figure 1: Graphs define kaon form factor

The electromagnetic form factors of kaons are determined by graphs shown in Figure 1 and can be written as :

$$F_{K^+}(t) = F_\Delta^+(t) + F_\rho(t) + F_\omega(t) - F_\phi(t) \tag{9}$$
$$F_{K^0}(t) = F_\Delta^0(t) - F_\rho(t) + F_\omega(t) - F_\phi(t) \tag{10}$$

where $F_\Delta(t)$ -contribution to the form factor of the triangle diagrams $F_{\rho,\omega,\phi}(t)$ - contributions from diagrams with intermediate vector mesons. Listed contributions received by us as follows:

$$
\begin{aligned}
F^{+}_{\Delta} &= \frac{h_K}{6}\left(2W_1(t)+W_2(t)\right)\\
F^{0}_{\Delta} &= \frac{h_K}{6}\left(W_1(t)-W_2(t)\right)\\
F_\rho(t) &= t\frac{h_K}{2}W_1(t)h_\rho D_\rho(t)F_{VV}\left(\frac{t}{4\Lambda_u^2}\right)\\
F_\omega(t) &= t\frac{h_K}{6}W_1(t)h_\omega D_\omega(t)F_{VV}\left(\frac{t}{4\Lambda_u^2}\right)\\
F_\phi(t) &= t\frac{h_K}{3}W_2(t)h_\phi D_\phi(t)F_{VV}\left(\frac{t}{4\Lambda_u^2}\right)
\end{aligned}
\tag{11}
$$

where

$$
\begin{aligned}
W_1(t) &= \frac{1}{2}F^{-}_{VPP}\left(t, m_K^2, m_K^2, \Lambda_u, \Lambda_u, \Lambda_s\right)\\
W_2(t) &= \frac{1}{2}F^{-}_{VPP}\left(t, m_K^2, m_K^2, \Lambda_s, \Lambda_s, \Lambda_u\right)
\end{aligned}
\tag{12}
$$

function $F^{-}_{VPP}(t, q_1^2, q_2^2, \Lambda_s, \Lambda_s, \Lambda_u)$ have been received as

$$
\begin{aligned}
&F^{-}_{VPP}\left(t, q_1^2, q_2^2, \Lambda_s, \Lambda_s, \Lambda_u\right) =\\
&= \int_0^\infty du b(u) + s\int_0^{u_\Delta} du b(-su)\sqrt{1-u+\frac{u^2\Delta^2}{4}} +\\
&+ \int_0^1 d^3\alpha\delta\left(1-\sum_{i=1}^{3}\alpha_i\right)P\left(q_1^2, q_2^2, \Lambda_1, \Lambda_2, \Lambda_3\right)b(-Q)
\end{aligned}
\tag{13}
$$

The following notation is introduced

$$
\Lambda^2 = \frac{\Lambda_1^2+\Lambda_2^2}{2}; \Delta = \frac{\Lambda_2^2-\Lambda_1^2}{\Lambda_1^2+\Lambda_2^2}; s = \frac{p^2}{4\Lambda^2}; u_\Delta = \frac{2}{1+\sqrt{1-\Delta^2}}. \tag{14}
$$

$$
\begin{aligned}
&P\left(q_1^2, q_2^2, \Lambda_1, \Lambda_2, \Lambda_3\right) = \qquad (15)\\
&= \frac{(\alpha_1+\alpha_2)(\Lambda_1-\Lambda_3)(\Lambda_2-\Lambda_3)\Lambda_3(\Lambda_1+\Lambda_2-\Lambda_3)}{\sum_{i=1}^{3}\alpha_i\Lambda_i^2}Q + \frac{q_1^2\alpha_1+q_2^2\alpha_2}{\sum_{i=1}^{3}\alpha_i\Lambda_i^2}
\end{aligned}
$$

$$Q = \frac{q_1^2\alpha_1\alpha_3 + q_2^2\alpha_2\alpha_3 + p^2\alpha_1\alpha_2}{\sum_{i=1}^3 \alpha_i\Lambda_i^2} \qquad (16)$$

It can be seen that the contribution to $F_{K^0}(t)$ from the triangular diagrams is linked to the difference between Λ_u and Λ_s and therefore vanishes in the limit of precision $SU(3)$ symmetry.

Structure integral $F_{VV}\left(\frac{t}{4\Lambda_u^2}\right)$ is defined by

$$F_{VV} = \int_0^\infty dub(u) + x\int_0^1 dub(-ux)\frac{1-\frac{u}{2}+\frac{u^2}{4}}{\sqrt{1-u}} \qquad (17)$$

The product of the coupling constants for the vector meson propagator is:

$$h_V D_V(t) = \frac{F_V\left(\frac{t}{4\Lambda^2}\right)}{s_V F_V(s_V) - \left(\frac{t}{4\Lambda^2}\right) F_V\left(\frac{t}{4\Lambda^2}\right)} \qquad (18)$$

for$V = \rho, \omega \Lambda = \Lambda_u$, in the case of ϕ-meson $\Lambda = \Lambda_s$.

$$F_V(x) = \int_0^\infty dub(u) + x\int_0^1 dub(-ux)\left(1+\frac{u}{2}\right)\sqrt{1-u} \qquad (19)$$

In the framework of the model used effective constant of kaon - quarks interaction is:

$$h_K = \frac{2}{F_{PP}\left(m_K^2, \Lambda_u, \Lambda_s\right)} \qquad (20)$$

where

$$F_{PP}\left(x, \Lambda_1, \Lambda_2\right) = \int_0^\infty dub(u) + \frac{x}{4\Lambda^2}\int_0^{u_\Delta} dub\left(-\frac{ux}{4\Lambda^2}\right)\frac{1-\frac{u}{2}+u\Delta}{\sqrt{1-u+\frac{u^2\Delta^2}{4}}} \qquad (21)$$

Figure 2 shows that the inclusion of the contribution of intermediate states significantly changes the behavior of the form factor.

Figure 3 shows that the contributions from ω and ϕ are small and opposite in sign. Therefore, the decisive role played by the contribution of the intermediate ρ meson.

Figure 5 shows that, in contrast to the form factor of the K^+ meson contributions from ω and ϕ have the same sign and are comparable to the contribution of the ρ meson. Because of this, the values of the K^0 form factor depends on the mixing angle δ.Figure 6 demonstrates this dependence.

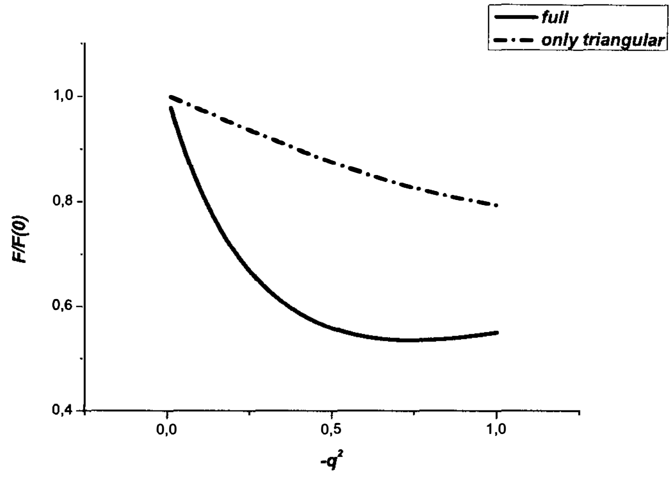

Figure 2: Form factor of K^+ mesons (solid line) and only the contribution of the triangle diagram, normalized to the value 0 ($\delta = 5°$).

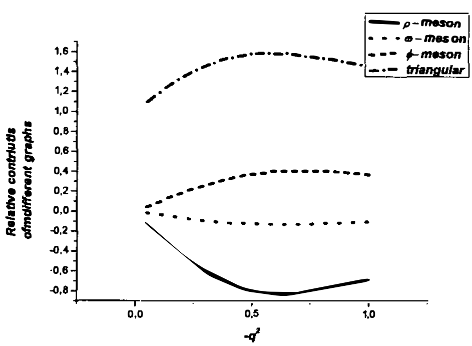

Figure 3: The relative contributions to K^+ of the individual diagrams appearing in (9).

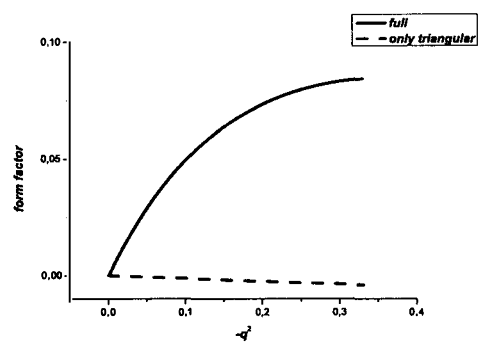

Figure 4: Form factor of K^0 mesons (solid line) and only the contribution of the triangle diagram, normalized to the value 0 ($\delta = 5°$).

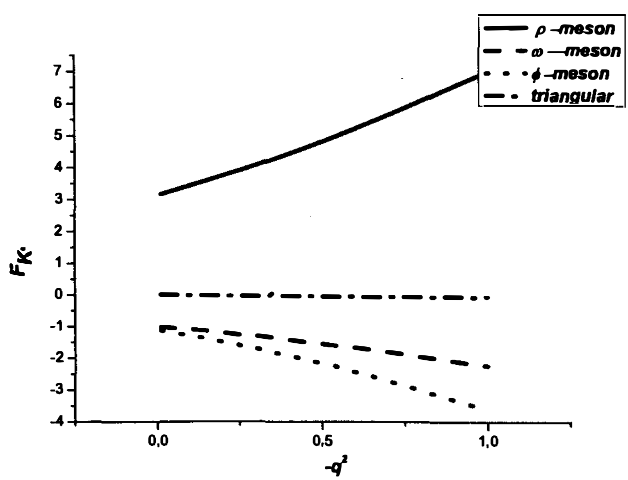

Figure 5: The contribution of different diagrams in the form factor of K^0 meson(10)

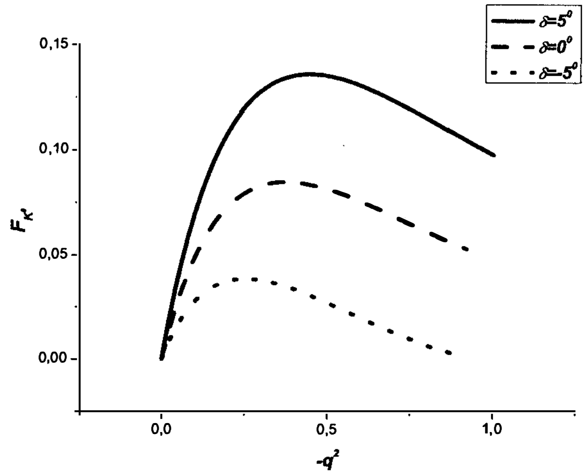

Figure 6: form factor $F_{K^0}(t)$, calculated for different values of the mixing angle δ

4 Electromagnetic Radii of Kaons

Electromagnetic radii of kaons one can calculate by the formula

$$\langle r_M \rangle^2 = 6\frac{dF_M}{dq^2}|_{q^2=0} \tag{22}$$

Table 1 shows the contributions of the various diagrams in $< r_{K^+} >^2$, obtained value of the electromagnetic radius, as well as the experimental value.

Table 1

Contributions from different graphs $\langle r_{K^+} \rangle^2$ (fm^2)				$< r_{K^+} >$ (fm)	exp [8] (fm)
Δ	ρ	ω	ϕ		
0.027	0.263	−0.129	0.111	0.501	0.560 ± 0.31

The square of the K^0 meson electromagnetic radius depends on the mixing angle δ. Table 2 shows the contributions of the various diagrams in $\langle r_{K^0} \rangle^2$ for different values of δ.

Table 2

δ	Contributions from different graphs $\langle r_{K^0}\rangle^2$ (fm^2)				$< r_{K^0} >^2$ (fm^2)	exp [8] (fm^2)
	Δ	ρ	ω	ϕ		
$-10°$	0.0015	−0.263	0.203	0.149	−0.02	
$-5°$	0.0015	−0.263	0.074	0.02	−0.051	
$0°$	0.0015	−0.263	0.085	0.094	−0.084	-0.077 ± 0.010
$5°$	0.0015	−0.263	0.196	0.099	−0.116	
$10°$	0.0015	−0.263	−0.0015	0.016	−0.146	

Table 2 shows that the best agreement with the experimental value of the electromagnetic radius of the neutral kaon is achieved at $\delta = 0°$.

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