The Polarized Deep Inelastic Lepton-Nucleon Scattering with Charged Current at Future Colliders

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Abstract

The new ways the extraction of the polarized structure function from the measurable asymmetries of the processes DIS with charged current are proposed.

In QCD the spin of the nucleon is

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_q + L_g,$$

where $\Delta\Sigma$, Δg; $L_{q,g}$ are the contributions the spin of the quarks, the gluons and their orbital angular moments respectively.

The data all the experiments give what only about 1/3 the nucleon spin carry the quarks and antiquarks ($\Delta\Sigma = 0,35$ from COMPASS, $\Delta\Sigma = 0,33$ from HERMES) and $|\Delta g| \leq 0,3$ [1–3].

The orbital angular moments L_q, L_g can to be accessible through the measurements the generalized parton distributions in the exclusive processes of the deep virtual Compton scattering and meson production [2].

New information on the contributions of the quarks and the antiquarks can to obtain from the experiments DIS with charged current [4–7]

$$\nu(\overline{\nu}) + N \longrightarrow l^-(l^+) + X,$$

$$l^-(l^+) + N \xrightarrow{W} \nu(\overline{\nu}) + X.$$

The processes have the following advantages:

- Two independent polarized structure functions (SF) $g_1(x, Q^2)$ and $g_6(x, Q^2)$ for the longitudinal polarization of the target. The parity-conserving polarized SF $g_1 \sim (\Delta q + \Delta\overline{q})$ but the parity-violating polarized SF $g_6 \sim (\Delta q - \Delta\overline{q})$. This allows to obtain separately the contributions of the valence and the sea quarks as in the semi-inclusive DIS. However, in last case the knowledge the fragmentation functions is necessary, which now badly known.

- The neutrino have 100% polarization.

The experiments on the processes lN-DIS with charged current are planned on ep-colliders in the frame of the projects LHeC, EIC, eRHIC.

The neutrino experiments on the polarized targets are possible as the high-focusing neutrino beams can to obtain from the decays of the muons (the neutrino factories).

The information about the spin structure of the nucleon contain the polarized SF $g_1(x, Q^2)$ and $g_6(x, Q^2)$, which extract from the measurable polarized asymmetries.

The extraction the polarized SF from the asymmetries is not trivial task since in case the processes DIS with charged current have two independent polarized SF in contrary from the lN-DIS with the exchange of the photon. Therefore, this task requires the separate of the consideration.

In present work we propose the new ways the extraction of the polarized SF g_1 and g_6 from the asymmetries of the processes DIS with charged current.

Now we consider the DIS the (anti)neutrino on the polarized nucleons with charged current

$$\nu(\overline{\nu}) + N \dashrightarrow l^-(l^+) + X \qquad (l = e, \mu) \tag{1}$$

in Born approximation (fig.1)

The differential cross sections (1) are

$$d\sigma_{\nu,\overline{\nu}} = d\sigma^a_{\nu,\overline{\nu}} + d\sigma^p_{\nu,\overline{\nu}}, \tag{2}$$

where

$$d\sigma^a_{\nu,\overline{\nu}} = \frac{G^2}{(2\pi)^2}\frac{M}{p\cdot k}(L^{\alpha\beta} \pm L_5^{\alpha\beta})W^{\nu,\overline{\nu}}_{\alpha\beta}\frac{d^3k'}{k'_0},$$

$$W^{\nu,\overline{\nu}}_{\alpha\beta} = -g_{\alpha\beta}W^{\nu,\overline{\nu}}_1 + \frac{1}{M^2}p_\alpha p_\beta W^{\nu,\overline{\nu}}_2 - \frac{i}{2M^2}e_{\alpha\beta\rho\sigma}p^\rho q^\sigma W^{\nu,\overline{\nu}}_3 \tag{3}$$

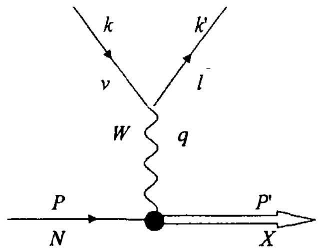

Figure 1. The process νN-DIS with charged current

and

$$d\sigma^{p}_{\nu,\overline{\nu}} = \frac{G^2}{(2\pi)^2}\frac{M}{p\cdot k}(L^{\alpha\beta} \pm L_5^{\alpha\beta})G^{\nu,\overline{\nu}}_{\alpha\beta}\frac{d^3k'}{k_0'}, \tag{4}$$

$$\begin{aligned} G^{\nu,\overline{\nu}}_{\alpha\beta} = &-\frac{i}{M}e_{\alpha\beta\rho\sigma}q^{\sigma}\left[M^2 s^{\sigma} G_1^{\nu,\overline{\nu}} + (p\cdot q s^{\sigma} - s\cdot q p^{\sigma})G_2^{\nu,\overline{\nu}}\right] - \\ &-\frac{i}{M}e_{\alpha\beta\rho\sigma}p^{\rho}s^{\sigma}G_3^{\nu,\overline{\nu}} + \frac{1}{M^2}(p_{\alpha}s_{\beta} + p_{\beta}s_{\alpha})G_4^{\nu,\overline{\nu}} + \\ &+\frac{2}{M^2}p_{\alpha}p_{\beta}(s\cdot q)G_5^{\nu,\overline{\nu}} + \frac{1}{M^2}g_{\alpha\beta}(s\cdot q)G_6^{\nu,\overline{\nu}}, \end{aligned}$$

$$\begin{aligned} L^{\alpha\beta}(k,k') &= k^{\alpha}k'^{\beta} + k'^{\alpha}k^{\beta} - g^{\alpha\beta}(kk'), \\ L_5^{\alpha\beta}(k,k') &= -ie^{\alpha\beta\rho\sigma}k_{\rho}k'_{\sigma}, \end{aligned}$$

G is Fermi constant, M is the mass of the nucleon, $q = k - k'$, s is the vector of the polarization of the nucleon.

$W^{\nu,\overline{\nu}}_{1,2,3}(\nu, q^2)$ and $G^{\nu,\overline{\nu}}_{1\ldots 6}(\nu, q^2)$ are the averaged and the polarized SF of the nucleon correspondingly.

$$\nu = p\cdot q/M.$$

The cross sections $\nu(\overline{\nu})N$-DIS on the unpolarized target are

$$\frac{d^2\sigma^{a}_{\nu,\overline{\nu}}}{dxdy} = \sigma_0\left[xy^2F_1^{\nu,\overline{\nu}}(x,Q^2) + (1-y)F_2^{\nu,\overline{\nu}}(x,Q^2) \pm xy\left(1-\frac{y}{2}\right)F_3^{\nu,\overline{\nu}}(x,Q^2)\right], \tag{5}$$

where

$$\sigma_0 = \frac{G^2}{\pi} ME; \qquad F_1^{\nu,\overline{\nu}} = MW_1^{\nu,\overline{\nu}}; \qquad F_{2,3}^{\nu,\overline{\nu}} = \nu W_{2,3}^{\nu,\overline{\nu}};$$

$$x = \frac{Q^2}{2M\nu}, \qquad y = \frac{\nu}{E}, \qquad Q^2 = -q^2,$$

E is the energy of the neutrino or the antineutrino.

For the longitudinal polarization of the nucleon ($s = \left(0, P_N \frac{\vec{k}}{|\vec{k}|}\right)$ in laboratory frame, P_N is the degree of the polarization) the cross sections (4) obtained in the form

$$\frac{d^2\sigma^p_{\nu,\overline{\nu}}}{dxdy} =$$
$$= P_N\sigma_0 \left[xy^2\left(1 + \frac{Mx}{E}\right) g_6^{\nu,\overline{\nu}}(x, Q^2) - \left(2 - 2y - \frac{Mxy}{E}\right) g_4^{\nu,\overline{\nu}}(x, Q^2) - \right.$$
$$-2\left(1 + \frac{Mx}{E}\right)\left(1 - y - \frac{Mxy}{2E}\right) g_5^{\nu,\overline{\nu}}(x, Q^2) \pm$$
$$\pm \left(xy\left(2 - y - \frac{Mxy}{E}\right) g_1^{\nu,\overline{\nu}}(x, Q^2) \right.$$
$$\left.\left. - \frac{2Mx^2y}{E} g_2^{\nu,\overline{\nu}}(x, Q^2) + \frac{Mxy}{E} g_3^{\nu,\overline{\nu}}(x, Q^2)\right)\right].$$

Here have introduced the dimensionless SF

$$g_1 = M^2\nu G_1, g_2 = M\nu^2 G_2, g_3 = \nu G_3, g_4 = \frac{\nu}{M} G_4, g_5 = \nu^2 G_5, g_6 = \frac{\nu}{M} G_6.$$

In deep inelastic region $\frac{M}{E} \ll 1$, therefore in this approximation have

$$\frac{d^2\sigma^p_{\nu,\overline{\nu}}}{dxdy} = P_N\sigma_0 \left[xy^2 g_6^{\nu,\overline{\nu}}(x, Q^2) - 2(1 - y)\left(g_4^{\nu,\overline{\nu}}(x, Q^2) + g_5^{\nu,\overline{\nu}}(x, Q^2)\right) \pm \right.$$
$$\left. \pm xy(2 - y) g_1^{\nu,\overline{\nu}}(x, Q^2)\right].$$

The interaction of the virtual boson with the quark conserve the helicity, when all the masses neglect, such what the hadron tensor $\mathrm{T}_{\alpha\beta} =$

$W_{\alpha\beta}+G_{\alpha\beta}$, if consider leading order, must disappear by the multiplication on the vector of the longitudinal polarization boson ε_α. Since ε_α can be represented as the linear combination p and q, this the condition implies $q^\alpha q^\beta \mathrm{T}_{\alpha\beta} = 0$. Then from (3) and (4) obtain

$$q^\alpha q^\beta \mathrm{T}_{\alpha\beta} = \nu\,(F_2 - 2xF_1) + 2(sq)\,(g_4 + g_5 + xg_6) = 0.$$

Therefore in leading order

$$g_4(x, Q^2) + g_5(x, Q^2) = -xg_6(x, Q^2).$$

With respect to this correlation the polarized part of the cross sections the processes (1) is

$$\frac{d^2\sigma^p_{\nu,\overline{\nu}}}{dxdy} = P_N x\sigma_0\Big(y_1^+ g_6^{\nu,\overline{\nu}}(x, Q^2) \pm y_1^- g_1^{\nu,\overline{\nu}}(x, Q^2)\Big), \tag{6}$$

where

$$y_1 = 1 - y, \quad y_1^\pm = 1 \pm y_1^2.$$

From (6) see, what in the case of the longitudinal polarization of the nucleon the cross sections $\nu(\overline{\nu})N$-DIS contain two independent polarized SF g_1 and g_6 in contrary from electromagnetic processes DIS, where only one SF g_1^γ.

The cross sections of the processes (1) DIS (anti)neutrino on the longitudinal target in accordance with (2), (5), (6) are

$$\frac{d^2\sigma_{\nu,\overline{\nu}}}{dxdy} = \sigma_0\Big[xy_1^+ F_1^{\nu,\overline{\nu}}(x, Q^2) \pm x\frac{1}{2}y_1^- F_3^{\nu,\overline{\nu}}(x, Q^2) + \\ + P_N x\Big(y_1^+ g_6^{\nu,\overline{\nu}}(x, Q^2) \pm y_1^- g_1^{\nu,\overline{\nu}}(x, Q^2)\Big)\Big]. \tag{7}$$

The polarized asymmetries for the processes (1) determine as the following combination of the cross sections (7):

$$A_{\nu,\overline{\nu}} = \frac{d^2\sigma_{\nu,\overline{\nu}}^{\downarrow\uparrow,\uparrow\uparrow} - d^2\sigma_{\nu,\overline{\nu}}^{\downarrow\downarrow,\uparrow\downarrow}}{d^2\sigma_{\nu,\overline{\nu}}^{\downarrow\uparrow,\uparrow\uparrow} + d^2\sigma_{\nu,\overline{\nu}}^{\downarrow\downarrow,\uparrow\downarrow}}. \tag{8}$$

The first arrow in (8) corresponds the helicity of the neutrino $\downarrow$ or antineutrino $\uparrow$, the second – the direction of the spin of the nucleon $\uparrow$

($P_N = 1$) and $\downarrow$ ($P_N = -1$). With the help (7) for the asymmetries obtain the expressions through SF

$$A_{\nu,\overline{\nu}}(x,Q^2) = \frac{y_1^+ g_6^{\nu,\overline{\nu}}(x,Q^2) \pm y_1^- g_1^{\nu,\overline{\nu}}(x,Q^2)}{y_1^+ F_1^{\nu,\overline{\nu}}(x,Q^2) \pm \frac{y_1^-}{2} F_3^{\nu,\overline{\nu}}(x,Q^2)}, \tag{9}$$

The extraction the spin-dependent SF in DIS with charged weak current is non-trivial task, since the asymmetries (9) contain two independent SF g_1 and g_6.

Obviously, what for the extraction SF $g_{1,6}$ from (9) necessary the complementary correlation connecting these functions. Here propose the new ways of the extraction SF g_1 and g_6 from the asymmetries only the processes with the participation of the neutrino and applicable for the any target. For that consider the processes $l^{\pm}N$-DIS with charged weak current inverse to the neutrino reactions (1)

$$l^-(l^+) + N \to \nu(\overline{\nu}) + X, \tag{10}$$

where $l = e, \mu$.

For the SF the processes (1) and (10) are following conformities in leading order

$$\begin{aligned} g_{1,6}^{\overline{\nu}}(x,Q^2) &= g_{1,6}^{l^-}(x,Q^2), \qquad & g_{1,6}^{\nu}(x,Q^2) &= g_{1,6}^{l^+}(x,Q^2),\\ F_{1,3}^{\overline{\nu}}(x,Q^2) &= F_{1,3}^{l^-}(x,Q^2), \qquad & F_{1,3}^{\nu}(x,Q^2) &= F_{1,3}^{l^+}(x,Q^2). \end{aligned} \tag{11}$$

The measurable asymmetries $A_{l^-,l^+}(x,Q^2)$ the processes (10) note in the form [8]

$$A_{l^-,l^+}(x,Q^2) = \frac{y_1^+ g_6^{l^-,l^+}(x,Q^2) \pm y_1^- g_1^{l^-,l^+}(x,Q^2)}{y_1^+ F_1^{l^-,l^+}(x,Q^2) \pm \frac{y_1^-}{2} F_3^{l^-,l^+}(x,Q^2)}.$$

With respect to the correlation (11) asymmetries $A_{l^-,l^+}(x,Q^2)$ present through SF $g_1^{\nu,\overline{\nu}}(x,Q^2)$ and $g_6^{\nu,\overline{\nu}}(x,Q^2)$

$$A_{l^-,l^+}(x,Q^2) = \frac{y_1^+ g_6^{\overline{\nu},\nu}(x,Q^2) \pm y_1^- g_1^{\overline{\nu},\nu}(x,Q^2)}{y_1^+ F_1^{\overline{\nu},\nu}(x,Q^2) \pm \frac{y_1^-}{2} F_3^{\overline{\nu},\nu}(x,Q^2)}. \tag{12}$$

The correlations (9), (12) allow to extract SF $g_1^{\nu,\overline{\nu}}(x,Q^2)$ and $g_6^{\nu,\overline{\nu}}(x,Q^2)$, since $F_1^{\nu,\overline{\nu}}(x,Q^2)$, $F_3^{\nu,\overline{\nu}}(x,Q^2)$ known from neutrino experiments on unpolarized targets. Thus obtain

$$
\begin{aligned}
g_1^{\nu,\overline{\nu}}(x,Q^2) = \frac{1}{2}\Bigg[& A_{\nu,\overline{\nu}}(x,Q^2)\left(\frac{y_1^+}{y_1^-}F_1^{\nu,\overline{\nu}}(x,Q^2) \pm \frac{F_3^{\nu,\overline{\nu}}(x,Q^2)}{2}\right) \mp \\
& \mp A_{l^+,l^-}(x,Q^2)\left(\frac{y_1^+}{y_1^-}F_1^{\nu,\overline{\nu}}(x,Q^2) \mp \frac{F_3^{\nu,\overline{\nu}}(x,Q^2)}{2}\right)\Bigg], \\
g_6^{\nu,\overline{\nu}}(x,Q^2) = \frac{1}{2}\Bigg[& A_{\nu,\overline{\nu}}(x,Q^2)\left(F_1^{\nu,\overline{\nu}}(x,Q^2) \pm \frac{y_1^-}{2y_1^+}F_3^{\nu,\overline{\nu}}(x,Q^2)\right) + \\
& + A_{l^+,l^-}(x,Q^2)\left(F_1^{\nu,\overline{\nu}}(x,Q^2) \mp \frac{y_1^-}{2y_1^+}F_3^{\nu,\overline{\nu}}(x,Q^2)\right)\Bigg].
\end{aligned}
\tag{13}
$$

The quantity $y_1^+ \neq 0$ by the any meaning scaling variable y in the limit it the change in the region DIS

$$
0 < y \leq 1.
$$

The $y_1^- = 0$ by $y = 0$, but this meaning not enter in the kinematical region variable y.

Note, what in proposed approach can to determine SF $g_{1,6}^{l^-}$ and $g_{1,6}^{l^+}$ the processes (10) with help asymmetries $A_{l^-,l^+}(x,Q^2)$, (9) and (10). In this case for $g_1^{l^-,l^+}$, $g_6^{l^-,l^+}$ we obtain

$$
\begin{aligned}
g_1^{l^-,l^+}(x,Q^2) = \frac{1}{2}\Bigg[& \pm A_{l^-,l^+}(x,Q^2)\left(\frac{y_1^+}{y_1^-}F_1^{l^-,l^+}(x,Q^2) \pm \frac{F_3^{l^-,l^+}(x,Q^2)}{2}\right) \mp \\
& \mp A_{\overline{\nu},\nu}(x,Q^2)\left(\frac{y_1^+}{y_1^-}F_1^{l^-,l^+}(x,Q^2) \mp F_3^{l^-,l^+}(x,Q^2)\right)\Bigg], \\
g_6^{l^-,l^+}(x,Q^2) = \frac{1}{2}\Bigg[& A_{l^-,l^+}(x,Q^2)\left(F_1^{l^-,l^+}(x,Q^2) \pm \frac{y_1^-}{2y_1^+}F_3^{l^-,l^+}(x,Q^2)\right) + \\
& + A_{\overline{\nu},\nu}(x,Q^2)\left(F_1^{l^-,l^+}(x,Q^2) \mp \frac{y_1^-}{2y_1^+}F_3^{l^-,l^+}(x,Q^2)\right)\Bigg].
\end{aligned}
\tag{14}
$$

Thus for the finding SF on the formula (13), (14) enough data only the experiments (1) and (10) the DIS with charged current with the participation of the neutrino and this method applicable for the any target in physical region $0 < y < 1$.

The another possibility of the extraction SF g_1 and g_6 consider on the example lN-DIS with charged current. Now we introduce the asymmetries as the constructions simultaneously the cross sections $l^{\pm}N$-DIS:

$$\begin{aligned} A_{\pm}(x,Q^2) &= \frac{\left(d^2\sigma_{l^-}^{\downarrow\uparrow} \pm d^2\sigma_{l^+}^{\uparrow\uparrow}\right) - \left(d^2\sigma_{l^-}^{\downarrow\downarrow} \pm d^2\sigma_{l^+}^{\uparrow\downarrow}\right)}{\left(d^2\sigma_{l^-}^{\downarrow\uparrow} \pm d^2\sigma_{l^+}^{\uparrow\uparrow}\right) + \left(d^2\sigma_{l^-}^{\downarrow\downarrow} \pm d^2\sigma_{l^+}^{\uparrow\downarrow}\right)} = \\ &= \frac{y_1^+\left[g_6^{l^-}(x,Q^2) \pm g_6^{l^+}(x,Q^2)\right] + y_1^-\left[g_1^{l^-}(x,Q^2) \mp g_1^{l^+}(x,Q^2)\right]}{y_1^+\left[F_1^{l^-}(x,Q^2) \pm F_1^{l^+}(x,Q^2)\right] + \frac{y_1^-}{2}\left[F_3^{l^-}(x,Q^2) \mp F_3^{l^+}(x,Q^2)\right]}. \end{aligned} \tag{15}$$

By small y will $y_1^- \ll y_1^+$ and from (15) follows

$$A_{\pm}(x,Q^2) = \frac{g_6^{l^-}(x,Q^2) \pm g_6^{l^+}(x,Q^2)}{F_1^{l^-}(x,Q^2) \pm F_1^{l^+}(x,Q^2)}. \tag{16}$$

Then from (16) obtain

$$\begin{aligned} g_6^{l^-}(x,Q^2) &= \frac{1}{2}\Big[A_+(x,Q^2)\Big(F_1^{l^-}(x,Q^2) + F_1^{l+}(x,Q^2)\Big) + \\ &\qquad + A_-(x,Q^2)\Big(F_1^{l^-}(x,Q^2) - F_1^{l^+}(x,Q^2)\Big)\Big], \\ g_6^{l^+}(x,Q^2) &= \frac{1}{2}\Big[A_+(x,Q^2)\Big(F_1^{l^-}(x,Q^2) + F_1^{l^+}(x,Q^2)\Big) - \\ &\qquad - A_-(x,Q^2)\Big(F_1^{l^-}(x,Q^2) - F_1^{l^+}(x,Q^2)\Big)\Big]. \end{aligned} \tag{17}$$

From the asymmetries

$$A_{l^-,l^+}(x,Q^2) = \frac{y_1^+ g_6^{l^-,l^+}(x,Q^2) \pm y_1^- g_1^{l^-,l^+}(x,Q^2)}{y_1^+ F_1^{l^-,l^+}(x,Q^2) \pm \frac{y_1^-}{2} F_3^{l^-,l^+}(x,Q^2)}$$

by known $g_6^{l^-}(x,Q^2)$, $g_6^{l^+}(x,Q^2)$ (see (17)) extract SF $g_1^{l^-}, g_1^{l^+}$:

$$\begin{aligned} g_1^{l^-}(x,Q^2) &= A_{l^-}(x,Q^2)\left[\frac{y_1^+}{y_1^-}F_1^{l^-}(x,Q^2) + \frac{1}{2}F_3^{l^-}(x,Q^2)\right] - \frac{y_1^+}{y_1^-}g_6^{l^-}(x,Q^2), \\ g_1^{l^+}(x,Q^2) &= \frac{y_1^+}{y_1^-}g_6^{l^+}(x,Q^2) - A_{l^+}(x,Q^2)\left[\frac{y_1^+}{y_1^-}F_1^{l^+}(x,Q^2) - \frac{1}{2}F_3^{l^+}(x,Q^2)\right]. \end{aligned}$$

Analogously the given scheme is applied for $\nu(\overline{\nu})N$-DIS with charged current.

THE CONCLUSIONS

The methods of the extraction the polarized SF $g_1(x, Q^2)$ and $g_6(x, Q^2)$ from the measurable asymmetries DIS with charged current have proposed:

1) on the base the joint application the asymmetries $A_{\nu,\overline{\nu}}(x, Q^2)$ and $A_{l^-,l^+}(x, Q^2)$;

2) with the help asymmetries the same the process, for example, $A_{l^-,l^+}(x, Q^2)$ and $A_{\pm}(x, Q^2)$ for $l^{\pm}N$-DIS (CC) (analogously for the neutrino processes DIS).

Both methods are applicable for the any target.

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