STUDY OF TARGET MASS EFFECTS BASED ON THE JOST-LEHMANN-DYSON INTEGRAL REPRESENTATION

*V.I. Lashkevich and O.P. Solovtsova**

International Center for Advanced Studies, GSTU, Belarus

Abstract

We develop a method for taking target mass effects into account for the structure functions of inelastic lepton-hadron scattering using analytic moments in the variable s instead of the well-known Nachtmann variable ξ and Bjorken variable x. We find new expressions for the structure functions F_1, F_2, and F_3 that depend on the target mass and agree with the spectral property. We demonstrate that these new expressions, which take mass corrections into account for the structure functions, lead to results that differ significantly from the results obtained both using the standard ξ method and using the method suggested by Steffens-Melnitchouk, especially for values Q^2 of the order of several GeV^2 and for large values of x.

1 Introduction

The operator product expansion is widely used in the theoretical analysis of inelastic lepton-hadron scattering, where it is applied to products of currents and leads to a twisted structure of the representation for the moments of the structure functions. For large values of the momentum transfer in this representation, the main contribution is determined by the term with the smallest twist, which is the difference between the dimension of the operator and its Lorentz spin. Using the renormalization group method [1] allows determining the Q^2-evolution law

*E-mail: solovtsova@gstu.gomel.by, olsol@theor.jinr.ru

for the momenta. In this approach, originally oriented at the Bjorken limit of high energy and transfer momenta, several general properties of the spectral functions, which follow from the fundamental principles of the theory, remain hidden.

The operator expansion method was first used to study target mass effects in [2]. Such an approach for considering mass corrections became known as the ξ-scaling method. The expressions for the structure functions obtained by this method have a difficulty arising from the violation of the spectral condition. It hence became a problem to describe the structure functions as the Bjorken variable x tends to unity. This problem has been widely discussed in the literature ever since its appearance [3, 4, 5, 6, 7, 8, 9, 10, 11] (also see [12]).

It was shown in [13] that this problem is similar to the problem that appears for an invariant charge in quantum chromodynamics, when the violation of the general principles of the theory, which are reflected in the Källén–Lehmann representation, leads to unphysical singularities. A solution of this problem was proposed in the Shirkov-Solovtsov analytic approach [14, 15], which was later generalized to more complicated objects, such as structure functions (see [16, 17]). For the inelastic lepton-hadron scattering process, the general principles of the theory are accumulated in the Jost-Lehmann-Dyson (JLD) integral representation. This representation was proposed for the symmetric case in [18] and for the general case in [19]. The proof of the JLD representation is based on the most general principles of the theory, such as the covariance, Hermiticity, spectrality, and causality (see [1]; certain mathematical aspects related to the JLD representation were also considered in [20, 21]).

Papers by Bogoliubov Vladimirov, and Tavkhelidze [22, 23] were devoted to using the JLD representation to study the automodel asymptotic behavior of the structure functions. The JLD representation was also used in [24]. It was shown in [13] that by using the JLD representation, the natural scaling variable is a new variable s, which leads to the moments $\mathcal{M}_n(Q^2)$ that are analytic functions.[1]

In this case, the spectral property for the structure functions is satisfied automatically, and no problem arises in the limit as the Bjorken variable x tends to unity.

Our purpose here is to use the integral JLD representation to study target mass effects. In Sec. 2, we give the basic relations and describe the essence of the method. In Sec. 3, we consider the nucleon structure functions F_1, F_2 and F_3 and compare our expressions with other approaches. In the conclusion, we

[1] Unlike the variable η introduced in [24], the variable s ranges from zero to one.

discuss the results of the analysis.

2 Method

For any structure function $W(\nu, Q^2)$ that satisfies the covariance, spectrality, realness, symmetry, and causality conditions, there exists a unique real-valued tempered distribution $\psi(\mathbf{u}, \lambda^2)$ such that the JLD integral representation holds. This representation can be written in the nucleon rest frame as [22, 23]

$$W(\nu, Q^2) = \varepsilon(q_0) \int d\mathbf{u} d\lambda^2 \delta \left[q_0^2 - (M\mathbf{u} - \mathbf{q})^2 - \lambda^2\right] \psi(\mathbf{u}, \lambda^2) . \quad (1)$$

The support of the function $\psi(\mathbf{u}, \lambda^2)$ is in the set

$$\rho = |\mathbf{u}| \leq 1 , \qquad \lambda^2 \geq M^2 \left(1 - \sqrt{1 - \rho^2}\right)^2 . \quad (2)$$

We use the standard notation [25]: $Q^2 = -q^2$, $\nu = P \cdot q$, and $M = \sqrt{P^2}$ is the nucleon mass.

We note that in addition to the JLD representation, the integral Deser-Hilbert-Sudarshan (DHS) representation, which is simpler in form, is also discussed in the literature [26]. The simpler form naturally makes the DHS representation easier to use, but its status in quantum field theory is not as clearcut as that of the JLD representation (see [29, 30, 31]).

It was shown in [13] that the natural scaling variable in representation (1) is the variable

$$s = \sqrt{\frac{Q^4/4 + M^2 Q^2}{\nu^2 + M^2 Q^2}} = x \sqrt{\frac{1 + 4\epsilon}{1 + 4\epsilon x^2}} , \quad \epsilon \equiv \frac{M^2}{Q^2} , \quad (3)$$

which accumulates the root structure from the argument of the δ-function. The Solovtsov variable s depends on the target (nucleon) mass and differs from not only the Bjorken variable $x = Q^2/2\nu$ but also the Nachtmann variable [32]

$$\xi = \frac{2x}{1 + \sqrt{1 + 4\epsilon x^2}} , \quad (4)$$

which in the parton interpretation comes from the $\delta\left[(\xi P + q)^2\right]$. In the physical region of the process, the variable s is used just like the variable x, in the range from zero to one.

The s moments of the structure functions were introduced in [13] using the variable s,

$$\mathcal{M}_n(Q^2) = \frac{1}{(1+4M^2/Q^2)^{(n-1)/2}} \int_0^1 ds\, s^{n-2} W(\nu, Q^2)\,, \tag{5}$$

for which Eq. (1) implies the representation

$$\mathcal{M}_n(Q^2) = \left(Q^2\right)^{n-1} \int\limits_0^\infty d\sigma \frac{m_n(\sigma)}{(\sigma+Q^2)^n}\,, \tag{6}$$

where $m_n(\sigma)$ is the weight function for the s moments,

$$m_n(\sigma) = \frac{1}{n} \int_0^1 d\rho\, \rho^{n+1}\, \theta\left(\sigma - \sigma_{\min}\right) \psi(\rho, \sigma - M^2\rho^2)\,, \tag{7}$$

$$\sigma_{\min} = 2\, M^2 \left(1 - \sqrt{1-\rho^2}\right)\,.$$

It follows from representation (6) that the function $\mathcal{M}_n(Q^2)$ is analytic in the Q^2 plane cut along the negative real half-axis, i.e., it is of the Källén–Lehmann analytic type. The relation between the analytic moments of $\mathcal{M}_n(Q^2)$ and the operator product expansion, in which the tensor structure of the matrix elements of the operators in the nucleon states must be fixed by the condition of their orthogonality to the nucleon momentum, was established in [16].

The relation between s moments $\mathcal{M}_n(Q^2)$ and the usual x moments

$$M_n(Q^2) = \int_0^1 dx\, x^{n-2}\, W(\nu, Q^2) \tag{8}$$

is [16]

$$\mathcal{M}_n(Q^2) = \frac{1}{\Gamma\left[\dfrac{n+1}{2}\right]} \sum_{k=0}^{\infty} \frac{\Gamma[k+(n+1)/2]}{k!} \left(-\frac{4M^2}{Q^2}\right)^k M_{n+2k}(Q^2)\,. \tag{9}$$

In the asymptotic region corresponding to large values of Q^2, we can neglect polynomial corrections of the form $1/(Q^2)^n$, and the s and x moments coincide.

One can write JLD representation in the form [13]

$$W(x, Q^2) = \int_0^1 d\beta \int_0^\infty d\sigma\, \delta\left[\, \sigma + Q^2 + \right. \tag{10}$$

$$\left. 2M^2\left(1 - \sqrt{1-\beta^2}\right) - \frac{\beta}{s} Q^2 \sqrt{1+4\epsilon}\,\right] H(\beta, \sigma)\,,$$

where the new weight function $H(\beta,\sigma))$ is supported in the domain $\{0 < \beta < 1; \sigma > 0\}$ and can be expressed in terms of the original function $\psi(\rho,\lambda^2)$ as

$$\begin{aligned} H(\beta,\sigma) &= \mathcal{H}(\beta,\sigma+2M^2\left(1-\sqrt{1-\beta^2}\right)), \\ \mathcal{H}(\beta,\sigma) &= \int_\beta^1 d\rho\,\rho\,\theta\left[\sigma-2M^2\left(1-\sqrt{1-\rho^2}\right)\right] \\ &\times \tilde{\psi}(\rho,\sigma-2M^2\left(1-\sqrt{1-\rho^2}\right)), \\ \tilde{\psi}(\rho,s) &= \psi(\rho,s+\lambda^2_{\rm min}), \quad \lambda^2_{\rm min}=M^2\left(1-\sqrt{1-\rho^2}\right)^2 . \end{aligned} \tag{11}$$

The relation between the functions $H(\beta,\sigma)$ and $m_n(\sigma)$ has the form

$$m_n(\sigma) = \int_0^1 d\beta\,\beta^{n-1}\,\tilde{H}(\beta,\sigma)\,, \tag{12}$$

where $\tilde{H}(\beta,\sigma) = H(\beta,\sigma-2M^2(1-\sqrt{1-\beta^2}))$.

Therefore, the function $m_n(\sigma)$ is the moment of the weight $H(\beta,\sigma)$ and $H(\beta,\sigma)$ can hence be reconstructed using the Mellin transform

$$\tilde{H}(\beta,\sigma) = \frac{1}{2\pi i}\int\limits_{\gamma-i\infty}^{\gamma+i\infty} dn\,\beta^{-n}\,m_n(\sigma)\,. \tag{13}$$

The representation [13]

$$\mathcal{F}(x,Q^2) = \int_x^1 dy\,H\left[y,\left(\frac{y}{x}-1\right)Q^2\right]. \tag{14}$$

holds for the function $\mathcal{F}(x,Q^2)$, which corresponds to the structure function $W(x,Q^2)$ if we can neglect the dependence on the target mass.

The parton distribution $F(x)$ is obtained from the function $\mathcal{F}(x,Q^2)$ in $Q^2 \to \infty$. We let $H(x)$ denote the limit of the weight function $H(x,\sigma)$ as the second argument tends to infinity. Then (14) implies

$$F(x) = \int_x^1 dy\,H\,(y)\,, \tag{15}$$

whence it follows that $H(x) = -dF(x)/dx$ in the Bjorken limit.

Therefore, knowing the expression for the weight function $m_n(\sigma)$, we can use the method described above to find the structure function $W(x,Q^2)$. This is clearly a difficult problem in the general case: solving it is essentially equivalent to finding an exact solution of (1) Eq. (2.1). Here, to find the weight function $m_n(\sigma)$, we use expressions for the structure functions obtained in the leading twist approximation [2]. Then, using a procedure similar to the Shirkov-Solovtsov analytic approach [15], we find expressions for the structure functions that agree with the spectral condition. The calculations for the case of scalar currents was done earlier (see [13, 27, 28]). In this paper we apply the method to find new expressions that depend on the target mass and agree with the spectral property for another structure function.

According to the method we can represent a structure function W as

$$W(x,Q^2) = W_0(x,Q^2) + w(x,Q^2)\,. \tag{16}$$

For $W_0(x,Q^2)$ it follows from JLD representation (10) that

$$W_0(x,Q^2) = \int_0^1 d\beta\,\theta[f(\beta;x,\epsilon)]\,H_0(\beta)\,, \tag{17}$$

where

$$f(\beta;x,\epsilon) = \frac{\beta}{s}\sqrt{1+4\epsilon} - 1 - 2\epsilon(1-\sqrt{1-\beta^2})\,. \tag{18}$$

We find the roots of the equation $f(\beta;x,\epsilon)=0$. If $x > \tilde{x} \equiv 1/\sqrt{1+4\epsilon^2}$, then we have the two roots

$$\beta_\pm = \frac{x\sqrt{1+4\epsilon x^2}}{1+4\epsilon x^2+4\epsilon^2x^2}\left(1+2\epsilon \pm 2\epsilon\sqrt{\frac{1-x^2}{1+4\epsilon x^2}}\right)\,, \tag{19}$$

if $x < \tilde{x}$, then there is one root β_-.

Substituting $H_0(\beta) = -dF(\beta)/d\beta$ in (17), we obtain

$$W_0(x,Q^2) = \begin{cases} F(\beta_-) - F(1)\,, & 0 \le x < \tilde{x}\,, \\ F(\beta_-) - F(\beta_+)\,, & \tilde{x} \le x \le 1\,. \end{cases} \tag{20}$$

The spectral property of the function $W_0(x,Q^2)$ (that it vanishes as $x \to 1$) and its continuity for $x = \tilde{x}$ follow because $\beta_-(x=1) = \beta_+(x=1)$ and $\beta_-(\tilde{x}) = 1$.

For the function $w(x,Q^2)$, we obtain

$$w(x,Q^2) = \int_0^1 d\beta\,\theta[f(\beta;x,\epsilon)]\,\theta[g(\beta;x,\epsilon)]\phi(\beta;x,\epsilon)\,, \tag{21}$$

where $f(\beta; x, \epsilon)$ is defined by (18), $g(\beta; x, \epsilon) = \tau - \beta^2$,

$$\tau \equiv \tau(\beta; x, \epsilon) = \left[(\beta/s)\sqrt{1+4\epsilon} - 1\right] / \epsilon \,. \tag{22}$$

The expression for $\phi(\beta; x, \epsilon)$ depends on a concrete structure function and can be found in [28].

The equation $\tau(\beta; x, \epsilon) = 1$ has a root

$$\beta_\tau = \frac{(1+\epsilon)s}{\sqrt{1+4\epsilon}} \,.$$

The equation $g(\beta; x, \epsilon) = 0$ has the two roots

$$\xi_\pm = \frac{\sqrt{1+4\epsilon x^2} \pm 1}{2\epsilon x} \,,$$

where the root ξ_- is equal to Nachtmann variable (4). We note that for $\epsilon \le 1/2$, the integration in (21) is performed from β_- β_τ for $x \in (0, \sqrt{\bar{y}}\,]$, where

$$\bar{y} = \frac{3}{4(1-\epsilon+\epsilon^2)} \,. \tag{23}$$

For $x > \sqrt{\bar{y}}$ the function $w(x, Q^2)$ in (21) is identically zero. For $\epsilon > 1/2$ the integration in (21) for $x \in (0, \sqrt{\bar{y}}\,]$ is performed in the same limits as for $\epsilon \le 1/2$. For $x \in [\sqrt{\bar{y}}\,, 1)$ the integration $w(x, Q^2)$ is not identically zero, and we integrate from β_- β_+.

Therefore, the sought expression for the structure function W is sum 16) in which the first term is given by expression (20) and the additional term is given by (21). To calculate, we must know the form of the parton distribution. In our analysis, we use the model function

$$F(x) = \sqrt{x}(1-x)^3, \tag{24}$$

which is used in processing experimental data (see, e.g., [33]).

3 Structure functions of the nucleon

The cross section of inelastic lepton-hadron scattering in the case of a unpolarized nucleon is determined by the hadron tensor $W_{\mu\nu}$, which is parameterized by the nucleon structure functions F_1, F_2, and F_3 as

$$W_{\mu\nu}(q,P) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) + \frac{1}{(q\cdot P)}\left(P_\mu - q_\mu \frac{(q\cdot P)}{q^2}\right) \tag{25}$$
$$\times \left(P_\nu - q_\nu \frac{(q\cdot P)}{q^2}\right) F_2(x,Q^2) - \frac{i}{2(q\cdot P)}\varepsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta F_3(x,Q^2)\,.$$

Let us begin by considering the function $F_3(x,Q^2)$, for which the analysis is less cumbersome. We represent the function $xF_3(x,Q^2)$ in the form

$$xF_3(x,Q^2) \equiv W_3(x,Q^2) = W_3^{(0)}(x,Q^2) + w_3(x,Q^2)\,,$$

where $W_3^{(0)}(x,Q^2)$ is expressed in terms of the parton distribution according to (20, and the final expression for $w_3(x,Q^2)$) has the form

$$w_3(x,Q^2) = 2\epsilon\eta^3\int_{z_-}^{1} dz\,\frac{F(z)}{z^2} - \frac{\epsilon\eta}{4}\Big[(\eta^2\epsilon^2 z_-^4 + (-1+2\eta^2\epsilon)z_-^2 + \eta^2)$$
$$\times\; F^{(1)}(z_-) + (\eta^2\epsilon^2 z_-^4 + 3(1+2\eta^2\epsilon)z_-^2 + 5\eta^2)\,\frac{F(z_-)}{z_-}\Big]\,, \tag{26}$$

where

$$z_- \equiv z(\beta_-) = \sqrt{\frac{1}{\epsilon}\left(\frac{\beta_-}{\eta} - 1\right)}\,, \tag{27}$$

and β_- is defined by (19). We note that the function $w_3(x,Q^2)$ contributes nontrivially for $x \in (0,\sqrt{\bar{y}})$, where $\bar{y}$ is given by formula (23).

The results of the calculations are presented in Fig. 1, which shows the behavior of $xF_3(x,Q^2)$ as a function of x for $Q^2 = 2$ GeV^2. The solid line shows the results of our calculations using formulas (20 and (26). The dash-dotted line separately shows the contribution of the function $w_3(x,Q^2)$. The dashed line corresponds to the ξ-scaling method [2]]. We can see from1 that both the solid line and the dashed line are close to the dotted line for values up to $x \simeq 0.5$, i.e. the mass corrections calculated by the two methods are insignificant. For $x > 0.5$, mass effects become significant, but the results of our calculations virtually coincide with those of the ξ-scaling method up to $x \simeq 0.9$. For x close to unity, the behavior of the curves becomes substantially different.

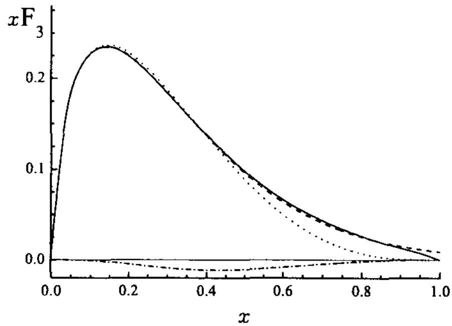

Figure 1. The behavior of the structure function $xF_3(x, Q^2)$ for Q^2=2 GeV^2. The solid line is the result of calculations using analytic formulas, the dashed line is the result of the ξ-scaling method [2], the dash-dotted line is the contribution of the function $w_3(x, Q^2)$, and the dotted line is the parton distribution.

To calculate the structure function F_2, we begin with the expression from the ξ-scaling method, which in this case has the form [9]

$$\begin{aligned} F_2(x, Q^2) &= \frac{x^2}{\xi^2(1+4x^2\epsilon)^{3/2}} F(\xi) + \frac{6\epsilon\, x^3}{(1+4x^2\epsilon)^2} \int_\xi^1 d\xi' \frac{F(\xi')}{\xi'^2} \\ &+ \frac{12\epsilon^2\, x^4}{(1+4x^2\epsilon)^{5/2}} \int_\xi^1 d\xi' \int_{\xi'}^1 d\xi'' \frac{F(\xi'')}{\xi''^2} \,. \end{aligned} \tag{28}$$

The corresponding x moments can be written as

$$M_n(Q^2) = n(n-1) \int_0^1 dz \, \frac{z^{n-2}\, G(z)}{(1-\epsilon z^2)^{n+1}} \,. \tag{29}$$

where

$$G(z) = \int_z^1 dy \int_y^1 du \, \frac{F(u)}{u^2} \,. \tag{30}$$

We use relation (9) between the analytic and the x moments. Then we obtain

$$\begin{aligned} \mathcal{M}_n(Q^2) &= \int_0^1 dz \, \frac{z^{n-2}\, G(z)}{(1+\epsilon z^2)^{n+5}} \left[n(n-1)((1-\epsilon z^2)^4 \right. \\ &- \left. 4(5n+1)\epsilon z^2 (1-\epsilon z^2)^2 + 32\epsilon^2 z^4) \right] \,. \end{aligned} \tag{31}$$

We represent $F_2(x, Q^2)$ as a sum in which the first summand is expressed in terms of the parton distribution according to (20). We find that

$$w_2(x, Q^2) = \frac{\epsilon\,\eta}{16}\left[3\int_{z_-}^{1} dz\,\phi_1(z)\int_{z}^{1} dy\,y\,F(y) + \int_{z_-}^{1} dz\,\phi_2(z)\,F(z)\right] \quad (32)$$
$$+ \frac{\epsilon\,\eta}{16}\left[\phi_3(z)F(z) + \phi_4(z)F'(z) + \phi_5(z)F''(z)\right]\Big|_{z_-}^{1} .$$

Expressions for ϕ_k can be found in [28].

Recently, a new method for taking mass corrections into account for the structure functions was presented in [11]. In this method, the standard parton distribution moments A_n are replaced with the moments

$$A_n^{(\mathrm{SM})} \equiv \int_0^{\xi_0} d\xi\,\xi^n\,F(\xi, \xi_0)\,, \qquad (33)$$

where $\xi_0 \equiv \xi(x = 1) = 2/(1 + \sqrt{1+4\epsilon}) < 1$, $F(\xi, \xi_0)$ depends on the two variables ξ and ξ_0. The parton distribution function is chosen in the form $F(\xi, \xi_0) = \xi_0^{-a-b-1}\,\xi^a\,(\xi_0 - \xi)^b$. As a result, the structure functions do indeed tend to zero as $x \to 1$, and the threshold problem is resolved. But the moment $A_n^{(\mathrm{SM})}$ now depends on Q^2, which contradicts the operator product expansion and makes the parton interpretation problematic. Below, we compare our results with the results obtained for the structure function F_2 in [11].

Figure 2 shows the behavior of $F_2(x, Q^2)$ as a function of x for Q^2 =2 GeV^2. As in the preceding figures, the solid line corresponds to the structure function obtained using formulas 20) and (32), the dashed line corresponds to the ξ-scaling method, the dash-dotted line corresponds to the result obtained in [11] using moments (33, and the dotted line corresponds to the parton distribution.

We see from Fig. 2 that for Q^2 = 2 GeV^2 and $x > 0.2$, there is a significant difference between the results obtained by the different methods. The structure function that we obtained and the function obtained in [11] tend to zero as $x \to 1$. But the function in [11] tends to zero much faster than the function we obtained using analytic moments. Our results are closer to the result of the ξ-scaling method except for values of x close to unity. One can see from Fig. 3 that for Q^2 = 10 GeV^2 mass corrections become insignificant. In our calculations of the structure function $F_1(x, Q^2)$, we use the relation between the structure functions [2]

$$6xF_1(x, Q^2) = \left(1 + 4x^2\epsilon\right)F_2(x, Q^2) + \frac{2x^2F(\xi)}{\xi^2\sqrt{1+4\epsilon x^2}}\,. \qquad (34)$$

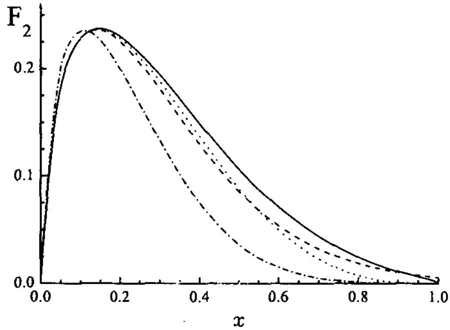

Figure 2. The behavior of the structure function $F_2(x, Q^2)$ for Q^2=2 GeV^2. The solid line is the result of calculations using analytic formulas, the dashed line is the result of the ξ-scaling method [2], the dash-dotted line is the result in [11], and the dotted line is the parton distribution.

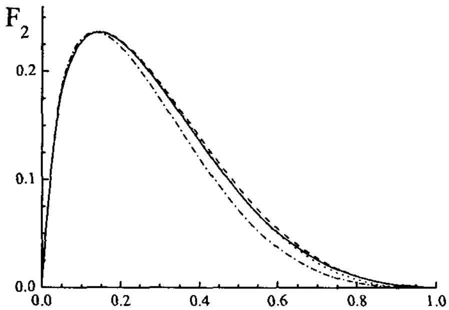

Figure 3. The behavior of the structure function $F_2(x, Q^2)$ for $Q^2 = 10$ GeV^2. The notation is the same as in Fig. 2.

We note that this expression in the limit $\epsilon \to 0$ gives the well-known Callan-Gross relation $2xF_1(x) = F_2(x)$ [34]. Furthermore, the second term in the right-hand side of (34) is twice the expression for the structure function in the scalar case, which simplifies the calculations. Using expression (28) and repeating calculations analogous to the ones above, we obtain the expression for $xF_1(x, Q^2)$.

The results of calculating $xF_1(x, Q^2)$ are presented in Figs. 4 and 5 (the notation is the same as in Fig.2). Figure 4 shows that mass corrections, just as for the structure function $F_2(x, Q^2)$, are significant for $x > 0.2$ but that their difference is not very large except for large values of x ($x > 0.8$), where the values of the functions become small. To show the difference in this region, Fig. 5 shows the behavior of the structure function $xF_1(x, Q^2)$ for Q^2= 5 GeV^2 and large values of x.

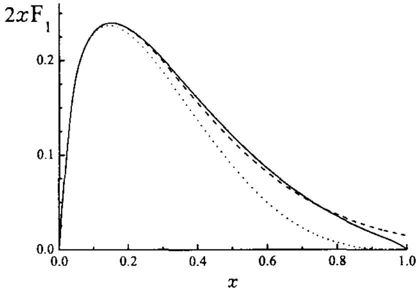

Figure 4. The behavior of the structure function $xF_1(x, Q^2)$ for Q^2=2 GeV^2. The notation is the same as in Fig. 2.

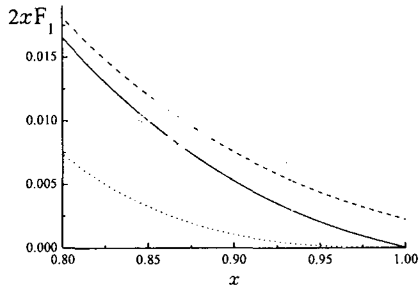

Figure 5. The structure function $xF_1(x, Q^2)$ for large values of x and Q^2= 5 GeV^2.

Therefore, the most significant difference between the structure functions obtained by the ξ-scaling method and using analytic moments is evident for large values of x. It is known (see, e.g., [9, 33, 35, 36, 37, 38, 39]) that target mass corrections (relating to "kinematic" contributions) influence on the dependence of the higher-twist contribution, which is connected with the process dynamics ("dynamical" contribution).

Processing experimental data (see, e.g., [35]) implies that the higher-twist contributions can increases sharply for large values of the Bjorken variable x. Therefore, it is reasonable to expect that using analytic moments will significantly affect the dependence of the higher-twist contribution on x obtained from experimental data. For example, if we use (to compare to the experimental data) the expression [35]

$$F^{exp}(x, Q^2) \Rightarrow F(x, Q^2)\left[1 + \frac{h_i^{HT}(x)}{Q^2}\right], \qquad (35)$$

where a structure function contains target mass corrections and the function $h^{HT}(x)$ corresponds to dynamical contribution, we can study a difference in dynamical contribution, $\Delta h_i^{HT}(x)$, obtained in depending on a method used. Figure 6 shows the changes of dynamical contributions at Q^2=2 GeV^2 for F_1, F_2

and F_3 at replacement of the ξ-scaling method including target mass corrections on the method using analytic moments.

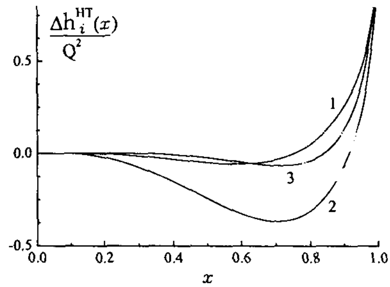

Figure 6. The behavior of the $\Delta h_i^{HT}(x)$ as a function of x for F_1, F_2, and F_3 at $Q^2 = 2$ GeV^2. The numbers near curves indicate what a structure function the curve corresponds.

4 Conclusion

The precision of experimental data is constantly increasing, and studying such subtle effects as the higher twists is becoming relevant. In this situation, it is expedient to base theoretical approaches on methods that are compatible with the general principles of quantum field theory. We have obtained expressions for the structure functions F_1, F_2, and F_3 of a massive nucleon that satisfy the spectral condition. We showed that these new expressions, which take mass corrections into account for the structure functions, lead to results that differ significantly from the results obtained both using the standard the ξ-scaling method and using the method suggested in [11], especially for values Q^2 of the order of several GeV^2 and for large values of x.

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