Radiative decays of vector mesons in point form of Poincare-invariant quantum mechanics

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In this work we present a scheme of obtaining parameters of the model based on point form of Poincare invariant quantum mechanics. Using the integral representation of pseudoscalar and vector meson decay constants the basic parameters for u, d and s - quark sector are defined. With these numbers it is possible to estimate quark magnetic moments from $V \to P\gamma$ decay.

Keywords: Poincare, point form, quark, integral representation, vector meson, radiative decay, decay constant, magnetic moment.

Introduction

Electroweak and semileptonic decays of pseudoscalar and vector mesons have always been convenient tools for approbation of various theoretical models and approaches for studying the structure of hadrons. The description of such processes in the framework of QCD, as a quantum theory, is impossible by properties of the SU(3) group operators; also, the behavior of the running QCD constant $\alpha_s(q^2)$ at low energies makes the perturbation theory inapplicable to calculations. These and other difficulties (see [1, 2]) motivated the development of alternative approaches for investigating bounded quark systems.

In this paper, based on the point form of Poincare-invariant quantum mechanics (PIQM), a procedure for obtaining model parameters from electroweak decays of pseudoscalar and vector mesons is proposed. A distinctive feature of this procedure authors note is obtaining constituent quark masses using the pseudoscalar density constant.

As a result, we demonstrate following procedure for calculation of the radiative decay constant and applying obtained model parameters for fixing quark magnetic moments in PIQM.

1. Leptonic decays of pseudoscalar and vector mesons in PIQM

Basic futures of the model is described in [3], so we use only results: the vector of a meson with momentum $\mathbf{Q}$, mass M, spin J and it's projection μ is defined as the direct product of the state vectors of quarks with momentums $\mathbf{p}_1, \mathbf{p}_2$:

$$
\begin{aligned}
\left|\mathbf{Q}, J\mu, M\right\rangle = \sum_{\lambda_1,\lambda_2}\sum_{\nu_1,\nu_2}\int \mathbf{dk}\sqrt{\frac{\omega_{m_q}(\mathbf{p}_1)\omega_{m_Q}(\mathbf{p}_2)M_0}{\omega_{m_q}(\mathbf{k})\omega_{m_Q}(\mathbf{k})\omega_{M_0}(\mathbf{P})}}\Phi^J_{LS}(\mathrm{k},\beta)Y_{Lm}(\theta_k,\varphi_k)\times \\
\times\Omega\begin{Bmatrix} L & S & J \\ \nu_1 & \nu_2 & \mu \end{Bmatrix}(\theta_k,\varphi_k)\, D^{1/2}_{\lambda_1,\nu_1}(\mathbf{n}_{W_1})D^{1/2}_{\lambda_2,\nu_2}(\mathbf{n}_{W_2})\left|\mathbf{p}_1,\lambda_1,\mathbf{p}_2,\lambda_2\right\rangle,
\end{aligned}
\tag{1}
$$

where $Y_{Lm}(\theta_k,\varphi_k)$ – the spherical functions determined by the angles of the vector $\mathbf{k}$ and $D^{1/2}_{\lambda,\nu}(\mathbf{n}_W)$ – Wigner function. In (1) we use abbreviation

$$
\Omega\begin{Bmatrix} L & S & J \\ \nu_1 & \nu_2 & \mu \end{Bmatrix}(\theta_\mathrm{k},\varphi_\mathrm{k}) = C\begin{Bmatrix} \mathrm{s}_1 & \mathrm{s}_2 & S \\ \nu_1 & \nu_2 & \lambda \end{Bmatrix} C\begin{Bmatrix} L & S & J \\ m & \lambda & \mu \end{Bmatrix} Y_{Lm}(\theta_\mathrm{k},\varphi_\mathrm{k})
\tag{2}
$$

and wave functions $\Phi^J_{LS}(\mathrm{k},\beta)$ taking into account the number of colors of quarks N_c is normalized by the condition

$$\sum_{L,s}\int_0^\infty \mathrm{dk}\ \mathrm{k}^2\ \left|\Phi^J_{LS}(\mathrm{k},\beta)\right|^2 = N_c. \tag{3}$$

Constants of the leptonic decays of the pseudoscalar $P(Q\bar{q}) \to \ell + \nu_\ell$ and vector $\ell \to V(Q\bar{q}) + \nu_\ell$ mesons, after removing the elements of the Kabayashi-Maskawa matrix, are determined by the expressions:

$$\left\langle 0\left|\hat{J}^\mu_P(0)\right|\mathbf{Q},0,M_P\right\rangle = i\frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{2\omega_{M_P}(\mathbf{Q})}}P^\mu f_P, \tag{4}$$

$$\left\langle 0\left|\hat{J}^\mu_V(0)\right|\mathbf{Q},1\ \lambda_V,M_V\right\rangle = i\frac{1}{(2\pi)^{3/2}}\frac{\varepsilon^\mu(\lambda_V)}{\sqrt{2\omega_{M_V}(\mathbf{Q})}}M_V f_V, \tag{5}$$

where the transition current $\hat{J}^\mu(0)$ and the state vector of mesons with masses are chosen in the Heisenberg representation.

The matrix element of the current in the quark basis in the case of leptonic decays of pseudoscalar and vector mesons is determined as

$$\left\langle 0\left|\hat{J}^\mu_P(0)\right|\mathbf{p}_1,\lambda_1,\mathbf{p}_2,\lambda_2\right\rangle = \frac{1}{(2\pi)^3}\frac{\bar{\upsilon}_{\lambda_2}(\mathbf{p}_2,m_Q)\gamma^\mu\gamma_5 \mathrm{u}_{\lambda_1}(\mathbf{p}_1,m_q)}{\sqrt{2\omega_{m_Q}(\mathbf{p_2})}\sqrt{2\omega_{m_q}(\mathbf{p_1})}}, \tag{6}$$

$$\left\langle 0\left|\hat{J}^\mu_V(0)\right|\mathbf{p}_1,\lambda_1,\mathbf{p}_2,\lambda_2\right\rangle = \frac{1}{(2\pi)^3}\frac{\bar{\upsilon}_{\lambda_2}(\mathbf{p}_2,m_q)\gamma^\mu \mathrm{u}_{\lambda_1}(\mathbf{p}_1,m_q)}{\sqrt{2\omega_{m_q}(\mathbf{p}_2)}\sqrt{2\omega_{m_q}(\mathbf{p}_1)}}. \tag{7}$$

Substituting the meson state vectors (1) into expressions (4) and (5), taking expressions (6), (7) and using relations

$$\Omega\begin{Bmatrix}0 & 0 & 0\\ \nu_1, & \nu_2, & \mu\end{Bmatrix}(\theta_\mathrm{k},\varphi_\mathrm{k}) = \delta_{\mu,0}\delta_{\nu_1,-\nu_2}\frac{\nu_1}{\sqrt{2\pi}}, \tag{8}$$

$$\Omega\begin{Bmatrix}0 & 1 & 1\\ \nu_1, & \nu_2, & \mu\end{Bmatrix}(\theta_\mathrm{k},\varphi_\mathrm{k}) = \delta_{\mu,\nu_1+\nu_2}\frac{\sqrt{3+4\nu_1\nu_2}}{4\sqrt{\pi}}, \tag{9}$$

leading us to integral representations of the leptonic decays constants of pseudoscalar and vector mesons f_P and f_V:

$$f_I(m_q,m_Q,\beta^\mathrm{I}_{Qq}) = \sqrt{\frac{3}{2}}\frac{1}{\pi}\int \mathrm{dk}\ \mathrm{k}^2\Phi(\mathrm{k},\beta^\mathrm{I}_{Qq})\sqrt{\frac{W^+_{m_q}(\mathbf{k})W^+_{m_Q}(\mathbf{k})}{M_0\omega_{m_q}(\mathbf{k})\omega_{m_Q}(\mathbf{k})}}\left(1+a_\mathrm{I}\frac{\mathrm{k}^2}{W^+_{m_q}(\mathbf{k})W^+_{m_Q}(\mathbf{k})}\right),$$

$$\mathrm{I}=P,V;\ a_P=-1,\ a_V=-\tfrac{1}{3}, \tag{10}$$

where

$$W^\pm_m(\mathbf{k}) = \omega_m(\mathbf{k})\pm\mathrm{k},\ \sqrt{\mathbf{k}^2} = \mathrm{k}. \tag{11}$$

For further fixation of the model parameters, based on point form of PIQM, we use the constant of pseudoscalar density, which determined by relation

$$\left\langle 0\,|\,\bar{Q}\gamma_5 q\,|\,\mathbf{Q},0,M_P\right\rangle = -\frac{\mathrm{i}}{(2\pi)^{3/2}}\frac{g_P}{\sqrt{2\omega_{M_P}(\mathbf{Q})}}, \tag{12}$$

where axial current $\hat{J}_P^\alpha(x) = \bar{Q}(x)\gamma^\alpha\gamma_5 q(x)$ and pseudoscalar density $j^5(x) = \mathrm{i}\bar{Q}(x)\gamma_5 q(x)$ are related by

$$\partial_\alpha \hat{J}_P^\alpha(x) = (\hat{m}_Q + \hat{m}_q)\, j^5(x), \tag{13}$$

with current masses of quarks $\hat{m}_{\bar{Q}}$ and $\hat{m}_q$. The equation (13) leads to the fact, that the constants f_P and g_P are related by

$$(\hat{m}_Q + \hat{m}_q) g_P = M_P^2 f_P \tag{14}$$

for u and $d-$quarks (sometimes and for $s-$quark).

Carrying out a similar procedure for substituting the meson state vector (1), after the calculations we obtain the integral representation decay constants g_P of pseudoscalar meson:

$$\begin{aligned} g_P(m_q, m_Q, \beta_{Qq}^P) = \sqrt{\frac{3}{2}}\frac{1}{\pi}\int \mathrm{dk}\ \mathrm{k}^2 \Phi(\mathrm{k}, \beta_{Qq}^P)\ \sqrt{\frac{M_0}{\omega_{m_q}(\mathbf{k})\omega_{m_Q}(\mathbf{k})}} \times \\ \times\left(\sqrt{W_{m_q}^+(\mathbf{k}) W_{m_Q}^+(\mathbf{k})} + \sqrt{W_{m_q}^-(\mathbf{k}) W_{m_Q}^-(\mathbf{k})}\right). \end{aligned} \tag{15}$$

2. Numerical calculation of the model parameters

The values of the constituent masses of light quarks (u, d and s) and the parameters of the wave function β_{Qq}^{I} can be fixed by the experimental values of the decay constants and the values of the current quark masses [4]. Using wave function

$$\Phi^{os}(\mathrm{k}, \beta) = N_{os} \exp\left[-\frac{\mathrm{k}^2}{2\beta^2}\right],\ \ N_{os} = \frac{2}{\pi^{1/4}\beta^{3/2}}; \tag{16}$$

and relations (10) , (15, one can find the values of the model parameters:

$$\begin{gathered} m_u = 218.3 \pm 4.2\ \mathrm{MeV},\ \bar{m}_{ud} = 219.6 \pm 4.2\ \mathrm{MeV}, \bar{m}_{us} = 226.2 \pm 59.7\ \mathrm{MeV}, \beta_{ud}^P = 370.8 \pm 9.3\ \mathrm{MeV}, \\ \beta_{us}^P = 373.2 \pm 20.9\ \mathrm{MeV}, \beta_{ud}^V = (310.9 \pm 2.2)\ \mathrm{MeV},\ \ \beta_{us}^V = (314.3 \pm 83.6)\ \mathrm{MeV} \end{gathered} \tag{17}$$

(more details of these calculations see in [5]).

3. Radiative decays of vector mesons in point form of PIQM

Let us consider the application of the technique, presented in the work, to radiative decays $V \to P\gamma$, which are widely used to study the structure of hadrons.

Parameterization of the matrix element for the vector meson transition V with the polarization vector $\varepsilon_V(\lambda)$ into a pseudoscalar meson P by emitting a virtual γ^* is given by [6]:

$$\left\langle \mathbf{Q}', 0, M_P \left| \hat{j}^\alpha(0) \right| \mathbf{Q}, 1\lambda, M_V \right\rangle = i\frac{g_{VP\gamma^*}(q^2)}{(2\pi)^3}\frac{\varepsilon^{\alpha\nu\rho\sigma}\varepsilon_V(\lambda) Q_\rho Q'_\sigma}{\sqrt{4\omega_{M_V}(\mathbf{Q})\omega_{M_P}(\mathbf{Q}')}} \tag{18}$$

Multiplying the expression (18) by $K^{*\alpha}(\lambda) = -i\varepsilon^{\alpha\nu\rho\sigma}\varepsilon_V^*(\lambda) V_\rho V'_\sigma$ and rewriting in terms of 4-violicities one can obtain:

$$g_{VP\gamma^*}(q^2) = (2\pi)^3\sqrt{4V_0V_0'}\langle \mathbf{Q}', 0, M_P | \frac{\left(K^*(\lambda)\hat{J}(0)\right)}{\sqrt{M_P M_V}\left(K(\lambda)K^*(\lambda)\right)} | \mathbf{Q}, 1\lambda, M_V \rangle. \tag{19}$$

Decay process $V \to P\gamma^*$ is due to the electromagnetic interaction of quarks Q and q with charges e_q, e_Q: in this approach, the current $\hat{J}^\alpha(0)$ will be written in the form

$$\hat{J}^\alpha(0) = \sum_{q=u,d,...} e_q \bar{\psi}_q \Gamma^\alpha \psi_q \,. \qquad (20)$$

For this decay in the Breit system, we have [3]

$$J^{'} = L^{'} = s^{'} = L = 0\,,\ J = s = 1,$$
$$m_q = m_q^{'}\,,\ m_Q = m_Q^{'},$$
$$K(\lambda) = \frac{\sqrt{\varpi^2 - 1}}{\sqrt{2}}\{0, 0, -i\lambda, 1, 0\}\,,\ \varpi = \left(V_Q V_{Q^{'}}\right) \qquad (21)$$

and

$$\Gamma^\alpha = F_1(q^2)\gamma^\alpha + \frac{1}{2m_{q,Q}} F_2(q^2)\sigma^{\mu\nu} q_\nu. \qquad (22)$$

Note, that in (22) form-factors are defined as

$$F_1(q^2 = 0) + F_2(q^2 = 0) = \mu_{q,Q}, \qquad F_2(q^2 = 0) = \kappa_{q,Q}, \qquad (23)$$

where $\mu_{q,Q}$ – quark magnetic moment and

$$\varpi = \varpi_{12} = \frac{M_0^{'2}\left(k^{'}\right) + M_0^2(k) + q^2}{2\, M_0(k) M_0^{'}\left(k^{'}\right)}\,. \qquad (24)$$

For fixing quark magnetic moments, we use experimental data for ρ^+, K^{*+} and K^{*0}-meson (this choice is because of fact, that the quark structure of these mesons doesn't depend on the mixing angles). Simple, but great calculations, lead us to following values:

Quark magnetic moment	**PIQM,** μ_N	**[8],** μ_N	**[9],** μ_N
μ_u	2.558 ± 0.08	2.066	2.08 ± 0.07
μ_d	-1.36 ± 0.015	-1.11	-1.31 ± 0.06
μ_s	-0.714 ± 0.011	-0.633	-0.77 ± 0.06

Table 1. Quark magnetic moments

Authors note, that in the case $m_q = m_Q = m$ one can get the integral representation of the radiative decay constant as follows:

$$g_{VP\gamma} = \int \mathrm{dk}\, \mathrm{k}^2 \Phi_V(\mathrm{k}) \Phi_P(\mathrm{k}) \left(e_q f_1(\mathrm{k}, m) + \frac{\kappa_q}{2m_q} f_2(\mathrm{k}, m) - e_Q f_1(\mathrm{k}, m) - \frac{\kappa_Q}{2m_Q} f_2(\mathrm{k}, m) \right), \qquad (25)$$

where

$$f_1(\mathrm{k},m)=\frac{2m+\omega_m(\mathrm{k})}{3\,\omega_m^2(\mathrm{k})},\quad f_2(\mathrm{k},m)=\frac{2(\mathrm{k}^2-\omega_m(\mathrm{k})\,(\,m+2\,\omega_m(\mathrm{k})\,)\,)}{3\,\omega_m^2(\mathrm{k})}. \tag{26}$$

Conclusion and remarks

The paper presents a technique for calculating the form factors and the radiative decay constants taking into account quark structure in the framework of the point-form of the PIQM. In this approach, the integral expressions for the observables do not depend on the meson masses: similar property observed in methods developed in on the light-front dynamics and in the instant form of PIQM [9].

The authors note, that this model can be used for a self-consistent description of the leptonic decays of hadrons and radiative decays of vector mesons: it is became possible only taking into account the quarks magnetic moments, which in the developed model correlate with other works.

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