Interactions of Strange Mesons at Low Energies

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Abstract

Analytical expressions for the vector and scalar form factors of semileptonic decays K_{l3} have been obtained in the Quark Confinement Model.The contribution from the direct diagrams, as well as that one from the intermediate vector states in the form factors are examined.The performed investigation proofs that the semileptonic kaon decays can be successfully described in the framework of QCM. We need no additional parameters and assumptions for adequate description of this kind of decays. Well known CTMOP relation, obtained in the current algebra approach,is reproduces in QCM with 10% accuracy.Numerical values for slope parameters $\lambda'_+ = 0.031$ and $\lambda'_0 = 0.0165$ are in satisfactory agreement with experimental data.We also study the radiative decays of neutral kaons $K^0_{L,S} \to \gamma\gamma$ in the framework of effective weak lagrangian approach. It is shown that the dominant contribution to $K^0_L \to \gamma\gamma$ amplitude is given by the weak transitions of kaons into π, η and η' mesons. It should also be noted that the amplitudes associated with the operator O_5,are strengthened in comparison with other. Decay $K^0_S \to \gamma\gamma$ is completely described by graphs with intermediate scalar states .Recived values $Br(K^0_L \to \gamma\gamma) = 5.58 \times 10^{-4}$ and $Br(K^0_S \to \gamma\gamma) = 2.083 \times 10^{-6}$ are in a good agreement with experimental data.

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1 Introduction

Study of kaon decays has attracted the attention of researchers for decades. The reason is that kaon decays involve an intricate interplay between weak, electromagnetic and strong interactions.This decays are of extraordinary interest as a source of information about a New Physics beyond Standard Model. From this point of view it is very important to have trustworthy quantitative estimations of parameters of mentioned decays in the framework of Standard model. The problem is that calculation of hadronic matrix elements in the most of theoretical approaches needs a great number of additional parameters and model assumptions . Kaon decays have been treated in several reviews and lecture notes during the past 20 years [1].

Pure leptonic and semileptonic decays are among the theoretically cleanest K decays. From this point of view it is very important to have trustworthy quantitative estimations of parameters of mentioned decays in the framework of Standard model.

The aim of this work is theoretical study of semileptonic and electromfgnetic interactions of kaons by means of effective Lagrangians proposed in [2], [3],[4]. The calculation of hadronic matrix elements are performed in the Quark Confinement Model (QCM) [5]. This model based on the certain assumptions about nature of quark confinement and hadronization allows to describe the electromagnetic,strong and weak interactions of light (nonstrange and strange)mesons from a unique point of view. Basic low-energy properties of kaons in QCM were considered by us in [6]. The undoubtful dignity of model is that further study of kaon decays doesn't need no more additional assumptions and no more additional parametres.

2 Quark Interactions

The hadronic interactions will be described in the QCM.This model is based on the following assumptions [5]:

The hadron fields are assumed to arise after integration over gluon and quark variables in the QCM generating function. The transition of hadrons to quarks and vice versa is given by the interaction Lagrangian. In particular necessary interaction Lagrangians for $\pi^{\pm}$and$K$ mesons look like:

$$\mathcal{L}_M = \frac{g_M}{\sqrt{2}} M \bar{q}^a \Gamma \lambda^m q^a \tag{1}$$

where Γ- Dirak matrix,λ^m - is a corresponding SU(3)-matrix,q- quark vector

$$q_j^a = \begin{pmatrix} u^a \\ d^a \\ s^a \end{pmatrix}$$

In order to quantify the mixing in the η, η' system, one have to define appropriate mixing parameters, which can be related to physical observables. In the [8] the best agreement with experimental data was achieved with the

$$\varphi = 39.3^\circ \tag{2}$$

The properties of scalars are not well established and its description needs an additional assumptions. We use the Lagrangian with additional interaction with derivative [9]:

$$L_S = \frac{g_s}{\sqrt{2}} s(x) \overline{q}(x) (I - i\frac{H}{\Lambda}(\overleftarrow{\hat{\partial}} - \overrightarrow{\hat{\partial}})) \lambda_S q(x) \tag{3}$$

with

$$\lambda_S = \begin{array}{c} diag(1,-1,0) \Rightarrow a_0(980) \\ diag(\cos\delta_s, \cos\delta_s, -\sqrt{2}\sin\delta_s) \Rightarrow \sigma(600) \\ diag(-\sin\delta_s, -\sin\delta_s, -\sqrt{2}\cos\delta_s) \Rightarrow f_0(980) \end{array}$$

We use the values of additional parameters H, δ_s fixed in [9]:

$$H = 0.54; \quad \delta_S = 17^\circ \tag{4}$$

The coupling constants g_M for meson-quark interaction are defined from so-called compositeness condition. It us convenient to use interaction constant in a form:

$$h_M = \frac{3g_M^2}{4\pi^2} = -\frac{1}{\tilde{\prod}'_M(m_M)} \tag{5}$$

instead of g_M in the further calculations. All hadron-quark interactions are described by quark diagrams induced by S matrix averaged over vacuum backgrounds.

The confinement ansatz in the case of one-loop quark diagrams consists in following replacement:

$$\int d\sigma_{VAC} Tr|M(x_1)S(x_1,x_2|B_{VAC})...M(x_n)S(x_n,x_1|B_{VAC})| \longrightarrow$$
$$\int d\sigma_v Tr|M(x_1)S_v(x_1-x_2)...M(x_n)S_v(x_n-x_1)|, \qquad (6)$$

where

$$S_v(x_1-x_2) = \int \frac{d^4p}{i(2\pi)^4} e^{-ip(x_1-x_2)} \frac{1}{v\Lambda_q - \hat{p}} \qquad (7)$$

The parameter Λ_q characterizes the confinement rang of quark with flavor number $q = u, d, s$. The measure $d\sigma_v$ is defined as:

$$\int \frac{d\sigma_v}{v - \hat{z}} = G(z) = a(-z^2) + \hat{z}b(-z^2) \qquad (8)$$

The function $G(z)$ is called the confinement function. $G(z)$ is independent on flavor or color of quark. $G(z)$ is an entire analytical function on the z-plane.$G(z)$ decreases faster then any degree of z in Euclidean region.The choice of $G(z)$,or as the same of $a(-z^2)$ andq $b(-z^2)$, is one of model assumptions.In the note [5] $a(-z^2)$ and $b(-z^2)$ are chosen as:

$$\begin{aligned} a(u) &= a_0 e^{-u^2 - a_1 u} \\ b(u) &= b_0 e^{-u^2 - b_1 u} \end{aligned} \qquad (9)$$

The request of satisfaction of Ward anomaly identity in QCM gives the additional correlation between $a(0)$ and $b(0)$: $b(0) = -a'(0)$, $a(0) = 2$. Using $a(u)$ and $b(u)$ as (9), one can receive: $a_0 = 2$, $a_1 = \frac{b_0}{4}$. So, the free parameters of the model are Λ_q, b_0, b_1. The model parameters for nonstrange quarks were fixed by fitting the well-established constants of low-energy physics in [6]

$$\begin{aligned} \Lambda_u &= \ 460\ MeV, & \Lambda_S &= 506\ MeV, \\ b_0 &= \ 2, & b_1 &= 0.2, \\ a_0 &= \ 2, & a_1 &= 0.5. \end{aligned} \qquad (10)$$

We put $\Lambda_u = \Lambda_d$ in the most of decays.

Semileptonic transitions are mediated by the effective Lagrangian

$$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}} S_{EW}^{1/2} [\bar{l}\gamma_\mu(1-\gamma_5)\nu_l][\bar{u}_i\gamma^\mu(1-\gamma_5)V_{ij}d_j] + h.c. \qquad (11)$$

where V_{ij} denotes the ij element of CKM matrix [10], $G_F = 1.1663788(7) \times 10^{-5} GeV^{-2}$ [11] is the Fermi constant as extracted from muon decay. The universal short distance factor

$$S_{EW} = 1 + \frac{2\alpha}{\pi}(1 - \frac{\alpha_s}{4\pi}) \ln \frac{M_Z}{M_\rho} + \mathcal{O}(\frac{\alpha\alpha_s}{\pi^2}) = 1.0223 \pm 0.0005 \qquad (12)$$

encodes electroweak corrections not included in G_F and small QCD effects [12].

Electromagnetic quark interaction is described in the standard form:

$$L_q^{em} = eA_\mu \overline{q} Q \gamma^\mu q. \qquad (13)$$

the notation is adopted

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$$Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}$$

The quark weak interaction is described by effective Lagrangian $\mathcal{L}_w^{eff}$ for $\Delta S = 1$ -transitions (the $K^+ \to \gamma\gamma$ decays are of this type). This Lagrangian is a sum of usual four-quark operators [3] :

$$\mathcal{L}_w^{eff} = \frac{G_F}{2\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{6} c_i O_i \qquad (14)$$

where four-quark local operators O_i are chosen in following way:

$$\begin{aligned}
O_1 &= (\overline{d} O_L^\mu s)(\overline{u} O_L^\mu u) - (\overline{d} O_L^\mu u)(\overline{u} O_L^\mu s) \qquad (15) \\
O_2 &= (\overline{d} O_L^\mu u)(\overline{u} O_L^\mu s) + (\overline{d} O_L^\mu s)(\overline{u} O_L^\mu u) + 2(\overline{d} O_L^\mu s)(\overline{d} O_L^\mu d) + 2(\overline{d} O_L^\mu s)(\overline{s} O_L^\mu s) \\
O_3 &= (\overline{d} O_L^\mu u)(\overline{u} O_L^\mu s) + (\overline{d} O_L^\mu s)(\overline{u} O_L^\mu u) - (\overline{d} O_L^\mu s)(\overline{s} O_L^\mu s) \\
O_4 &= (\overline{d} O_L^\mu u)(\overline{u} O_L^\mu s) + (\overline{d} O_L^\mu s)(\overline{u} O_L^\mu u) - (\overline{d} O_L^\mu s)(\overline{d} O_L^\mu d) \\
O_5 &= (\overline{d} O_L^\mu \lambda^a s) \sum_{q=u,d,s} (\overline{q} O_R^\mu \lambda^a q) \\
O_5 &= (\overline{d} O_L^\mu s) \sum_{q=u,d,s} (\overline{q} O_R^\mu q)
\end{aligned}$$

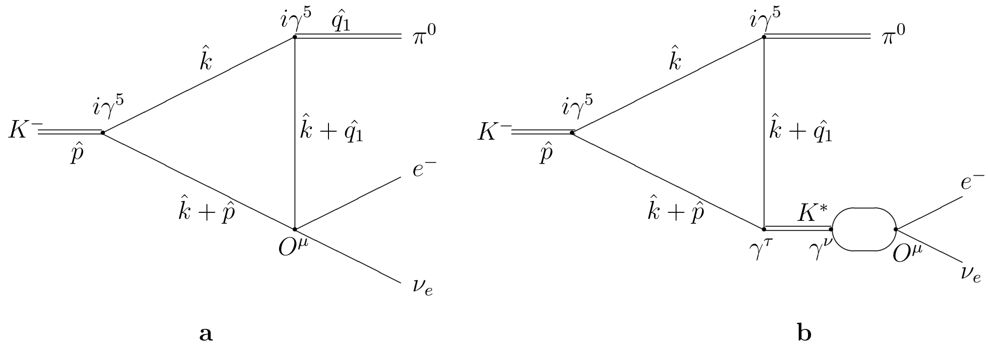

Figure 1: Graphs define K_{l_3} matrix element. Indexes a and b indicate the contributions of direct graph (a) and graph with intermediate K^* resonanse(b).

Here $O^\mu_{R,L} = \gamma^\mu(1 \pm \gamma^5)$, λ^a-Gell-Mann matrices, acting in colour space. The numerical values of c_i depend on QCD parameters μ_s α_s [4]. In this note we use the set of coefficients c_i corresponding $\mu_s = 0.25$ GeV, $\alpha_s = 0.45$:

$$c_1 = -1.97 \quad c_2 = 0.12 \quad c_3 = 0.093 \quad c_4 = 0.47 \quad c_5 = -0.036 \tag{16}$$

3 K_{l3} Decay

Matrix element of K_{l3} decay is determined by graphs shown in Figure 1. and can be written as

$$M^\mu = F_+(t)(p_1+p_2)^\mu + F_-(t)(p_1-p_2)^\mu \tag{17}$$

where

$$\begin{aligned} F_+(t) &= F_+^a(t) + F_+^b(t) \\ F_-(t) &= F_-^a(t) + F_-^b(t) \\ t &= (p_1-p_2)^2 \end{aligned} \tag{18}$$

Contribution from graph (1a) have been obtained in following form:

$$\begin{aligned} F_+^a(t) &= \sqrt{2h_K h_\pi} F^-_{VPP}(t, m_K^2, m_\pi^2, \Lambda_s, \Lambda_u, \Lambda_u) \\ F_-^a(t) &= \sqrt{2h_K h_\pi} F^+_{VPP}(t, m_K^2, m_\pi^2, \Lambda_s, \Lambda_u, \Lambda_u) \end{aligned} \tag{19}$$

where h_K, h_π- K, π-quark interaction constants calculated by (5).

$F^{\pm}_{VPP}(t, m_K^2, m_\pi^2, \Lambda_s, \Lambda_u, \Lambda_u)$-loop integrals for triangle graph, describing $V \to PP$ transition:

$$F^{+}_{VPP}(p^2, k_1^2, k_2^2, \Lambda_1, \Lambda_2, \Lambda_3) = \tag{20}$$
$$= \frac{\Delta p^2}{16\Lambda^2}\int_0^{u_\Delta} duub(-\frac{p^2}{4\Lambda^2})\sqrt{1-u+(\frac{\Delta u}{2})^2}+$$
$$+\frac{1}{2}\int\int\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta(1-\alpha_1-\alpha_2-\alpha_3)\times$$
$$\times\frac{P\cdot[(\alpha_1-\alpha_2)(\Lambda_1-\Lambda_2)(\Lambda_2-\Lambda_3)+\Lambda_3(\Lambda_1-\Lambda_2)]+\alpha_1 k_1^2+\alpha_2 k_2^2}{\alpha_1\Lambda_1^2+\alpha_2\Lambda_2^2+\alpha_3\Lambda_3^2}$$

$$F^{-}_{VPP}(p^2, k_1^2, k_2^2, \Lambda_1, \Lambda_2, \Lambda_3) = \tag{21}$$
$$= \frac{1}{2}\int_0^{\infty} dub(u) + \frac{p^2}{8\Lambda^2}\int_0^{u_\Delta} dub(-\frac{p^2}{4\Lambda^2})\sqrt{1-u+(\frac{\Delta u}{2})^2}+$$
$$+\frac{1}{2}\int\int\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta(1-\alpha_1-\alpha_2-\alpha_3)\times$$
$$\times\frac{P\cdot[(\alpha_1+\alpha_2)(\Lambda_1-\Lambda_3)(\Lambda_2-\Lambda_3)+\Lambda_3(\Lambda_1+\Lambda_2-\Lambda_3)]+\alpha_1 k_1^2+\alpha_2 k_2^2}{\alpha_1\Lambda_1^2+\alpha_2\Lambda_2^2+\alpha_3\Lambda_3^2}$$

The following notations have been introduced:

$$\Lambda^2 = \frac{1}{2}(\Lambda_1^2+\Lambda_2^2) \tag{22}$$
$$\Delta = \frac{\Lambda_2^2-\Lambda_1^2}{\Lambda_1^2+\Lambda_2^2}$$
$$u_\Delta = \frac{2}{1+\sqrt{1-\Delta^2}}$$
$$P = \frac{\alpha_1\alpha_2 p^2+\alpha_1\alpha_3 k_1^2+\alpha_2\alpha_3 k_2^2}{\alpha_1\Lambda_1^2+\alpha_2\Lambda_2^2+\alpha_3\Lambda_3^2}$$

For sequential account of the intermediate vector meson the contribution the so-called chain approximation have been used for its propagator:

$$h_V G^{\mu\nu}(p^2) = \frac{1}{\Pi_1(p^2)-\Pi_1(m_V^2)}\{-g^{\mu\nu}+\frac{p^\mu p^\nu \Pi_2(p^2)}{\Pi_1(p^2)-\Pi_1(m_V^2)+p^2\Pi_2(p^2)}\} \tag{23}$$

where $\Pi_{1,2}(p^2)$ are transverse and longitudinal parts of vector polarization operator.

So, after standard transformations, we obtain the following expressions for $F^b_\pm(t)$:

$$F^b_+(t) = -F^b_+(t)\frac{t}{\Pi_1(t) - \Pi_1(m^2_{K^*})}F_{VV}(t) \tag{24}$$
$$F^b_-(t) = F^b_-(t)\frac{m_k^2 + m_\pi^2}{\Pi_1(t) - \Pi_1(m^2_{K^*})}F_{VV}(t)$$

$F_{VV}(t)$ is a loop integral, corresponding the transverse part of vector polarization operator.

The very simple relationship between $F_+(m_K^2)$ and $F_-(m_K^2)$ was established by C.G. Callan and S.B.Treiman [13],V.Mattur, S.Okubo and L.Pandit [14] by means of current algebra:

$$F_+(m_K^2) + F_-(m_K^2) = f_k/f_\pi \tag{25}$$

In QCM we can obtain analogous relationship without any additional without any additional assumptions. So after calculating $F_+(t)$ and $F_-(t)$ with $t = m_K^2, (m_\pi^2 = 0)$, one obtains

$$F_+(m_K^2) + F_-(m_K^2) = 0.9 f_k/f_\pi \tag{26}$$

i.e. QCM with 10% accuracy reproduces CTMOP relation. One have to mention the cancelation of resonance graphs.

The vector form factor $F_+(t)$ deined in (17) represents the p-wave projection of the crossed-channel matrix element $\langle 0|\bar{s}\gamma^\mu u|K\pi\rangle$ whereas the s-wave projection is described by the scalar form factor [15]

$$F_0(t) = F_+(t) + \frac{t}{m_K^2 - m_\pi^2}F_-(t) \tag{27}$$

It is convenient to normalize all the form factors to $F_+(0)$, so

$$f_{+,0}(t) = \frac{F_{+,0}(t)}{F_+(0)} \tag{28}$$

In the analysis of experimental data form factors usually parameterized in the form [16]

$$f_{+,0}(t) = 1 + \lambda'_{+,0}\frac{t}{m_\pi^2} + \frac{1}{2}\lambda''_{+,0}(\frac{t}{m_\pi^2})^2 + \dots \tag{29}$$

Slope parameters can be calculated as follows:

$$\lambda'_{+,-,0} = m_\pi^2 f'_{+,-,0}(0) \tag{30}$$

For K_{e3}decays, recent measurements of the quadratic slope parameters of the vector form factor (λ', λ'') from (refeq:tey) are available from KTeV [17], KLOE [18], ISTRA+ [19], and NA48 [20]. Calculated values for slope parameters λ' and averaged experimental data are displayed in table 1

Table 1.

λ'	QCM	Experiment
$\lambda'_+ \times 10^{-3}$	31	25.2 ± 0.9
$\lambda'_- \times 10^{-3}$	3	0
$\lambda'_0 \times 10^{-3}$	16.5	11.7 ± 1.4

4 $K(p) \to \gamma^*\gamma^*$ Transition

The amplitude of a transition

$$K \to \gamma^*(q_1)\gamma^*(q_2) \tag{31}$$

can be written in the general form compatible with gauge invariance as [15]

$$\begin{aligned} M^{\mu\nu} = {} & [g^{\mu\nu} - \frac{q_1^\mu q_1^\nu}{q_1^2} - \frac{q_2^\mu q_2^\nu}{q_2^2} + \frac{q_1 \cdot q_2}{q_1^2 q_2^2} q_1^\mu q_2^\nu] m_K^2 f_1(q_1^2, q_2^2) + \\ & +[q_2^\mu q_1^\nu - q_1 \cdot q_2 (\frac{q_1^\mu q_1^\nu}{q_1^2} + \frac{q_2^\mu q_2^\nu}{q_2^2} - \frac{q_1 \cdot q_2}{q_1^2 q_2^2} q_1^\mu q_2^\nu) f_2(q_1^2, q_2^2) + \\ & + i\varepsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma} f_3(q_1^2, q_2^2) \end{aligned} \tag{32}$$

When one of the photons is on-shell ($q_1^2 = 0$ for instance) $M^{\mu\nu}$ is described by two invariant amplitudes

$$M^{\mu\nu} = (q_2^\mu q_1^\nu - q_1 \cdot q_2 g^{\mu\nu}) f_2(0, q_2^2) + i\varepsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma} f_3(0, q_2^2) \tag{33}$$

The (33)remains valid for both photons on-shell.

The $K_L^0 \to \gamma\gamma$ decay produces photons with perpendicular polarisation $(\varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma})$ and then only $f_3(0,0)$ in (33)contributes to the width.Let denote it as

$$f_3(0,0) = M(K_L^0 \to \gamma\gamma)$$

Amplitude of the studied decay can be written in the form:

$$M(K_L^0 \to \gamma\gamma) = \frac{G_F}{2\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{6} c_i [T^i_{K_L^0\gamma\gamma} + \sum_{P=\pi,\eta,\eta'} T^i_{KP} D_P(m_K^2) g_{P\gamma\gamma}(m_K^2)] \tag{34}$$

The following notation has been adopted:

$$T^i_{K_L^0\gamma\gamma} = \int dy dx_1 dx_2 dx_3 e^{ip_1x_1+ip_2x_2+ip_3x_3} \langle 0|T(L_q^{em}(x_1)L_q^{em}(x_2)L_K(x_3)O^i(y))|0\rangle \tag{35}$$

$$T^i_{KP} = \int dy dx_1 dx_2 e^{ip_1x_1+ip_2x_2} \langle 0|T(L_P(x_1)\mathcal{L}_K(x_2)O^i(y))|0\rangle \tag{36}$$

$L_q^{em}(x)$, $\mathcal{L}_P(x)$ are defined by (13) and (1) correspondingly. We use the chain approximation for propagator of pseudoscalar meson P $D_P(m_K^2)$:

$$h_P D_P(p^2) = \frac{1}{\Pi_P(p^2) - \Pi_P(m_P^2)}. \tag{37}$$

where $\Pi_P(p^2)$-mass operator of P.

$g_{P\gamma\gamma}(m_K^2)$ -form factor of $P \to \gamma\gamma$ decay:

$$g_{P\gamma\gamma}(x) = \frac{1}{\Lambda}\frac{\sqrt{3h_P}}{9\pi} F_{PVV}(\frac{x}{\Lambda^2}) Tr\{\lambda_P Q^2\} \tag{38}$$

$F_{PVV}(\frac{x}{\Lambda^2})$ is the loop integral (20) with $p^2 = x, q_1^2 = q_2^2 = 0, \Lambda_1 = \Lambda_2 = \Lambda_u$.

Analytical expressions for invariant amplitudes $T^i_{K_L^0\gamma\gamma}$ and T^i_{KP} obtained in the QCM are given in the Table 2 and the numerical values are shown in Table 3.

The table 3 shows that the dominant contribution is given by the weak transitions of kaons into π, η and η' mesons. This is consistent with [15]. It should also be noted that the amplitudes associated with the operator O_5,are strengthened in comparison with other.

T	Analytical expression
$T^1_{K^0_L\gamma\gamma}$	$-\frac{16}{3}\sqrt{\frac{h_K}{6}}\frac{\alpha}{3\pi^2}\Pi_{PA}(m_K^2,\Lambda_u,\Lambda_s)F_{AVV}(m_K^2)$
$T^5_{K^0_L\gamma\gamma}$	$\sqrt{\frac{h_K}{6}}\frac{\alpha}{3\pi^2}[\frac{\Lambda_u^3+\Lambda_s^3}{2\Lambda^2}\cdot(F_{PVVI}(m_K^2,\Lambda_s,\Lambda_s,\Lambda_s,\Lambda_u)+$ $F_{PVVI}(m_K^2,\Lambda_s,\Lambda_s,\Lambda_s,\Lambda_u)+F_{PVIV}(m_K^2,\Lambda_s,\Lambda_s,\Lambda_u,\Lambda_u))-$ $-\Lambda_s\frac{4}{3}\Pi_{PA}(m_K^2,\Lambda_u,\Lambda_s)F_{PVV}(\frac{m_K^2}{\Lambda_u^2})+$ $+\frac{\Lambda_s^2}{\Lambda_u}\frac{4}{27}\Pi_{PP}(m_K^2,\Lambda_u,\Lambda_s)F_{PVV}(\frac{m_K^2}{\Lambda_u^2})]$
$T^1_{K^0_L\pi}$	$\sqrt{\frac{h_\pi h_K}{2}}/\pi^2\Pi_{PA}(m_K^2,\Lambda_u,\Lambda_s)F_P(\frac{m_K^2}{\Lambda_u^2})m_K^2\Lambda_u\sqrt{\Lambda_u^2+\Lambda_s^2}$
$T^5_{K^0_L\pi}$	$\frac{\sqrt{\Lambda_u^2+\Lambda_s^2}}{\pi^2}\frac{16}{3}[\Pi_{PP}(m_K^2,\Lambda_u,\Lambda_s)\Pi_{PP}(m_K^2,\Lambda_u,\Lambda_u)\frac{\Lambda_u^2(\Lambda_u^2+\Lambda_s^2)}{2}+$ $+F_{IPP}(0,m_K^2,m_\pi^2,\Lambda_u,\Lambda_u,\Lambda_s)(\Lambda_u^3+\Lambda_s^3)C_A^{(1)}]$
$T^1_{K^0_L\eta'}$	$\sqrt{h_{\eta'}h_K}/\pi^2\Pi_{PA}(m_K^2,\Lambda_u,\Lambda_s)F_P(\frac{m_K^2}{\Lambda_u^2})m_K^2\Lambda_u\sqrt{\frac{\Lambda_u^2+\Lambda_s^2}{2}}\cos\varphi$
$T^2_{K^0_L\eta'}$	$\sqrt{h_{\eta'}h_K}/\pi^2 m_K^2\sqrt{\frac{\Lambda_u^2+\Lambda_s^2}{2}}\Pi_{PA}(m_K^2,\Lambda_u,\Lambda_s)\times$ $\times[-6\cos\varphi\Lambda_u F_P(\frac{m_K^2}{\Lambda_u^2})+4\sqrt{2}\sin\varphi\Lambda_s F_P(\frac{m_K^2}{\Lambda_s^2})]$
$T^3_{K^0_L\eta'}$	$\sqrt{h_{\eta'}h_K}/\pi^2 m_K^2\sqrt{\frac{\Lambda_u^2+\Lambda_s^2}{2}}\Pi_{PA}(m_K^2,\Lambda_u,\Lambda_s)\times$ $\times[-6\cos\varphi\Lambda_u F_P(\frac{m_K^2}{\Lambda_u^2})-6\sqrt{2}\sin\varphi\Lambda_s F_P(\frac{m_K^2}{\Lambda_s^2})]$
$T^5_{K^0_L\eta'}$	$\frac{\sqrt{h_{\eta'}h_K}}{\pi^2}\frac{16}{3}[\cos\varphi(\Lambda_u^2\frac{\Lambda_u^2+\Lambda_s^2}{2}\Pi_{PP}(m_K^2,\Lambda_u,\Lambda_s)\times$ $\times\Pi_{PP}(m_K^2,\Lambda_s,\Lambda_s)+$ $+F_{IPP}(0,m_K^2,m_{\eta'}^2,\Lambda_u,\Lambda_u,\Lambda_s)(\Lambda_u^3+\Lambda_s^3)C_A^{(1)})-$ $-\sqrt{2}\sin\varphi(\Lambda_s^2\frac{\Lambda_u^2+\Lambda_s^2}{2}\Pi_{PP}(m_K^2,\Lambda_u,\Lambda_s))F_{PP}(\frac{m_K^2}{\Lambda_s^2})+$ $+F_{IPP}(0,m_K^2,m_{\eta'}^2,\Lambda_s,\Lambda_s,\Lambda_u)(\Lambda_u^3+\Lambda_s^3)C_A^{(1)})]$
$T^1_{K^0_L\eta}$	$-\sqrt{h_\eta h_K}/\pi^2\Pi_{PA}(m_K^2,\Lambda_u,\Lambda_s)F_P(\frac{m_K^2}{\Lambda_u^2})m_K^2\Lambda_u\sqrt{\frac{\Lambda_u^2+\Lambda_s^2}{2}}\sin\varphi$
$T^2_{K^0_L\eta}$	$\sqrt{h_\eta h_K}/\pi^2 m_K^2\sqrt{\frac{\Lambda_u^2+\Lambda_s^2}{2}}\Pi_{PA}(m_K^2,\Lambda_u,\Lambda_s)\times$ $\times[6\sin\varphi\Lambda_u F_P(\frac{m_K^2}{\Lambda_u^2})+4\sqrt{2}\cos\varphi\Lambda_s F_P(\frac{m_K^2}{\Lambda_s^2})]$
$T^3_{K^0_L\eta}$	$\sqrt{h_\eta h_K}/\pi^2 m_K^2\sqrt{\frac{\Lambda_u^2+\Lambda_s^2}{2}}\Pi_{PA}(m_K^2,\Lambda_u,\Lambda_s)\times$ $\times[\sin\varphi\Lambda_u F_P(\frac{m_K^2}{\Lambda_u^2})-6\sqrt{2}\cos\varphi\Lambda_s F_P(\frac{m_K^2}{\Lambda_s^2})]$

Table 2: Analytical expressions for invariant amplitudes

T	Analytical expression
$T^5_{K^0_L\eta}$	$\frac{\sqrt{h_\eta h_K}}{\pi^2}\frac{16}{3}[\sin\varphi(\Lambda_u^2\frac{\Lambda_u^2+\Lambda_s^2}{2}\Pi_{PP}(m_K^2,\Lambda_u,\Lambda_s)\Pi_{PP}(m_K^2,\Lambda_u,\Lambda_u)-$ $-F_{IPP}(0,m_K^2,m_{\eta'}^2,\Lambda_u,\Lambda_u,\Lambda_s)(\Lambda_u^3+\Lambda_s^3)C_A^{(1)})+$ $+\sqrt{2}\cos\varphi(\Lambda_s^2\frac{\Lambda_u^2+\Lambda_s^2}{2}\Pi_{PP}(m_K^2,\Lambda_u,\Lambda_s))\Pi_{PP}(m_K^2,\Lambda_s,\Lambda_s)-$ $-F_{IPP}(0,m_K^2,m_{\eta'}^2,\Lambda_s,\Lambda_s,\Lambda_u)(\Lambda_u^3+\Lambda_s^3)C_A^{(1)})]$
$T^5_{K^0_s a_0}$	$\frac{\sqrt{h_k h_{a_0}}}{\pi^2}\frac{16}{3}(\Pi_{PP}(m_K^2,\Lambda_u,\Lambda_s)\Pi_{IS}(\frac{m_K^2}{\Lambda_u^2})\Lambda_u^2\frac{\Lambda_u^2+\Lambda_s^2}{2}+$ $+F_{SPP}(m_{a_0}^2,0,m_K^2,\Lambda_u,\Lambda_s,\Lambda_u)(\Lambda_s^3-\Lambda_u^3)C_a^{(1)})$
$T^5_{K^0_s\sigma}$	$\frac{\sqrt{h_k h_\sigma}}{\pi^2}\frac{16}{3}[\cos\delta_S(\Pi_{PP}(m_K^2,\Lambda_u,\Lambda_s)\Pi_{IS}(m_K^2,\Lambda_u,\Lambda_u)\Lambda_u^2\frac{\Lambda_u^2+\Lambda_s^2}{2}+$ $+F_{SPP}(m_\sigma^2,0,m_K^2,\Lambda_u,\Lambda_s,\Lambda_u)(\Lambda_s^3-\Lambda_u^3)C_a^{(1)})-$ $-\sqrt{2}\sin\delta_S(\Pi_{PP}(m_K^2,\Lambda_u,\Lambda_s)\Pi_{IS}(m_K^2,\Lambda_s,\Lambda_s)\Lambda_s^2\frac{\Lambda_u^2+\Lambda_s^2}{2}+$ $+F_{SPP}(m_\sigma^2,0,m_K^2,\Lambda_s,\Lambda_u,\Lambda_s)(\Lambda_s^3-\Lambda_u^3)C_a^{(1)})]$
$T^5_{K^0_s f_0}$	$\frac{\sqrt{h_k h_\sigma}}{\pi^2}\frac{16}{3}[\sin\delta_S(\Pi_{PP}(m_K^2,\Lambda_u,\Lambda_s)\Pi_{IS}(m_K^2,\Lambda_u,\Lambda_u))\Lambda_u^2\frac{\Lambda_u^2+\Lambda_s^2}{2}+$ $+F_{SPP}(m_{f_0}^2,0,m_K^2,\Lambda_u,\Lambda_s,\Lambda_u)(\Lambda_s^3-\Lambda_u^3)C_a^{(1)})-$ $+\sqrt{2}\cos\delta_S(\Pi_{PP}(m_K^2,\Lambda_u,\Lambda_s)\Pi_{IS}(m_K^2,\Lambda_s,\Lambda_s)\Lambda_s^2\frac{\Lambda_u^2+\Lambda_s^2}{2}+$ $+F_{SPP}(m_{f_0}^2,0,m_K^2,\Lambda_s,\Lambda_u,\Lambda_s)(\Lambda_s^3-\Lambda_u^3)C_a^{(1)})]$

Table 2: Analytical expressions for invariant amplitudes (continue)

T	Numerical value	T	Numerical value
$T^1_{K^0_L\gamma\gamma}$	$-1.06\cdot10^{-5}$ GeV	$T^5_{K^0_L\gamma\gamma}$	$-1.43\cdot10^{-4}$ GeV
$T^1_{K^0_L\pi}$	$9.69\cdot10^{-3}GeV^4$	$T^5_{K^0_L\pi}$	$2.1\cdot10^{-1}GeV^4$
$T^1_{K^0_L\eta'}$	$5.49\cdot10^{-3}GeV^4$	$T^2_{K^0_L\eta'}$	$-6.6\cdot10^{-2}GeV^4$
$T^3_{K^0_L\eta'}$	$1.71\cdot10^{-2}GeV^4$	$T^1_{K^0_L\eta}$	$6.3\cdot10^{-3}GeV^4$
$T^2_{K^0_L\eta}$	$-2.04\cdot10^{-2}GeV^4$	$T^3_{K^0_L\eta}$	$9.15\cdot10^{-2}GeV^4$
$T^5_{K^0_L\eta}$	$4.07\cdot10^{-1}GeV^4$	$T^5_{K^0_s a_0}$	$0.22GeV^4$
$T^5_{K^0_s\sigma}$	$0.36GeV^4$	$T^5_{K^0_s f_0}$	$0.22GeV^4$

Table 3: Numerical values for invariant amplitudes

The photons $K^0_S\to\gamma\gamma$ decay have parallel polarisation ($F_{\mu\nu}F^{\mu\nu}$),so its amplitude is determined by $f_2(0,0)$ from (33)and, up to one loop, there is no short- distance contribution due to Furry's theoreme . We denote

$$f_s(0,0)=M(K^0_S\to\gamma\gamma)$$

Matrix element of the studied decay can be written in the form:

$$M(K_s^0 \to \gamma\gamma) = \frac{G_F}{2\sqrt{2}} V_{ud} V_{us}^* \cdot c_5 \cdot \sum_{S=a_0,\sigma(600),f_0(980)} T_{KS}^5 D_S(m_K^2) g_{S\gamma\gamma}(m_K^2) \tag{39}$$

where $g_{S\gamma\gamma}(m_K^2)$-form factor of scalar radiative decay at $m_S^2 = m_K^2$:

$$g_{s\gamma\gamma}\left(m_s^2\right) = \alpha\sqrt{6h_s(H)}\frac{1}{\Lambda}Tr\left\{Q^2\lambda_s\right\}\left[F_{SVV}^1\left(m_s^2\right) + HF_{SVV}^2\left(m_s^2\right)\right] \tag{40}$$

$F_{SVV}^{1,2}\left(m_s^2\right)$ are defined in following way:

$$F_{SVV}^1\left(m_s^2\right) = \frac{x}{4}\int_0^1 dua\left(-u\frac{x}{4}\right)(1+u)\ln\left(\frac{1+\sqrt{1-u}}{1-\sqrt{1-u}}\right) \tag{41}$$

$$F_{SVV}^2\left(m_s^2\right) = \frac{x}{4}\int_0^1 dub\left(-u\frac{x}{4}\right)u\ln\left(\frac{1+\sqrt{1-u}}{1-\sqrt{1-u}}\right) \tag{42}$$

Table 2 represents analytical expressions obtained in QCM for T_{KS}^5,while the numerical value are displayed in Table 3.

The decay width of $K^0 \to \gamma\gamma$ decay is given by

$$\Gamma(K^0 \to \gamma\gamma) = \frac{m_K^3}{64\pi}|M((K^0 \to \gamma\gamma)|^2 \tag{43}$$

Matrix elements $M((K^0 \to \gamma\gamma)$ is defined (34) and (39).Numerical value of obtained in the QCM invariant amplitudes $T^i_{K_L^0\gamma\gamma}$, T^i_{KP} and T^5_{KS} are given in Tables 2,3. We use the set of coefficients c_i (16) and numerical values $G_F = 1.1664 \times 10^{-5} GeV^{-2}$ for Fermi constant [11] ,$|V_{ud}| = 0.97425$, $|V_{us}| = 0.2253$ for CKM matrix elements [7].

Table 4 summarizes our values of branching ratios $K_{L,S}^0 \to \gamma\gamma$

Decay	QCM	Experiment[7]
$K_L^0 \to \gamma\gamma$	5.58×10^{-4}	$(5.47 \pm 0.04) \times 10^{-4}$
$K_S^0 \to \gamma\gamma$	2.083×10^{-6}	$(2.63 \pm 0.17) \times 10^{-6}$

Table 4: The values of branching ratios $K_{L,S}^0 \to \gamma\gamma$

The table shows that the obtained numerical values are in good agreement with modern experimental data. It should be noted that intermediate hadron states give the main contribution to the amplitude. We were able to describe $K_S^0 \to \gamma\gamma$ due to the correct account of the intermediate scalar mesons.

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