Form Factor of the Relativistic Two-particle System in the Relativistic Quasipotential Approach: The Case of Arbitrary Masses and Vector Current

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Abstract

A new relativistic form factor for a bound two-particle system was obtained for the case of a vector current. The present consideration was performed within the relativistic quasipotential approach based on the covariant Hamiltonian formulation of quantum field theory by going over to the three dimensional relativistic configuration representation for the case of interaction between two relativistic spinless particles of arbitrary mass.

1 Introduction

The study of hadrons electromagnetic form factors allows to obtain the information about spatial hadrons structure. The idea of the composite quark nature of hadrons and suggestion about scale invariant behavior in the region of large momentum transfers has allowed to reveal regularity of the elastic hadrons form factors behavior [1]. To describe the behavior of the form factors the different pole vector-dominance models (VDM) were used. These models successfully reproduce the behavior of the pion form factor as in space-like, so and at time-like regions [2], and behavior of the nucleon form factor in the space-like regions [3]. However the models VDM fail in description experimently of the observed for large importances of the momentum transfer of the system $-t = Q^2$ the quick decrease of electromagnetic form factor at time-like region according to the law of dipole $\sim t^{-2}$. The reason is that the model VDM assume that the virtual photon flying in the nucleon "sees" only the vector mesons which there are the quark-antiquark bound-states while the structure of nucleon study at small distances where the momentum transfer of the system there is enough large value and quarks move quasifree (the asymptotic freedom).

However the problem of covariant description of form factor in the whole, rather then only in asymptotic region energy within the framework of relativistic quark model taking into account differences of their masses, continues remain interesting and at present. For this we must know the dynamics of the interacting quarks more in detail, in particular, we must know the covariant wave functions their of relative motion.

Within the quantum field theory the covariant wave functions of the relative motion can be obtained using the relativistic covariant two-particle quasipotential equations of Logunov–Tavkhelidze [4] and Kadyshevsky [5, 6]. The using of three-dimensional relativistic quasipotential (RQP) equation of Logunov-Tavkhelidze for description of the form factors of composite systems was executed in [7–11]. However, use of the equation Logunov-Tavkhelidze for wave function in the momentum representation has not allowed to research the behavior of the form factor in broad interval of importances of the momentum transfer of the relativistic two-particle bound system. The other model of the account of the contribution small the distances in form factor of the proton was considered in [12]. This model is based on invariant description of the structure of the particles in relativistic configurational space that was carried in [13] in the case of interaction between two relativistic spinless particles that have equal masses m in which the Compton wavelength of particle plays role of the natural scale. In this model is taken into account both the contribution to the proton form factor of vector mesons and the contribution from its the central part having radius of the Compton wavelength. The method of transition to the relativistic configurational representation in the case of interaction between two relativistic spinless particles with equal masses proposed in [13] was used in [14] to construct the three-dimensional covariant formalism for the description of relativistic two-particle systems. Within the framework of this formalism the expressions for the form factors of relativistic two-particle systems [15, 16] were obtained.

The aim of this work is to obtain the expression for the elastic form factor of relativistic two-particle system in the case of vector current on the basis of covariant Hamiltonian formulation of quantum field theory [5, 6] by transition to the three-dimensional relativistic configurational representation for the interaction of two relativistic spinless particles having arbitrary masses m_1, m_2 [17, 18].

2 Equation for the wave function

In the case of interaction between two relativistic particles with arbitrary masses m_1 and m_2, the RQP approach developed in [17, 18] permitted introducing the concept of an effective relativistic particle whose mass is $m' = \sqrt{m_1 m_2}$ and which plays the role of a bound two-particle system. Whereby one reduces the two-body problem in question to a one-body problem treated in terms of the RQP wave function $\Psi_{M_\mathcal{Q}}(\boldsymbol{\Delta}_{p',m'\lambda_\mathcal{Q}})$ describing the effective relativistic particle and satisfying the fully covariant RQP Kadyshevsky equation in angular momentum space with the velocity 4-vector $\lambda_\mathcal{Q} = (\lambda^0_\mathcal{Q}; \boldsymbol{\lambda}_\mathcal{Q})$; [1)] that is,

$$(2\Delta^0_{q',m'\lambda_\mathcal{Q}} - 2\Delta^0_{p',m'\lambda_\mathcal{Q}})\Psi_{M_\mathcal{Q}}(\boldsymbol{\Delta}_{p',m'\lambda_\mathcal{Q}}) = \tag{1}$$

$$= \frac{2\mu}{m'}\frac{1}{(2\pi)^3}\int d\Omega_{\boldsymbol{\Delta}_{k',m'\lambda_\mathcal{Q}}} \widetilde{V}(\boldsymbol{\Delta}_{p',m'\lambda_\mathcal{Q}}, \boldsymbol{\Delta}_{k',m'\lambda_\mathcal{Q}}; \Delta^0_{q',m'\lambda_\mathcal{Q}})\Psi_{M_\mathcal{Q}}(\boldsymbol{\Delta}_{k',m'\lambda_\mathcal{Q}}),$$

where $\mu = m_1 m_2/(m_1 + m_2)$ is the ordinary reduced mass of two particles that have arbitrary masses and $d\Omega_{\boldsymbol{\Delta}_{k',m'\lambda_\mathcal{Q}}} = m' d\boldsymbol{\Delta}_{k',m'\lambda_\mathcal{Q}}/\Delta^0_{k',m'\lambda_\mathcal{Q}}$ is the relativistic three-dimensional volume element in Lobachevsky space, all 4-momenta now belonging to the upper sheet of the mass hyperboloid:

$$\Delta^{02}_{k',m'\lambda_\mathcal{Q}} - \boldsymbol{\Delta}^2_{k',m'\lambda_\mathcal{Q}} = m'^2. \tag{2}$$

This sheet, embedded in 4-dimensional momentum space, serves as a model of relativistic non-Euclidean momentum space. On the mass-hyperboloid sheet (2), the Lorentz group is the motion group for this space. Upon choosing the pure Lorentz transformation (boost) $\Lambda^{-1}_{\lambda_\mathcal{Q}}$ corresponding to the composite particle 4-velocity $\lambda_\mathcal{Q}$, $\Lambda^{-1}_{\lambda_\mathcal{Q}}\mathcal{Q} = (M_\mathcal{Q}; \mathbf{0})$, the 4-vector components $\Delta_{k',m'\lambda_\mathcal{Q}}$ from the Lobachevsky space assume the form

$$\Delta^0_{k',m'\lambda_\mathcal{Q}} = (\Lambda^{-1}_{\lambda_\mathcal{Q}} k')^0 = k'_0\lambda^0_\mathcal{Q} - \mathbf{k}' \cdot \boldsymbol{\lambda}_\mathcal{Q} = \sqrt{m'^2 + \boldsymbol{\Delta}^2_{k',m'\lambda_\mathcal{Q}}}, \tag{3}$$

$$\boldsymbol{\Delta}_{k',m'\lambda_\mathcal{Q}} = \Lambda^{-1}_{\lambda_\mathcal{Q}}\mathbf{k}' = \mathbf{k}'(-)m'\boldsymbol{\lambda}_\mathcal{Q} = \mathbf{k}' - \boldsymbol{\lambda}_\mathcal{Q}\left(k'_0 - \frac{\mathbf{k}' \cdot \boldsymbol{\lambda}_\mathcal{Q}}{1 + \lambda^0_\mathcal{Q}}\right).$$

[1)] We use the system of units where $\hbar = c = 1$.

Equation (1) can be considered as a direct relativistic generalization of the Schrödinger equation in the spirit of Lobachevsky geometry arising on the upper mass-hyperboloid sheet (2). This equation describes scattering on the quasipotential $\widetilde{V}(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{\Delta}_{k',m'\lambda_Q}; \Delta^0_{q',m'\lambda_Q})$ for an effective relativistic particle that plays the role of a two-particle system, has a mass m' and a relative 3-momentum $\mathbf{\Delta}_{q',m'\lambda_Q}$, and carries the total energy of two free relativistic particles of arbitrary mass. This energy $\sqrt{s_q} = M_Q$ is proportional to the energy $\Delta^0_{q',m'\lambda_Q}$ for one effective relativistic particle of mass m'; that is,

$$\sqrt{s_q} = \sqrt{(q_1+q_2)^2} = \frac{m'}{\mu}\Delta^0_{q',m'\lambda_Q},\ \Delta^0_{q',m'\lambda_Q} = \sqrt{m'^2 + \mathbf{\Delta}^2_{q',m'\lambda_Q}}. \qquad (4)$$

In the equation (1) it is convenient to expand over the complete system of functions [17, 18]

$$\xi(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r}) = \left(\frac{\Delta^0_{p',m'\lambda_Q} - \mathbf{\Delta}_{p',m'\lambda_Q}\cdot\mathbf{n}}{m'}\right)^{-1-ir/\lambda'}, \qquad (5)$$

which realize the principal series of unitary irreducible representations of the Lorentz group, i.e. the group of motions of the Lobachevsky space momentum, realized on upper sheet of the mass hyperboloid (2). The group parameter r in (5) plays the role of the modulus of the relativistic relative coordinate $\mathbf{r}$ ($\mathbf{r} = r\mathbf{n}, |\mathbf{n}| = \mathbf{1}$), and $\lambda' = 1/m'$ is the Compton wavelength associated with the effective relativistic particle of mass m' [13, 18]. This parameter enumerates the eigenvalues of the invariant Casimir operator of the Lorentz group $\hat{C}_L = (1/4)M_{\mu\nu}M^{\mu\nu}$ ($M_{\mu\nu} = p_\mu\partial/\partial p^\nu - p_\nu\partial/\partial p^\mu$ are the group generators):

$$\widehat{C}_L\xi(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r}) = \left(\frac{1}{m'^2} + r^2\right)\xi(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r}), 0 \le r < \infty, \qquad (6)$$

and, therefore, it is a relativistic invariant.

The functions in (5) obey the following conditions of completeness and orthogonality [18]:

$$\frac{1}{(2\pi)^3}\int d\Omega_{\mathbf{\Delta}_{p',m'\lambda_Q}}\xi(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r})\xi^*(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r}') = \delta(\mathbf{r}' - \mathbf{r}), \qquad (7)$$

$$\frac{1}{(2\pi)^3}\int d\mathbf{r}\xi(\mathbf{\Delta}_{q',m'\lambda_Q}, \mathbf{r})\xi^*(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r}) = \frac{\Delta^0_{q',m'\lambda_Q}}{m'}\delta(\mathbf{\Delta}_{p',m'\lambda_Q} - \mathbf{\Delta}_{q',m'\lambda_Q}),$$

and these the functions satisfy the equation in terms of finite differences [18]

$$(2\Delta^0_{p',m'\lambda_Q} - \widehat{H}_0)\xi(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r}) = 0. \tag{8}$$

Here

$$\widehat{H}_0 = 2m'\left[\cosh\left(i\lambda'\frac{\partial}{\partial r}\right) + \frac{i\lambda'}{r}\sinh\left(i\lambda'\frac{\partial}{\partial r}\right) - \frac{\lambda'^2}{2r^2}\Delta_{\theta,\varphi}\exp\left(i\lambda'\frac{\partial}{\partial r}\right)\right] \tag{9}$$

is the operator of the free Hamiltonian, while $\Delta_{\theta,\varphi}$ is its angular part.

The wave RQP-functions in the momentum space and the $\mathbf{r}$-representation, called the relativistic configuration representation [17, 18], are related by

$$\psi_{M_Q}(\mathbf{r}) = \frac{1}{(2\pi)^3}\int d\Omega_{\mathbf{\Delta}_{p',m'\lambda_Q}}\xi(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r})\Psi_{M_Q}(\mathbf{\Delta}_{p',m'\lambda_Q}), \tag{10}$$

$$\Psi_{M_Q}(\mathbf{\Delta}_{p',m'\lambda_Q}) = \int d\mathbf{r}\xi^*(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r})\psi_{M_Q}(\mathbf{r}).$$

For the local quasipotential

$$\widetilde{V}(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{\Delta}_{k',m'\lambda_Q}; \Delta^0_{q',m'\lambda_Q}) \equiv \widetilde{V}\left((\mathbf{\Delta}_{p',m'\lambda_Q}(-)\mathbf{\Delta}_{k',m'\lambda_Q})^2; \Delta^0_{q',m'\lambda_Q}\right) \tag{11}$$

square of the vector of momentum transfer in the Lobachevsky space $\mathbf{\Delta}_{p',k'} = \mathbf{p}'(-)\mathbf{k}'$ is the Loretz invariant that allows to present it in the form

$$\mathbf{\Delta}^2_{p',k'} = \left(\Delta^0_{p',k'}\right)^2 - m'^2 = \left(\mathbf{\Delta}_{p',m'\lambda_Q}(-)\mathbf{\Delta}_{k',m'\lambda_Q}\right)^2 = \mathbf{\Delta}^2_{\Delta_{p',m'\lambda_Q},\Delta_{k',m'\lambda_Q}}.$$

Thus, the quasipotential (11) depends on the invariant quantity the square of vector of difference in the Lobachevsky space of two momentum vectors $\mathbf{\Delta}_{\Delta_{p',m'\lambda_Q},\Delta_{k',m'\lambda_Q}} = \mathbf{\Delta}_{p',m'\lambda_Q}(-)\mathbf{\Delta}_{k',m'\lambda_Q}$. With this quasipotential, the right-hand side of equation (1) represents a convolution in the Lobachevsky space that allows to use the expansion over the matrix elements of group of motions of this space, i.e. transformations (10). By using transformations (10) and eq. (8), equation (1) with the quasipotential (11) local in the Lobachevsky space takes the form

$$(2\Delta^0_{q',m'\lambda_Q} - \widehat{H}_0)\psi_{M_Q}(\mathbf{r}) = \frac{2\mu}{m'}V(\mathbf{r}; \Delta^0_{q',m'\lambda_Q})\psi_{M_Q}(\mathbf{r}), \tag{12}$$

where the quasipotential $V(\mathbf{r};\Delta^0_{q',m'\lambda_\mathcal{Q}})$ is given in terms of the same relativistic plane waves as

$$V(\mathbf{r};\Delta^0_{q',m'\lambda_\mathcal{Q}}) = \frac{1}{(2\pi)^3}\int d\Omega_{\boldsymbol{\Delta}_{p',k'}}\xi(\boldsymbol{\Delta}_{p',k'},\mathbf{r})\widetilde{V}\left((\boldsymbol{\Delta}_{p',k'})^2;\Delta^0_{q',m'\lambda_\mathcal{Q}}\right).$$

For spherically symmetric potentials, expanding the quasipotential wave RQP-function $\psi_{M_\mathcal{Q}}(\mathbf{r})$ in the Legendre functions $P^\nu_\mu(z)$ of the first kind as

$$\psi_{M_\mathcal{Q}}(\mathbf{r}) = \sum_{\ell=0}^{\infty}(2\ell+1)i^\ell\frac{\varphi_\ell(r,\chi)}{r}P_\ell\left(\frac{\boldsymbol{\Delta}_{q',m'\lambda_\mathcal{Q}}\cdot\mathbf{r}}{|\boldsymbol{\Delta}_{q',m'\lambda_\mathcal{Q}}|r}\right), \tag{13}$$

we obtain equation for the partial wave function in the form

$$\left[\cosh\left(i\lambda'\frac{d}{dr}\right)+\frac{\lambda'^2\ell(\ell+1)}{2r(r+i\lambda')}\exp\left(i\lambda'\frac{d}{dr}\right)-X(r)\right]\varphi_\ell(r,\chi)=0, \tag{14}$$

where

$$X(r) = \frac{\mu}{m'^2}\left(M_\mathcal{Q}-V(r;\chi)\right),$$

and χ is the rapidity related with the relative 3-momentum and energy of effective relativistic particle by the formulas

$$\boldsymbol{\Delta}_{q',m'\lambda_\mathcal{Q}} = m'\sinh\chi\,\mathbf{n}_{\Delta_{q',m'\lambda_\mathcal{Q}}},\ |\mathbf{n}_{\Delta_{q',m'\lambda_\mathcal{Q}}}|=1,$$
$$M_\mathcal{Q} = \frac{m'}{\mu}\Delta^0_{q',m'\lambda_\mathcal{Q}},\ \Delta^0_{q',m'\lambda_\mathcal{Q}} = m'\cosh\chi.$$

3 Form factor of the relativistic two-particle system

For simplicity we consider here only the case of spinless field when the Hamiltonian density is given by the expression

$$H(x) = -z_1\varphi_1^+(x)\varphi_1(x)A(x) - z_2\varphi_2^+(x)\varphi_2(x)A(x). \tag{15}$$

In ref. [15] founded on refs. [7–11], the form factor of two-particle system was defined as the matrix element of the local current operator between bound states with the 4-momentum $\mathcal{P}$, $\mathcal{Q}$ through the covariant wave RQP-functions satisfying eq. (1). Then, as follows from refs. [15, 16], the invariant expression in the momentum representation for the matrix element of the local vector-current operator near poles of bound states for the interaction of two relativistic spinless particles with arbitrary masses m_1, m_2 has the form

$$< \mathcal{P}|J_\nu|\mathcal{Q} >= \frac{z_1}{(2\pi)^3}\int d\tau_{\mathcal{P}} d\tau_{\mathcal{Q}} d^{(4)}k_2 d^{(4)}k_1 d^{(4)}k_1' \theta(k_{20})\delta(k_2^2 - m_2^2)\times \quad (16)$$

$$\times\Gamma^+_{\mathcal{P}}\left(k_1', k_2; \lambda_{\mathcal{P}}\tau_{\mathcal{P}}\right)\frac{(k_1+k_1')_\nu}{(\tau_{\mathcal{P}}+i\varepsilon)(\tau_{\mathcal{Q}}-i\varepsilon)}\Gamma_{\mathcal{Q}}\left(k_1, k_2; \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}\right)\theta(k_{10})\delta({k_1}^2 - m_1^2)\times$$

$$\times\theta(k_{10}')\delta(k_1'^2 - m_1^2)\delta^{(4)}(-\mathcal{Q}+k_1+k_2+\lambda_{\mathcal{Q}}\tau_{\mathcal{Q}})\,\delta^{(4)}(\mathcal{P}-k_2-k_1'-\lambda_{\mathcal{P}}\tau_{\mathcal{P}})+$$
$$+(1\leftrightarrow 2),$$

where all the momenta of the particles belong to the mass shells

$$k_i^2 = k_{i0}^2 - \mathbf{k}_i^2 = m_i^2, i = 1, 2. \quad (17)$$

As a vectors $\lambda_{\mathcal{P}}$ and $\lambda_{\mathcal{Q}}$, it is convenient to choose the 4-velocities of the system: $\lambda_{\mathcal{P}} = \mathcal{P}/\sqrt{\mathcal{P}^2}, \mathcal{P}^2 = (p_1+p_2)^2 = s_p = M^2_{\mathcal{P}}$ and $\lambda_{\mathcal{Q}} = \mathcal{Q}/\sqrt{\mathcal{Q}^2}, \mathcal{Q}^2 = (q_1+q_2)^2 = s_q = M^2_{\mathcal{Q}}$. This expression answers the diagram on fig. 1. Here follows to emphasize that because of transition to different own timeses of the system before ($\tau_{\mathcal{Q}} = \lambda_{\mathcal{Q}}X, X = x_1 + x_2$) and after interaction ($\tau_{\mathcal{P}} = \lambda_{\mathcal{P}}X$) the diagram on fig. 1 differ from diagrams, which appear in approach of the Kadyshevsky for S-matrix. The 4-velocities of the com-

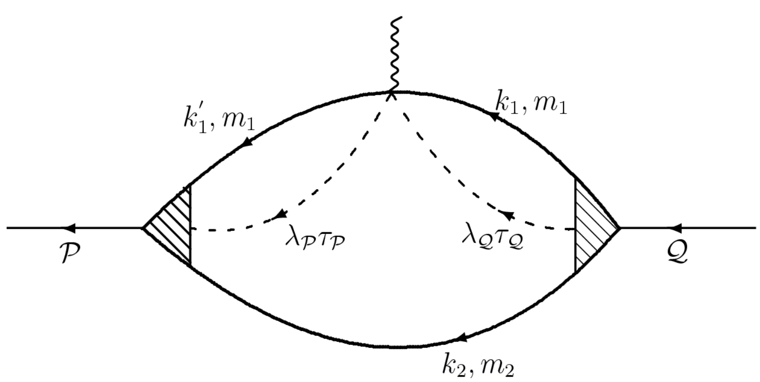

Figure 1: *The diagram for the matrix element of the local current operator between bound states with the 4-momentum $\mathcal{P}, \mathcal{Q}$ for the interaction of two relativistic spinless particles with arbitrary masses.*

posite particle before, $\lambda_{\mathcal{Q}}$, and after interaction, $\lambda_{\mathcal{P}}$, will differ also.

In the case of equal quark masses ($m_1 = m_2 = m$) and for real-valued wave functions, expression (16) for the matrix element of the vector-current operator satisfies the transverseness condition

$$(\mathcal{P} - \mathcal{Q})^\nu < \mathcal{P}|J_\nu|\mathcal{Q} >= 0, \tag{18}$$

This circumstance was used in [16]. In the case of unequal quark masses ($m_1 \neq m_2$), expression (16) for the matrix element of the vector-current operator features additionally its transverse component, which breaks the transverseness condition in (18). Therefore, the 4-vector in expression (16) can be represented in the form

$$< \mathcal{P}|J_\nu|\mathcal{Q} >= F^{(+)}(t)(\mathcal{P} + \mathcal{Q})_\nu + iF^{(-)}(t)(\mathcal{P} - \mathcal{Q})_\nu. \tag{19}$$

In the case of unequal masses ($m_1 \neq m_2$), expression (16) for the matrix element of the local vector-current operator can be reduced to a one-body problem. The respective expression will be the convolution of the RQP wave functions for a single effective relativistic particle in this space. Thus, it is necessary to multiply expression (16) by $(\mathcal{P} \pm \mathcal{Q})^\nu$ and to consider that, at $\mathcal{Q}^2 = M_\mathcal{Q}^2$, $\mathcal{P}^2 = M_\mathcal{P}^2$, the following relation holds:

$$t{=}(\mathcal{P}{-}\mathcal{Q})^2{=}{-}Q^2{=}M_\mathcal{Q}^2{+}M_\mathcal{P}^2{-}2\mathcal{P}\mathcal{Q}, 2\mathcal{P}\mathcal{Q}{=} \tag{20}$$

$$={M_\mathcal{Q}^2}{+}M_\mathcal{P}^2{-}t, (\mathcal{P}{+}\mathcal{Q})^2{=}2(M_\mathcal{Q}^2{+}M_\mathcal{P}^2){-}t. \tag{21}$$

Performing integration with respect to $dk_{20}, dk_{10}, dk'_{10}$ and taking into account Eqs. (19) and (20), we obtain the following expressions for the form-factor components:

$$F^{(+)}(t) = \frac{z_1}{(2M_\mathcal{Q}^2 + 2M_\mathcal{P}^2 - t)(4\pi)^3} \int \frac{d\tau_\mathcal{P} d\tau_\mathcal{Q} d\mathbf{k}_2 d\mathbf{k}_1 d\mathbf{k}'_1}{\sqrt{m_2^2 + \mathbf{k}_2^2}\sqrt{m_1^2 + \mathbf{k}_1^2}\sqrt{m_1^2 + \mathbf{k}_1'^2}} \times \tag{22}$$

$$\times \Gamma^+_\mathcal{P}\left(k'_1, k_2; \lambda_\mathcal{P}\tau_\mathcal{P}\right) \frac{(\mathcal{P} + \mathcal{Q})(k_1 + k'_1)}{(\tau_\mathcal{P} + i\varepsilon)(\tau_\mathcal{Q} - i\varepsilon)} \Gamma_\mathcal{Q}\left(k_1, k_2; \lambda_\mathcal{Q}\tau_\mathcal{Q}\right) \times$$

$$\times \delta^{(4)}\left[\left(-1 + \frac{\tau_\mathcal{Q}}{M_\mathcal{Q}}\right)\mathcal{Q} + k_1 + k_2\right] \delta^{(4)}\left[\left(1 - \frac{\tau_\mathcal{P}}{M_\mathcal{P}}\right)\mathcal{P} - k_2 - k'_1\right] +$$

$$+(1 \leftrightarrow 2),$$

$$F^{(-)}(t) == \frac{z_1}{it(4\pi)^3}\int \frac{d\tau_{\mathcal{P}} d\tau_{\mathcal{Q}} d\mathbf{k}_2 d\mathbf{k}_1 d\mathbf{k}'_1}{\sqrt{m_2^2+\mathbf{k}_2^2}\sqrt{m_1^2+\mathbf{k}_1^2}\sqrt{m_1^2+\mathbf{k}_1'^2}}\times \tag{23}$$

$$\times\Gamma^+_{\mathcal{P}}\left(k'_1,k_2;\lambda_{\mathcal{P}}\tau_{\mathcal{P}}\right)\frac{(\mathcal{P}-\mathcal{Q})(k_1+k'_1)}{(\tau_{\mathcal{P}}+i\varepsilon)(\tau_{\mathcal{Q}}-i\varepsilon)}\Gamma_{\mathcal{Q}}\left(k_1,k_2;\lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}\right)\times$$

$$\times\delta^{(4)}\left[\left(-1+\frac{\tau_{\mathcal{Q}}}{M_{\mathcal{Q}}}\right)\mathcal{Q}+k_1+k_2\right]\delta^{(4)}\left[\left(1-\frac{\tau_{\mathcal{P}}}{M_{\mathcal{P}}}\right)\mathcal{P}-k_2-k'_1\right]+(1\leftrightarrow 2).$$

Within this approach, for the bounded system of spinless particles which are found in the motion with moment $J=0$ the vertex functions $\Gamma_{\mathcal{Q}}\left(k_2,k_1;\lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}\right)$ and $\Gamma_{\mathcal{P}}\left(k_2,k'_1;\lambda_{\mathcal{P}}\tau_{\mathcal{P}}\right)$ when $\lambda_{\mathcal{Q}}\upuparrows\mathcal{Q}$ and $\lambda_{\mathcal{P}}\upuparrows\mathcal{P}$ will depend each only on one the Lorentz invariant scalar parameter, as which we choose accordingly $\mathcal{Q}k_2$ and $\mathcal{P}k_2$. According to Eqs. (3), these parameters are invariant under the pure Lorentz transformations $\Lambda^{-1}_{\lambda_{\mathcal{Q},\mathcal{P}}}$: $\Lambda^{-1}_{\lambda_{\mathcal{Q}}}\mathcal{Q}=(M_{\mathcal{Q}};\mathbf{0})$, $\Lambda^{-1}_{\lambda_{\mathcal{P}}}\mathcal{P}=(M_{\mathcal{P}};\mathbf{0})$; therefore, we have

$$\mathcal{Q}k_2=\Lambda^{-1}_{\lambda_{\mathcal{Q}}}(\mathcal{Q}k_2)=M_{\mathcal{Q}}\Delta^0_{k_2,m_2\lambda_{\mathcal{Q}}},\mathcal{P}k_2=\Lambda^{-1}_{\lambda_{\mathcal{P}}}(\mathcal{P}k_2)=M_{\mathcal{P}}\Delta^0_{k_2,m_2\lambda_{\mathcal{P}}}.$$

Moreover, the application of the Lorentz transformation $\Lambda^{-1}_{\lambda_{\mathcal{Q},\mathcal{P}}}$ to the conservation laws

$$-\mathcal{Q}+k_1+k_2+\lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}=0,\mathcal{P}-k_2-k'_1-\lambda_{\mathcal{P}}\tau_{\mathcal{P}}=0, \tag{24}$$

yields

$$\tau_{\mathcal{Q}}=M_{\mathcal{Q}}-\Delta^0_{k_2,m_2\lambda_{\mathcal{Q}}}-\Delta^0_{k_1,m_1\lambda_{\mathcal{Q}}},\boldsymbol{\Delta}_{k_1,m_1\lambda_{\mathcal{Q}}}=-\boldsymbol{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}; \tag{25}$$

$$\tau_{\mathcal{P}}=M_{\mathcal{P}}-\Delta^0_{k_2,m_2\lambda_{\mathcal{P}}}-\Delta^0_{k'_1,m_1\lambda_{\mathcal{P}}},\boldsymbol{\Delta}_{k'_1,m_1\lambda_{\mathcal{P}}}=-\boldsymbol{\Delta}_{k_2,m_2\lambda_{\mathcal{P}}}.$$

From Eqs. (24) and (25), it also follows that

$$k_1+k'_1=\lambda_{\mathcal{Q}}\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}}+\lambda_{\mathcal{P}}\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}}}-2k_2,$$

where we have used the invariance of the total energy under Lorentz transformations; that is,

$$\sqrt{s_k}=\sqrt{(k_2+k_1)^2}=\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}}=\sqrt{m_1^2+\boldsymbol{\Delta}^2_{k_2,m_2\lambda_{\mathcal{Q}}}}+\sqrt{m_2^2+\boldsymbol{\Delta}^2_{k_2,m_2\lambda_{\mathcal{Q}}}}, \tag{26}$$

$$\sqrt{s_{k'}}=\sqrt{(k_2+k'_1)^2}=\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}}}=\sqrt{m_1^2+\boldsymbol{\Delta}^2_{k_2,m_2\lambda_{\mathcal{P}}}}+\sqrt{m_2^2+\boldsymbol{\Delta}^2_{k_2,m_2\lambda_{\mathcal{P}}}}.$$

Taking into account Eq. (20), we find from here that

$$(\mathcal{P}\pm\mathcal{Q})(k_1+k_1')=\frac{M_{\mathcal{Q}}^2+M_{\mathcal{P}}^2-t}{2M_{\mathcal{Q}}}\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}}\pm \tag{27}$$

$$\pm\frac{M_{\mathcal{Q}}^2+M_{\mathcal{P}}^2-t}{2M_{\mathcal{P}}}\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}}}\pm\frac{(m_1^2-m_2^2)M_{\mathcal{Q}}}{\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}}}+\frac{(m_1^2-m_2^2)M_{\mathcal{P}}}{\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}}}}.$$

Now in (22) and (23) we execute the integrations respecting of $\mathbf{k}_1$,$\mathbf{k}_1'$,$\tau_{\mathcal{P}}$,$\tau_{\mathcal{Q}}$. For that we execute the pure Lorentz transformations $\Lambda^{-1}_{\lambda_{\mathcal{Q}}}$ and $\Lambda^{-1}_{\lambda_{\mathcal{P}}}$ by formulas (3) in the integrals with respect to $\mathbf{k}_1$ and $\mathbf{k}_1'$ accordingly, and take into account Eq. (27) and the invariance of the delta functions involved and the integration measures $d\Omega_{\mathbf{k}_i}=m_i d\mathbf{k}_i/\sqrt{m_i^2+\mathbf{k}_i^2}, i=1,2$ on the mass hyperboloids (17) under Lorentz transformations. Expressions (22) and (23) for the form-factor components can then be recast into the form

$$F^{(+)}(t)=\frac{z_1}{\left(2M_{\mathcal{Q}}^2+2M_{\mathcal{P}}^2-t\right)(4\pi)^3}\int\frac{d\boldsymbol{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}}{\sqrt{m_2^2+\boldsymbol{\Delta}^2_{k_2,m_2\lambda_{\mathcal{Q}}}}}\times \tag{28}$$

$$\times\frac{\Gamma^+_{M_{\mathcal{P}}}\left(\boldsymbol{\Delta}_{k_2,m_2\lambda_{\mathcal{P}}}\right)}{\sqrt{m_1^2+\boldsymbol{\Delta}^2_{k_2,m_2\lambda_{\mathcal{P}}}}\left(M_{\mathcal{P}}-\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}}}+i\varepsilon\right)}\left[\frac{M_{\mathcal{Q}}^2+M_{\mathcal{P}}^2-t}{2M_{\mathcal{Q}}}\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}}+\right.$$

$$+\frac{M_{\mathcal{Q}}^2+M_{\mathcal{P}}^2-t}{2M_{\mathcal{P}}}\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}}}+\left(m_1^2-m_2^2\right)\left(\frac{M_{\mathcal{Q}}}{\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}}}+\right.$$

$$\left.\left.+\frac{M_{\mathcal{P}}}{\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}}}}\right)\right]\frac{\Gamma_{M_{\mathcal{Q}}}\left(\boldsymbol{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}\right)}{\sqrt{m_1^2+\boldsymbol{\Delta}^2_{k_2,m_2\lambda_{\mathcal{Q}}}}\left(M_{\mathcal{Q}}-\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}}-i\varepsilon\right)}+(1\leftrightarrow 2),$$

$$F^{(-)}(t)=\frac{z_1}{it(4\pi)^3}\int\frac{d\boldsymbol{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}}{\sqrt{m_2^2+\boldsymbol{\Delta}^2_{k_2,m_2\lambda_{\mathcal{Q}}}}}\times \tag{29}$$

$$\times\frac{\Gamma^+_{M_{\mathcal{P}}}\left(\boldsymbol{\Delta}_{k_2,m_2\lambda_{\mathcal{P}}}\right)}{\sqrt{m_1^2+\boldsymbol{\Delta}^2_{k_2,m_2\lambda_{\mathcal{P}}}}\left(M_{\mathcal{P}}-\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}}}+i\varepsilon\right)}\left[\frac{M_{\mathcal{Q}}^2+M_{\mathcal{P}}^2-t}{2M_{\mathcal{Q}}}\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}}-\right.$$

$$-\frac{M_{\mathcal{Q}}^2+M_{\mathcal{P}}^2-t}{2M_{\mathcal{P}}}\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}}}-\left(m_1^2-m_2^2\right)\left(\frac{M_{\mathcal{Q}}}{\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}}}-\right.$$

$$\left.\left.-\frac{M_{\mathcal{P}}}{\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}}}}\right)\right]\frac{\Gamma_{M_{\mathcal{Q}}}\left(\boldsymbol{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}\right)}{\sqrt{m_1^2+\boldsymbol{\Delta}^2_{k_2,m_2\lambda_{\mathcal{Q}}}}\left(M_{\mathcal{Q}}-\sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}}-i\varepsilon\right)}+(1\leftrightarrow 2),$$

where we have introduced the notation $\Gamma_{\mathcal{Q}}\left(k_1,k_2;\lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}\right)=\Gamma_{M_{\mathcal{Q}}}\left(\mathbf{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}\right)$, $\Gamma_{\mathcal{P}}\left(k_1',k_2;\lambda_{\mathcal{P}}\tau_{\mathcal{P}}\right)=\Gamma_{M_{\mathcal{P}}}\left(\mathbf{\Delta}_{k_2,m_2\lambda_{\mathcal{P}}}\right)$.

Within the RQP approach being considered, the two body problem under study reduces to a one-body problem formulated in terms of the RQP wave function $\Psi_{M_{\mathcal{Q}}}(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}}})$ describing an effective relativistic particle and satisfying the fully covariant RQP Kadyshevsky equation (1) in the angular-momentum space. The 4-vector k' is chosen as

$$k'=(k_0';\mathbf{k}')=\sqrt{\frac{\mathcal{K}^2}{\mathcal{K}_\perp^2}}\mathcal{K}_\perp, \tag{30}$$

where $\mathcal{K}=(m_1k_2-m_2k_1)/(m_1+m_2)$, the vector $\mathcal{K}_\perp=\mathcal{K}-\lambda_{\mathcal{K}}(\lambda_{\mathcal{K}}\mathcal{K})$ is the Wightman–Gording vector, and $\lambda_{\mathcal{K}}=(k_1+k_2)/\sqrt{s_k}=\lambda_{\mathcal{Q}}$. Signifies, $(\lambda_{\mathcal{K}}\mathcal{K}_\perp)=0$, but from (30) we find:

$$k'^2={k_0}'^2-\mathbf{k}'^2=\mathcal{K}^2=\frac{m_1m_2}{(m_1+m_2)^2}\left[(m_1+m_2)^2-s_k\right]. \tag{31}$$

Under the Lorentz transformations (3) follows that

$$\Lambda_{\lambda_{\mathcal{Q}}}^{-1}\mathcal{K}_\perp=(0;\mathbf{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}), \tag{32}$$

$$\Lambda_{\lambda_{\mathcal{Q}}}^{-1}\mathcal{K}=\left(\frac{m_1\sqrt{m_2^2+\mathbf{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}^2}-m_2\sqrt{m_1^2+\mathbf{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}^2}}{m_1+m_2};\mathbf{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}\right).$$

Then from (26), (31) and (32) we get expression $(\Lambda_{\lambda_{\mathcal{Q}}}^{-1}k_0'=0)$

$$\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}}}^2=-(\Lambda_{\lambda_{\mathcal{Q}}}^{-1}k')^2=-(\Lambda_{\lambda_{\mathcal{Q}}}^{-1}\mathcal{K})^2=\frac{m_1m_2}{(m_1+m_2)^2}\left[s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}-(m_1+m_2)^2\right].$$

As direction of the vector $\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}}}$ in correspondence to (30) and (32), we choose the direction of the vector $\mathbf{\Delta}_{k_2m_2\lambda_{\mathcal{Q}}}$:

$$\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}}}=\sqrt{\frac{(\Lambda_{\lambda_{\mathcal{Q}}}^{-1}\mathcal{K})^2}{(\Lambda_{\lambda_{\mathcal{Q}}}^{-1}\mathcal{K}_\perp)^2}}(\Lambda_{\lambda_{\mathcal{Q}}}^{-1}\mathcal{K}_\perp)=\frac{\mathbf{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}}{|\mathbf{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}|}\times$$

$$\times\left[\mathbf{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}^2-\left(\frac{m_1\sqrt{m_2^2+\mathbf{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}^2}-m_2\sqrt{m_1^2+\mathbf{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}^2}}{m_1+m_2}\right)^2\right]^{1/2}. \tag{33}$$

The inverse transformation have the form

$$\mathbf{\Delta}_{k_2,m_2\lambda_\mathcal{Q}} = \mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}} \frac{m'}{2\mu}\sqrt{\frac{4\mu^2+\mathbf{\Delta}^2_{k',m'\lambda_\mathcal{Q}}}{m'^2+\mathbf{\Delta}^2_{k',m'\lambda_\mathcal{Q}}}}. \tag{34}$$

Farther, in expression (28) and (29) we shall perform the change of variables according to Eqs. (33), (34) and take into account Eq. (4). The expressions for the components of the elastic form factor ($M_\mathcal{P} = M_\mathcal{Q} = M$) then takes the form

$$F^{(+)}(t) = \frac{(z_1+z_2)(2M^2-t)}{M(4M^2-t)(2\pi)^3}\frac{2\mu}{m'}\int d\Omega_{\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}}}\Psi^*_M(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})\times \tag{35}$$

$$\times\left[\frac{f_+(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})+f_-(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})}{2f(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})}\right]\left(\Delta^0_{k',m'\lambda_\mathcal{P}}+\Delta^0_{k',m'\lambda_\mathcal{Q}}\right)\Psi_M(\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}})+$$

$$+\frac{(z_1-z_2)(m_1^2-m_2^2)M}{2(4M^2-t)(2\pi)^3}\left(\frac{2\mu}{m'}\right)^3\int d\Omega_{\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}}}\Psi^*_M(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})\times$$

$$\times\left[\frac{f_+(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})+f_-(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})}{2f(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})}\right]\left(\frac{\Delta^0_{k',m'\lambda_\mathcal{P}}+\Delta^0_{k',m'\lambda_\mathcal{Q}}}{\Delta^0_{k',m'\lambda_\mathcal{P}}\Delta^0_{k',m'\lambda_\mathcal{Q}}}\right)\Psi_M(\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}}),$$

$$F^{(-)}(t) = \frac{(z_1+z_2)(2M^2-t)}{iM(-t)(2\pi)^3}\frac{2\mu}{m'}\int d\Omega_{\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}}}\Psi^*_M(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})\times \tag{36}$$

$$\times\left[\frac{f_+(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})+f_-(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})}{2f(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})}\right]\left(\Delta^0_{k',m'\lambda_\mathcal{P}}-\Delta^0_{k',m'\lambda_\mathcal{Q}}\right)\Psi_M(\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}})+$$

$$+\frac{(z_1-z_2)(m_1^2-m_2^2)M}{2i(-t)(2\pi)^3}\left(\frac{2\mu}{m'}\right)^3\int d\Omega_{\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}}}\Psi^*_M(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})\times$$

$$\times\left[\frac{f_+(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})+f_-(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})}{2f(\mathbf{\Delta}_{k',m'\lambda_\mathcal{P}})}\right]\left(\frac{\Delta^0_{k',m'\lambda_\mathcal{P}}-\Delta^0_{k',m'\lambda_\mathcal{Q}}}{\Delta^0_{k',m'\lambda_\mathcal{P}}\Delta^0_{k',m'\lambda_\mathcal{Q}}}\right)\Psi_M(\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}}),$$

where

$$f_\pm(\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}}) = \frac{\sqrt{m'^2+\mathbf{\Delta}^2_{k',m'\lambda_\mathcal{Q}}}}{m'^2+\mathbf{\Delta}^2_{k',m'\lambda_\mathcal{Q}}\pm m'\sqrt{m'^2-4\mu^2}},$$

$$f(\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}}) = \frac{\sqrt{4\mu^2+\mathbf{\Delta}^2_{k',m'\lambda_\mathcal{Q}}}}{m'^2+\mathbf{\Delta}^2_{k',m'\lambda_\mathcal{Q}}},$$

and we have defined the wave function for the system in the angular-momentum space as

$$\Psi_M(\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{Q}}}) = \frac{f(\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{Q}}})\Gamma_M(\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{Q}}})}{2^{3/2}\sqrt{m'}\left(\frac{2\mu M}{m'} - 2\Delta^0_{k',m'\lambda_{\mathcal{Q}}}\right)},$$

and are introduced the notations

$$\Gamma_{M_{\mathcal{Q}}}(\boldsymbol{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}) = \Gamma_M(\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{Q}}}), \Gamma_{M_{\mathcal{P}}}(\boldsymbol{\Delta}_{k_2,m_2\lambda_{\mathcal{P}}}) = \Gamma_M(\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{P}}}).$$

It should be noted that the factor $[f_+ + f_-]/2f(\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{P}}})$ possible to be simplified to the form

$$\frac{f_+(\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{P}}}) + f_-(\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{P}}})}{2f(\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{P}}})} \approx 1 + \frac{m'^2 - 4\mu^2}{2\Delta^{02}_{k',m'\lambda_{\mathcal{P}}}},$$
$$\frac{m'\sqrt{m'^2 - 4\mu^2}}{\Delta^{02}_{k',m'\lambda_{\mathcal{P}}}} < 1,\ \ \frac{m'^2 - 4\mu^2}{\Delta^{02}_{k',m'\lambda_{\mathcal{P}}}} < 1,$$

and the vector $\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{P}}}$ from the Lobachevsky space arising on the upper mass-hyperboloid sheet (2) can be represented in the form

$$\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{P}}} = \Lambda^{-1}_{\lambda_{\mathcal{P}}}\mathbf{k}' = \left(\Lambda^{-1}_{\lambda_{\mathcal{P}}}\Lambda_{\lambda_{\mathcal{Q}}}\Lambda_{\Delta_{\mathcal{P},\mathcal{Q}}}\right)\left(\Lambda^{-1}_{\Delta_{\mathcal{P},\mathcal{Q}}}\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{Q}}}\right) =$$
$$= V(\Lambda_{\lambda_{\mathcal{Q}}}, \mathcal{P})\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{Q}}}(-)\frac{m'}{M}\boldsymbol{\Delta}_{\mathcal{P},\mathcal{Q}}. \quad (37)$$

Here $\Delta_{\mathcal{P},\mathcal{Q}} = \Lambda^{-1}_{\lambda_{\mathcal{Q}}}\mathcal{P}$ is the 4-momentum transfer in the Lobachevsky space; that is,

$$\boldsymbol{\Delta}_{\mathcal{P},\mathcal{Q}} = \Lambda^{-1}_{\mathcal{Q}}\boldsymbol{\mathcal{P}} = \boldsymbol{\mathcal{P}} - \frac{\boldsymbol{\mathcal{Q}}}{M}\left(\mathcal{P}_0 - \frac{\boldsymbol{\mathcal{P}}\cdot\boldsymbol{\mathcal{Q}}}{\mathcal{Q}_0 + M}\right) = M\sinh\chi_\Delta\mathbf{n}_\Delta, \quad (38)$$

$$\Delta^0_{\mathcal{P},\mathcal{Q}} = \left(\Lambda^{-1}_{\mathcal{Q}}\mathcal{P}\right)^0 = \frac{\mathcal{P}_0\mathcal{Q}_0 - \boldsymbol{\mathcal{P}}\cdot\boldsymbol{\mathcal{Q}}}{M} = \frac{\mathcal{P}\mathcal{Q}}{M} = M\cosh\chi_\Delta,$$

$$\boldsymbol{\mathcal{P}} = M\sinh\chi_{\mathcal{P}}\mathbf{n}_{\mathcal{P}},\ \ \boldsymbol{\mathcal{Q}} = M\sinh\chi_{\mathcal{Q}}\mathbf{n}_{\mathcal{Q}},$$

$$\mathcal{P}_0 = M\cosh\chi_{\mathcal{P}},\ \ \mathcal{Q}_0 = M\cosh\chi_{\mathcal{Q}},$$

$$|\mathbf{n}_{\mathcal{P}}| = |\mathbf{n}_{\mathcal{Q}}| = |\mathbf{n}_\Delta| = 1,\ \ \Delta^{02}_{\mathcal{P},\mathcal{Q}} - \boldsymbol{\Delta}^2_{\mathcal{P},\mathcal{Q}} = M^2,$$

where $V(\Lambda_{\lambda_\mathcal{Q}},\mathcal{P}) = \Lambda^{-1}_{\lambda_\mathcal{P}}\Lambda_{\lambda_\mathcal{Q}}\Lambda_{\Delta_{\mathcal{P},\mathcal{Q}}}$ is Wigner's rotation matrix and χ_Δ, $\chi_\mathcal{P}$, $\chi_\mathcal{Q}$ are the respective rapidities . From Eqs. (3) and (38), it follows that

$$\Delta^0_{k',m'\lambda_\mathcal{P}} \gtrsim \frac{m'^2\Delta^0_{\mathcal{P},\mathcal{Q}}}{2M\Delta^0_{k',m'\lambda_\mathcal{Q}}}, \tag{39}$$

and the square of the 4-momentum transfer, $Q^2 = -t = -(\mathcal{P}-\mathcal{Q})^2$, is related to the 3-momentum transfer $\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}}$ by the equation

$$Q^2 = -t = -2M^2 + 2M\sqrt{M^2 + \mathbf{\Delta}^2_{\mathcal{P},\mathcal{Q}}} = 2M^2\left(\cosh\chi_\Delta - 1\right). \tag{40}$$

Consequently, the components $F^{(\pm)}(t)$ of the elastic form factor in (35) and (36) can be considered as functions of the invariant variable $\mathbf{\Delta}^2_{\mathcal{P},\mathcal{Q}}$, which is the square of the momentum-transfer vector in the Lobachevsky space. Then, taking into consideration Eqs. (3), (3) and (39), they are convolutions of the wave functions in this space. It follows that, by employing the Shapiro transformation in (10), the addition theorem for relativistic plane waves (5) in the form [18]

$$\int d\omega_n \xi\left(\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}}(-)\frac{m'}{M}\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}},\mathbf{r}\right) = \int d\omega_n \xi(\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}},\mathbf{r})\xi^*\left(\frac{m'}{M}\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}},\mathbf{r}\right), \tag{41}$$

the completeness condition in (7), equation (8), and the Hermitian of operator of the free Hamiltonian (9), one can recast expressions (35) and (36) into the form of relativistic Fourier transforms of covariant RQP wave functions in the coordinate representation[2]:

$$F^{(+)}(Q^2) \approx \tag{42}$$

$$\approx \left\{\frac{(z_1+z_2)(2M^2+Q^2)}{M(4M^2+Q^2)}\frac{2\mu}{m'} + \frac{2M^3(z_1-z_2)(m_1^2-m_2^2)}{m'^2(4M^2+Q^2)(2M^2+Q^2)}\left(\frac{2\mu}{m'}\right)^3\right\}\times$$

$$\times\left\{\int d\mathbf{r}\xi^*\left(\frac{m'}{M}\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}},\mathbf{r}\right)\mathrm{Re}\big[\psi^*_M(\mathbf{r})\widehat{H}_0\psi_M(\mathbf{r})\big]+\right.$$

$$\left.+\frac{2M^4(m'^2-4\mu^2)}{m'^4(2M^2+Q^2)^2}\int d\mathbf{r}\xi^*\left(\frac{m'}{M}\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}},\mathbf{r}\right)\mathrm{Re}\big[\left(\widehat{H}_0\psi_M(\mathbf{r})\right)^*\widehat{H}_0^2\psi_M(\mathbf{r})\big]\right\},$$

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[2] A similar expression for the case of two particles of equal mass was obtained in [16].

$$F^{(-)}(Q^2) \approx \tag{43}$$

$$\approx \left\{ \frac{(z_1+z_2)(2M^2+Q^2)}{MQ^2}\frac{2\mu}{m'} + \frac{2M^3(z_1-z_2)(m_1^2-m_2^2)}{m'^2Q^2(2M^2+Q^2)}\left(\frac{2\mu}{m'}\right)^3 \right\}\times$$

$$\times\left\{ \int d\mathbf{r}\xi^*\left(\frac{m'}{M}\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}},\mathbf{r}\right)\mathrm{Im}\left[\psi_M(\mathbf{r})\big(\widehat{H}_0\psi_M(\mathbf{r})\big)^*\right]+\right.$$

$$\left.+\frac{2M^4(m'^2-4\mu^2)}{m'^4(2M^2+Q^2)^2}\int d\mathbf{r}\xi^*\left(\frac{m'}{M}\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}},\mathbf{r}\right)\mathrm{Im}\left[\left(\widehat{H}_0\psi_M(\mathbf{r})\right)\big(\widehat{H}_0^2\psi_M(\mathbf{r})\big)^*\right]\right\},$$

where possibility to applicability of the addition theorem (41) follows from independence of the wave RQP-function $\psi_M(\mathbf{r})$ in the case of $J=0$ from direction of the vector $\mathbf{r}$.

We note that, if the RQP wave function $\psi_M(\mathbf{r})$ is a real-valued function of the variable r and corresponds to a real-valued quasipotential $V(r)$, then, according to Eq. (12), the quantity $\psi_M(\mathbf{r})\big(\widehat{H}_0\psi_M(\mathbf{r})\big)^*$ is also real-valued. It follows that, in this case and at equal masses ($m_1=m_2=m$), the transverse component $F^{(-)}(t)$ of the elastic form factor vanishes. For s-state ($\ell=0$) the radial wave function $\varphi_0(r,\chi_n)$ corresponding to a real-valued quasipotential $V(r)$ is real-valued, the quantities $\frac{\varphi_0^*(r,\chi_n)}{r}\widehat{H}_{0,\ell=0}\frac{\varphi_0(r,\chi_n)}{r}$ and $\left(\widehat{H}_{0,\ell=0}\frac{\varphi_0(r,\chi_n)}{r}\right)^*\widehat{H}_{0,\ell=0}^2\frac{\varphi_0(r,\chi_n)}{r}$ are also real-valued. It follows that, in this case, the transverse component $F^{(-)}(t)$ of the elastic form factor for the s-wave state vanishes even for unequal masses ($m_1\neq m_2$).

For s-state ($\ell=0$) of the composite system the integrations in (42) respecting of angular variables gives

$$F^{(+)}_{\ell=0}(Q^2)= \tag{44}$$

$$=\left\{\frac{8\pi\mu(z_1+z_2)(2M^2+Q^2)}{m'M(4M^2+Q^2)}+\frac{8\pi M^3(z_1-z_2)(m_1^2-m_2^2)}{m'^2(4M^2+Q^2)(2M^2+Q^2)}\left(\frac{2\mu}{m'}\right)^3\right\}\frac{\chi_\Delta}{\sinh\chi_\Delta}\times$$

$$\times\left\{\int_0^\infty dr\frac{r\sin(rm'\chi_\Delta)}{m'\chi_\Delta}\mathrm{Re}\left[\frac{\varphi_0^*(r,\chi_n)}{r}\widehat{H}_{0,\ell=0}\frac{\varphi_0(r,\chi_n)}{r}\right]+\frac{2M^4(m'^2-4\mu^2)}{m'^4(2M^2+Q^2)^2}\times\right.$$

$$\left.\times\int_0^\infty dr\frac{r\sin(rm'\chi_\Delta)}{m'\chi_\Delta}\mathrm{Re}\left[\left(\widehat{H}_{0,\ell=0}\frac{\varphi_0(r,\chi_n)}{r}\right)^*\widehat{H}_{0,\ell=0}^2\frac{\varphi_0(r,\chi_n)}{r}\right]\right\},$$

where are used decompositions (13) for wave function $\psi_M(\mathbf{r})$ and the expansion for the relativistic plane wave (5) in the form

$$\xi(\mathbf{p}',\mathbf{r})=\sum_{\ell=0}^{\infty}(2\ell+1)i^{\ell}p_{\ell}(r,\cosh\chi_{p'})P_{\ell}\left(\frac{\mathbf{p}'\cdot\mathbf{r}}{p'r}\right).$$

Here the rapidity χ_n corresponds to the level n bound state with energy $M=M_n=(m'^2/\mu)\cosh\chi_n$; the function

$$p_{\ell}(\rho,\cosh\chi_{p'})=\sqrt{\frac{\pi}{2\sinh\chi_{p'}}}\frac{(-1)^{\ell+1}}{\rho}(-\rho)^{(\ell+1)}P^{-1/2-\ell}_{-1/2+i\rho}(\cosh\chi_{p'}),\ \rho=rm',$$

is a solution of the equation (8), where the function $(-\rho)^{(l+1)}=i^{l+1}\Gamma(l+1+i\rho)/\Gamma(i\rho)$ is called the generalized power [18], and $\Gamma(z)$ is a gamma function.

4 Root-mean-square radius and form factor for the Coulomb interaction

Now let us consider the expression for the invariant root-mean-square radius of a composite system, which has the group-theoretical meaning of an eigenvalue of the Casimir operator of the Loretz group and according to Eqs. (6) and (44) has the form

$$<r_0^2>=$$

$$=\frac{6\partial F^{(+)}_{\ell=0}(t)/\partial t|_{t=0}}{F^{(+)}_{\ell=0}(0)}=\frac{1}{M^2}+\left(\frac{m'}{M}\right)^2\frac{\int\limits_0^{\infty}drr^2\left(r^2-\frac{3}{2m'^2}\right)\left(R_1+\frac{m'^2-4\mu^2}{2m'^4}R_2\right)}{\int\limits_0^{\infty}drr^2\left(R_1+\frac{m'^2-4\mu^2}{2m'^4}R_2\right)}+$$

$$+\frac{3(m'^2-4\mu^2)\int\limits_0^{\infty}drr^2R_2}{m'^4M^2\int\limits_0^{\infty}drr^2\left(R_1+\frac{m'^2-4\mu^2}{2m'^4}R_2\right)}+\frac{3(z_1-z_2)(m_1^2-m_2^2)\left(2\mu/m'\right)^2}{m'^2M^2\left[z_1+z_2+\frac{(z_1-z_2)(m_1^2-m_2^2)(2\mu/m')^2}{2m'^2}\right]},$$

where

$$R_1=\mathrm{Re}\left[\frac{\varphi_0^*(r,\chi)}{r}\widehat{H}_{0,\ell=0}\frac{\varphi_0(r,\chi)}{r}\right],\ R_2=\mathrm{Re}\left[\left(\widehat{H}_{0,\ell=0}\frac{\varphi_0(r,\chi)}{r}\right)^*\widehat{H}^2_{0,\ell=0}\frac{\varphi_0(r,\chi)}{r}\right].$$

Thus, it is necessary to consider the composite particle as a dipole and that the wave function of s-state describes not all structure of the composite particle, but only the region which be upon distances that larger its of the Compton wavelength $1/M$. The root-mean-square radius of the composite system includes not only the central sphere of radius $r_0 = 1/M$, where the relative motion of the quarks forming this system proceeds, but also terms generated by the difference in the masses of the quarks and in their coupling constants. At $m_1 = m_2$, these terms vanish.

As example, we consider the form factor in the case of the attractive Coulomb field

$$V(r) = -\frac{\alpha}{r}, \alpha > 0. \tag{45}$$

The radial wave function of exact solution of the RQP-equation (14) with interaction (45) for the s-state and ground level ($n = 0$) with the energy M_0 has the form [19–21]

$$\varphi_0(r, i\kappa_0) = N_{0,0}(\kappa_0) r m' \exp\left[-r m' \kappa_0 + \frac{i\tilde{\alpha}\kappa_0}{2\sin\kappa_0}\right],$$

where $\tilde{\alpha} = 2\mu\alpha/m'$, $M_0 = (m'^2/\mu)\cos\kappa_0$, κ_0 defines by the following quantization condition $\tilde{\alpha}/(2\sin\kappa_0) = 1, 0 \leq \kappa_0 < \pi/2$, and $N^2_{0,0}(\kappa_0) = m'\kappa_0^3/\pi$ is the normalization factor.

The form factor (44) for the ground level of the bound s-state with the energy M_0 then assumes the form

$$F^{(+)}_{\ell=0,n=0}(Q^2) = \frac{16\mu\kappa_0^3 \sin\kappa_0 (2M_0^2 + Q^2)}{M_0(4M_0^2 + Q^2)\chi_\Delta \sinh\chi_\Delta} \times \tag{46}$$

$$\times \left[z_1 + z_2 + \frac{2(z_1 - z_2) M_0^4 (m_1^2 - m_2^2)}{m'^2 (2M_0^2 + Q^2)^2} \left(\frac{2\mu}{m'}\right)^2\right] \left\{ \frac{1}{1 + (2\kappa_0/\chi_\Delta)^2} + \right.$$

$$+ \frac{4\kappa_0}{\chi_\Delta^2 \tan\kappa_0 \, (1 + (2\kappa_0/\chi_\Delta)^2)^2} + \frac{4\pi M_0^4 (m'^2 - 4\mu^2) \chi_\Delta \sin 2\kappa_0}{m'^2 (2M_0^2 + Q^2)^2} \times$$

$$\times \left[1 - \frac{2}{\pi} \arctan\frac{2\kappa_0}{\chi_\Delta} + \frac{3}{\pi\chi_\Delta \tan\kappa_0 \, (1 + (2\kappa_0/\chi_\Delta)^2)} + \right.$$

$$\left.\left. + \frac{4\kappa_0}{\pi\chi_\Delta^3 \tan^2\kappa_0 \, (1 + (2\kappa_0/\chi_\Delta)^2)^2}\right]\right\}.$$

For large Q^2 the rapidity $\chi_\Delta \approx \ln(Q^2/M_0^2)$ and, consequently, the leading behavior of form factor (46) gives by expression

$$F^{(+)}_{\ell=0,n=0}(Q^2) \approx 8(z_1+z_2)\left(\frac{2\mu}{m'}\right)^2 \frac{\kappa_0^3 \tan\kappa_0}{(Q/M_0)^2 \ln(Q/M_0)^2}, \tag{47}$$

i.e. either as in [16]. Such behavior of the form factor under large $t = -Q^2$ differs from prediction of the nonrelativistic model based on the Coulomb potential, which gives the dipole decrease of the pion form factor: $F_\pi \sim t^{-2}$. However, the nonrelativistic result contradicts the prediction of the dimensional quark counting rules [1], which gives the decrease of the pion form factor under the law $F_\pi \sim t^{-1}$.

5 Conclusions

For the case of a vector current, the new covariant expressions for the components of the elastic form factor for a bound system of two relativistic spinless particles of arbitrary masses m_1 and m_2 are obtained. The components of the elastic form factor are functions of the invariant variable $\mathbf{\Delta}^2_{\mathcal{P},\mathcal{Q}}$, which is the square of the momentum-transfer vector in the Lobachevsky space. The consideration is conducted within the framework of relativistic quasipotential approach on the basis of covariant Hamiltonian formulation of quantum field theory [5, 6] by transition to the three-dimensional relativistic configurational representation in the case of two interacting relativistic spinless particles of arbitrary masses m_1, m_2 [17, 18]. In this approach, the invariant relativistic relative coordinate r is conjugated to the rapidity $m'\chi_\Delta$, and it is the distance in the Lobachevsky space.

It has been shown that expressions (35) and (36) for the form-factor components are convolutions of the RQP wave functions in the space of Lobachevsky angular momenta. This makes it possible to express them in terms of relativistic Fourier transforms of covariant RQP wave functions in the configuration representation [expressions (42)–(44))]. It has also been found that, for a real-valued RQP wave function $\psi_M(\mathbf{r})$, corresponding to a real-valued quasipotential $V(r)$ and in the case of equal masses ($m_1 = m_2 = m$), the transverse component $F^{(-)}(t)$ of the elastic form factor vanishes. Under the same real-valued conditions for the s-state ($\ell = 0$) of the radial wave function $\varphi_0(r, \chi_n)$ and the quasipotential, the transverse component $F^{(-)}_{\ell=0}(t)$ of the elastic form factor also vanishes even in the case of unequal masses ($m_1 \neq m_2$).

Using of the three-dimensional relativistic configurational representation for the system of two relativistic spinless particles with arbitrary masses has allowed to install that the wave function of s-state describes not whole structure of the composite particle, but only the region which be upon distances that larger its of the Compton wavelength $1/M$. The executed analysis has shown, that the leading contribution to structure of the composite particle from the central sphere of radius $r_0 = 1/M$ is proportional to $\chi_\Delta / \sinh \chi_\Delta$ and that the correction terms correspond to the dipole contribution associated with the difference in the masses of the particles constituting this system and in their coupling constants. In the nonrelativistic limit this the relativistic geometric factor go to 1, while the correction terms in expression (44) under $m_1 = m_2$ vanish.

As example, the expression (46) for the longitudinal component of the form factor for relativistic two-particles bound system that have arbitrary masses and in the case of Coulomb quasipotential was obtained. It is installed that the covariant wave RQP-function of attractive Coulomb quasipotential for larges Q^2 gives the decrease for this form factor under the law $F_\pi \sim t^{-1}$, which predicts the dimensional quark counting rules [1].

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