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Low Energy Hadronic Decays of τ-lepton

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Abstract. Hadronic decays of τ-lepton with two or three particles in the final state are investigated in the Quark Confinement Model. Consideration of intermediate states allows one to receive results in a good agreement with experimental data.

Key words: hadronic decays, quark confinement.

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The electroweak meson decays are of extraordinary interest as a source of information about structure of hadrons. Among a great number of this processes the leptonic and semileptonic decays are especially important, because leptons are known as "ideal" probes for study the hadronic matter.
Since the time it was found in 1975 τ-lepton is known as a very important one for investigations of fundamental properties of electroweak interactions. Because of τ-mass value , the hadronic , namely, mesonic decays are allowed. This fact gives one the opportunity to use the hadronic decays of τ-lepton as additional and very powerful tool for study both strong and electroweak phenomena.

Another problem is the evaluation of the hadronic matrix elements.We perform the calculations in the Quark Confinement Model (QCM) [1]. This model based on the certain assumptions about nature of quark confinement and hadronization allows to describe the electromagnetic,strong and weak interactions of light (nonstrange and strange)mesons from a unique point of view.
The following results were obtained for branching ratios of two and three particle τ- lepton decays

DECAY MODE	Br_{QCM}	Br_{exp}
$\tau \to \pi\nu_\tau$	$11,25\%$	$11,31 \pm 0,15$
$\tau \to \rho\nu_\tau$	$23,5\%$	$23,31 \pm 0,05$
$\tau \to a_1\nu_\tau$	$10,4\%$	-
$\tau \to \pi^-\pi^0\nu_\tau$	$23,7\%$	$25,24 \pm 0,16$
$\tau - \pi\eta\nu_\tau$	$1,52 \times 10^{-4}\%$	-
$\tau - K\eta\nu_\tau$	$5,72 \times 10^{-4}\%$	-

Obtained results show us the QCM to be able to describe hadron interactions of heavy lepton. Good agreement with experimental data was achieved by consistent account of intermediate hadron states.

1. Quark Confinement Model

The hadronic interactions will be described in the QCM.This model is based on the following assumptions [1]:

The hadron fields are assumed to arise after integration over gluon and quark variables in the QCM generating function. The transition of hadrons to quarks and vice versa is given by the interaction Lagrangian. In particular necessary interaction Lagrangians for $\pi^{\pm},^{\pm}$ and η mesons look like:

$$\mathcal{L}_{\pi^{\pm}} = \frac{g_{\pi^{\pm}}}{\sqrt{2}}\pi^{\pm}\bar{q}^{a}\gamma_5\frac{\lambda^1 \pm i\lambda^2}{\sqrt{2}}q^a \tag{1}$$

$$\mathcal{L}_{K^{\pm}} = \frac{g_{K^{\pm}}}{\sqrt{2}}K^{\pm}\bar{q}^{a}\gamma_5\frac{\lambda^4 \pm i\lambda^5}{\sqrt{2}}q^a \tag{2}$$

$$\mathcal{L}_{\eta} = \frac{g_{\eta^{\pm}}}{\sqrt{2}}\eta\bar{q}^{a}\gamma_5\frac{\lambda^8 sin\theta + \lambda^0 cos\theta}{\sqrt{2}}q^a. \tag{3}$$

Angle θ in (3) , is a mixing angle for η - mesons. The coupling constants g_M for meson-quark interaction are defined from so-called compositeness condition [3] It us convenient to use interaction constant in a form:

$$h_M = \frac{3g_M^2}{4\pi^2} = -\frac{1}{\tilde{\prod}'_M(m_M)} \tag{4}$$

instead of g_M in the further calculations. All hadron-quark interactions are described by quark diagrams induced by S matrix averaged over vacuum backgrounds.

The second QCM assumption is that the quark confinement is provided by nontrivial gluon vacuum background. The averaging of quark diagrams generated by S-matrix over vacuum gluon fields $\hat{B}_{VAC}$ is suggested to provide quark confinement and to make the ultraviolet finite theory. The confinement ansatz in the case of one-loop quark diagrams consists in following replacement:

$$\int d\sigma_{VAC} Tr|M(x_1)S(x_1,x_2|B_{VAC})...M(x_n)S(x_n,x_1|B_{VAC})| \longrightarrow$$
$$\int d\sigma_v Tr|M(x_1)S_v(x_1-x_2)...M(x_n)S_v(x_n-x_1)|, \tag{5}$$

where

$$S_v(x_1-x_2) = \int \frac{d^4p}{i(2\pi)^4}e^{-ip(x_1-x_2)}\frac{1}{v\Lambda_q - \hat{p}} \tag{6}$$

The parameter Λ_q characterizes the confinement rang of quark with flavour number $q = u, d, s$. The measure $d\sigma_v$ is defined as:

$$\int \frac{d\sigma_v}{v-\hat{z}} = G(z) = a(-z^2) + \hat{z}b(-z^2) \tag{7}$$

The function $G(z)$ is called the confinement function. $G(z)$ is independent on flavor or color of quark. $G(z)$ is an entire analytical function on the z-plane.$G(z)$ decreases faster then any degree of z in Eucidean region.The choice of $G(z)$,or as the same of $a(-z^2)$ $b(-z^2)$, is one of model assumptions.In the notes [1], [2] $a(-z^2)$ and $b(-z^2)$ are chosen as:

$$a(u) = a_0 e^{-u^2 - a_1 u}$$
$$b(u) = b_0 e^{-u^2 - b_1 u} \tag{8}$$

The request of satisfaction of Ward anomaly identity in QCM gives the additional correlation between $a(0)$ and $b(0)$: $b(0) = -a'(0)$, $a(0) = 2$. Using $a(u)$ and $b(u)$ as (8), one can receive: $a_0 = 2$, $a_1 = \frac{b_0}{4}$. So, the free parameters of the model are Λ_q, b_0, b_1. The model parameters were fixed in the [2] by fitting the well-established constants of low-energy physics.(f_π, f_K, $g_{\rho\gamma}$, $g_{\pi\gamma\gamma}$, $g_{\omega\pi\gamma}$, $g_{\rho\pi\pi}$, $g_{K^*\pi\gamma}$)

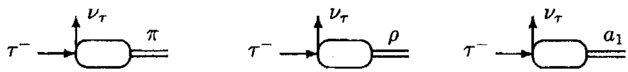
$$\begin{aligned} \Lambda_u &= 460\ MeV \\ \Lambda_s &= 506\ MeV \\ b_0 = 2 \qquad & b_1 = 0.2 \\ a_0 = 2 \qquad & a_1 = 0.5 \end{aligned} \tag{9}$$

We put $\Lambda_u = \Lambda_d$ in the most of decays, but we in the case of decays with η-mesons in the final state we consider Λ_d as an additional free parameter.

2. Two Particle τ-Decays

2.1. $\tau \to \pi\nu_\tau$-Decay

The amplitude of the $\tau \to \pi\nu_\tau$-decay is described by the diagram represented in Fig.1a.



a b c

*Fig.*1

It's analytical expression is written as:

$$M = \frac{G_F}{\sqrt{2}} f_\pi k^\alpha cos\theta_C \phi_\pi \bar{u}_{\nu_\tau} \gamma_\alpha (1+\gamma_5) u_\tau \tag{10}$$

where

$$f_\pi = \frac{\Lambda}{\pi} \frac{\sqrt{3} F_P(\mu_\pi^2)}{\sqrt{2 F_{PP}(\mu_\pi^2)}} \tag{11}$$

The functions $F_P(\mu_\pi^2)$ and $F_{PP}(\mu_\pi^2)$ have the following form:

$$F_P(x) = \int_0^\infty a(u)du + \frac{x}{4}\int_0^1 dua(-u\frac{x}{4})\sqrt{1-u}, \tag{12}$$

$$F_{PP}(x) = \int_0^\infty b(u)du + \frac{x}{4}\int_0^1 dub(-u\frac{x}{4})\frac{1-u/2}{\sqrt{1-u}} \tag{13}$$

here $\mu_\pi^2 = \frac{m_\pi^2}{\Lambda^2}$
The total decay rate for the $\tau \to \pi\nu_\tau$-decay is:

$$\Gamma(\tau \to \pi\nu_\tau) = \frac{G_F^2 cos^2\theta_C f_\pi^2 m_\tau^3}{16\pi}(1 - \frac{m_\pi^2}{m_\tau^2})^2 \tag{14}$$

The obtained numerical values for branching ratio

$$Br(\tau \to \pi\nu_\tau) = \frac{\Gamma(\tau \to \pi\nu_\tau)}{\Gamma_{tot}} = 11,25\%$$

turned out to be in a good agreement with experimental data[8] :

$$Br^{exp}(\tau \to \pi\nu_\tau) = 11,31 \pm 0,15$$

2.2. $\tau \to \rho\nu_\tau$-Decay

The theoretical study of the τ-lepton decay into vector particles is very important because it is the main decay mode of heavy lepton. Also calculation of $\tau \to \rho\nu_\tau$-width is an additional test of model of strong interaction , pretending to describe matrix elements momentum dependence . One have to stress that calculation of this amplitudes in the another theoretical approaches , for example in chiral one [6] , needs the $\rho \to \gamma$ constant which is known to be calculated at zero momentum.
Matrix element of $\tau \to \rho\nu_\tau$-decay is defined by diagram represented in Fig.1b. Amplitude of this decay is obtained as:

$$M^{\mu\nu}(\tau \to \rho\nu_\tau) = [g^{\mu\nu}q^2 - q^\mu q^\nu]M_{\tau\to\rho\nu_\tau}(q^2), \tag{17}$$

where $M_{\tau\to\rho\nu_\tau}(q^2)$ looks like:

$$M_{\tau\to\rho\nu_\tau}(q^2) = \frac{G_F}{\sqrt{2}} cos\Theta_C \sqrt{h_\rho}\frac{\sqrt{3}}{2\pi}\Lambda^2 \Pi_V(q^2) \tag{18}$$

Momentum dependent form factor $\Pi_V(q^2)$ is:

$$\Pi_V(x) = \frac{1}{3\Lambda^2}(\int_0^\infty b(u)du + \frac{x}{4}\int_0^1 dub(-u\frac{x}{4})\sqrt{1-u}) \tag{19}$$

The decay width have been calculated in standard way with the account of (17), (18) is given by::

$$\Gamma(\tau \to \rho\nu_\tau) = \frac{1.}{16\pi}G_F^2 cos^2\theta_C \frac{3h_\rho}{8\pi^2 m_\rho^2}\Lambda^4 m_\tau^3(1 - \frac{m_\rho^2}{m_\tau^2})^2(1 + \frac{2m_\rho^2}{m_\tau^2})\Pi_V(m_\rho^2) \tag{20}$$

The calculated by (20)branching ratio value for $\tau \to \rho\nu_\tau$

$$Br(\tau \to \rho\nu_\tau) = \frac{\Gamma(\tau \to \rho\nu_\tau)}{\Gamma_{tot}} = 23,5\%$$

is in a good agreement with experimental data [8]

$$Br^{exp}(\tau \to \rho\nu_\tau) = 23,31 \pm 0,05$$

2.3. $\tau \to a_1 \nu_\tau$-Decay

The study of τ-lepton decay into axial - vector meson is important because of study of heavy lepton physics (study of decays into $(2n+1)$ π-mesons). Also it is important for the development of model of strong interactions. Last statement is connected with the problem of calculation of constant of mentioned decay in the most approaches. QCM allows one to calculate this matrix element without any additional assumptions and phenomenological parameters.
Diagram defines the matrix element of $\tau \to \rho\nu_\tau$-decay is shown in Fig.1c:

$$M^{\mu\nu}(\tau \to a_1\nu_\tau) = \frac{G_F}{\sqrt{2}} cos\Theta_C \sqrt{h_{a_1}} \frac{\sqrt{3}}{2\pi} \Lambda^2 [g^{\mu\nu} q^2 F_1^A(q^2) - q^\mu q^\nu F_2^A(q^2)], \tag{23}$$

where form-factors $F_1^A(q^2)$ and $F_2^A(q^2)$ can be represented as follows:

$$F_1^A(x) = -2. \int_0^\infty b(u) u du - \frac{4}{3}\frac{x}{4} \left(\int_0^\infty b(u) du + \frac{x}{4} \int_0^1 du b(-u\frac{x}{4})(2u-1)\sqrt{1-u} \right) \tag{24}$$

$$F_2^A = \frac{1}{3\Lambda^2} \left(\int_0^\infty b(u) du + \frac{x}{4} \int_0^1 du b(-u\frac{x}{4})\sqrt{1-u} \right) \tag{25}$$

The width of $\tau \to a_1\nu_\tau$ can be written as:

$$\Gamma(\tau \to a_1\nu_\tau) = \frac{1.}{16\pi} G_F^2 cos^2\theta_C \frac{3h_{a_1}}{8\pi^2 m_{a_1}^2} \Lambda^4 m_\tau^3 (1 - \frac{m_{a_1}^2}{m_\tau^2})^2 (1 + \frac{2m_{a_1}^2}{m_\tau^2}) F_1^A(m_{a_1}^2) \tag{26}$$

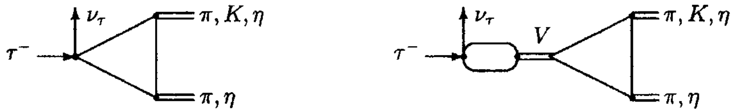
Branching ratio numerical value calculated with(26) turned out to be:

$$Br(\tau \to a_1\nu_\tau) = \frac{\Gamma(\tau \to a_1\nu_\tau)}{\Gamma_{tot}} = 10,4\%$$

We have used the value of a_1 -meson mass equal $m_{a_1} = 1.275$ GeV.

3. $\tau \to P_1 P_2 \nu_\tau$-Decays

Decay $\tau \to \pi\pi\nu_\tau$ is one of the main decay modes of heavy charged lepton. So calculation of it's decay width is necessary for study of τ-lepton hadronic decays [7]. Besides this, study of $\tau \to \pi\pi\nu_\tau$ decay provides one with important information about nonstrange vector mesons properties. The $\tau \to \pi\eta\nu_\tau, \tau \to K\eta\nu_\tau$ decays are of extraordinary interest as a source of information about strong and electroweak interactions.
The amplitude of the decay is described by the diagrams represented in Fig.2.



a) *Fig.2* b)

$$M^\mu(\tau \to P_1 P_2 \nu_\tau) = M^\mu_{dir}(\tau \to P_1 P_2 \nu_\tau) + M^\mu_V(\tau \to P_1 P_2 \nu_\tau), \tag{28}$$

where M^μ_{dir} M^μ_V corresponds to the contribution of direct diagram (Fig.2a) and M^μ_V -contribution of diagram with intermediate vector state (Fig.2b).P_1 means π, or η meson,P_2 - πor$K$.

Matrix element for direct diagram (Fig.2a) was obtained as :

$$M^\mu_{dir} = -\frac{G_F}{\sqrt{2}}\sqrt{h_{\pi^\pm}}\sqrt{\eta}\frac{1}{2}SpJ_C[\lambda_{P_1}, \lambda_{P_2}]((q_1 - q_2)^\mu F_- + (q_1 + q_2)^\mu F_+) \tag{29}$$

where G_F-Fermi constant, $h_{P_{1,2}}$-quark-mesons interaction constants defined by (4). J_C-Cabibbo matrix, defined in standard way.
Form factors $F_- and F_+$ have been received in QCM with account of Λ_u, $\Lambda_d and \Lambda_s$ difference.

Matrix element corresponds the diagram with intermediate vector state can be written in following way:

$$\begin{aligned} M^\mu_{im} &= \frac{G_F}{\sqrt{2}}\sqrt{h_{P_1}}\sqrt{h_{P_2}}\frac{1}{2}Sp(J_C\lambda_V)Sp\lambda_V[\lambda_{P_1}, \lambda_{P_2}] \\ &\times ((q_1+q_2)^\mu (q_1+q_2)^\nu - g^{\mu\nu}(q_1+q_2)^2)h_V G_V^{\nu\sigma}((q_1+q_2)^2) \\ &\times D_{VV}((q_1+q_2)^2)[(q_1-q_2)^\sigma F_- + (q_1+q_2)^\sigma F_+] \end{aligned} \tag{30}$$

Function $D_{VV}((q_1+q_2)^2)$ is a structure integral of the $\tau \to \nu_\tau V$ -decay matrix element(19) It was also evaluated with the account of Λ_u andΛ_d difference.

Finally, the expression (30)for matrix element corresponding diagram Fig.2 can be written as:

$$\begin{aligned} M^\mu_{im} &= \frac{G_F}{\sqrt{2}}\sqrt{h_{P_2}}\sqrt{h_{P_1}}\frac{1}{2}Sp(J_C\lambda_V)Sp\lambda_V[\lambda_{P_2}, \lambda_{P_1}]D_{VV}((q_1+q_2)^2) \\ &[(q_1+q_2)^2(q_1-q_2)^\sigma - (q_1+q_2)^\sigma(q_1^2 - q_2^2)]F_- \end{aligned} \tag{31}$$

Matrix element for three particle decay of heave lepton with η-meson was evaluated with (29) and(31):

$$M^\mu(\tau^- \to P_1 P_2 \nu_\tau) = \frac{G_F}{\sqrt{2}}\sqrt{h_{P_1}}\sqrt{h_{P_2}}(q_1-q_2)^\mu F_1 + (q_1+q_2)^\mu F_2 \tag{32}$$

where form factors F_1 and F_2 have the following form :

$$\begin{aligned} F_1 &= F_-[-\frac{1}{2}SpJ_C[\lambda_{P_1}, \lambda_{P_2}] + \frac{1}{4}Sp(J_C\lambda_V)Sp\lambda_V[\lambda_{P_1}, \lambda_{P_2}] \\ &\times D_{VV}((q_1+q_2)^2)(q_1+q_2)^2\frac{1}{D_{VV}(x) - D_{VV}(m_V^2) + im_V\Gamma_V}] \end{aligned} \tag{33}$$

$$\begin{aligned} F_2 &= -\frac{1}{2}SpJ_C[\lambda_{P_1}, \lambda_{P_2}]F_+ - \frac{1}{4}Sp(J_C\lambda_V)Sp\lambda_V[\lambda_\eta, \lambda_P] \\ &\times D_{VV}((q_1+q_2)^2)\frac{q_1^2 - q_2^2}{D_{VV}(x) - D_{VV}(m_V^2) + im_V\Gamma_V}F_- \end{aligned} \tag{34}$$

Decay width have been calculated in standard way :

$$\Gamma(\tau^- \to P_1 P_2 \nu_\tau) = \frac{G_F^2}{128 m_\tau \pi^3} h_{P_1} h_{P_2} \int\limits_{(m_{P_1}+m_{P_2})^2}^{m_\tau{}^2} dq^2 q^4 \lambda(1, \frac{m_\tau^2}{q^2}, 0) \lambda^{\frac{1}{2}}(1, \frac{m_{P_2}^2}{q^2}, \frac{m_{P_1}^2}{q^2}) \times \quad (35)$$

$$\times \{F_1^2 + 2F_2 F_1 \frac{m_{P_2}^2 - m_{P_1}^2}{q^2} + \frac{1}{3} F_2^2 [1 + \frac{2q^2}{m_\tau^2} - \frac{2}{q^2}(1 + \frac{2q^2}{m_\tau^2})(m_{P_2}^2 + m_{P_1}^2) +$$

$$+ \frac{2}{q^4}(m_{P_2}^2 - m_{P_1}^2)^2 (2 - \frac{q^2}{m_\tau^2})]\}$$

where$\lambda(x, y, z)$ - is defined in standard way

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz) \quad (36)$$

Form factorsF_1, F_2 are defined by (33) and (34).
We have used the following values for ρ-meson parameters [8] :

$$m_\rho = 768,5 \pm 0,6 MeV, \qquad \Gamma_\rho^{full} = 150,7 \pm 1.2 MeV$$

The following branching ratio values were received:

$$\begin{aligned} Br(\tau \to \pi\pi\nu_\tau) &= 23{,}7\ \% \\ Br(\tau \to \pi\eta\nu_\tau) &= 1,52 \times 10^{-4}\% \\ Br(\tau \to K\eta\nu_\tau) &= 5,72 \times 10^{-4}\% \end{aligned}$$

The performed investigation proofs that the "long distance " contribution to the matrix elements is important for proper description of this kind of processes .

References

[1] G. V. Efimov, M. A. Ivanov, Internat. J. Mod. Phys.**A, 3**, (1988) 1290.

[2] E. Z. Avakyan, et al. Fortschr. Phys. **38**, 8,(1990) 611.

[3] T. Eguchi, Phys. Rev. D **14** (1976) 2755

[4] E.Z. Avakyan,et al. Soviet Jorn.of Nucl.Phys.v.46(1987)576

[5] E.Z. Avakyan,et al. Soviet Jorn.of Nucl.Phys.v.49(1989)1398

[6] Nasreddinov C.Soviet Jorn.of Nucl.Phys.v.579(1994)1141

[7] Golutvin A. Tau Decays DESY 94-168

[8] Particle Data Group. Phys. Rev. 1994.V. D50.