

Interactions of η, η' - Mesons with Heavy Quarks

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Abstract

The η, η', η_c system was investigated in the $q\bar{q} - s\bar{s} - c\bar{c}$ basis. Numerical values for mixing angles φ, θ_c and θ_y were fixed by the experimental data of two photon η -decays. The decay constants of radiative $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ ($P \equiv \eta, \eta'$; $V \equiv \rho, \omega, \phi$) decays were calculated and turned out to be in agreement with experimental data.

1 Introduction.

The investigation of simple quark-antiquark systems such as pseudoscalar mesons π^0, η, η' ... is of extraordinary interest as a source of information about structure of hadrons. CLEO [1] and L3[2]experiments call the additional interest to this phenomena. It's well known that SU(3)-symmetry predicts the existence of massless pseudoscalar octet $\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - s\bar{s})$ and massive singlet $\eta_0 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$. Physical states η, η' are the mixture of η_8 and η_0 . The study of $\eta - \eta'$ mixing is very important for understanding of basic properties of quark - hadron matter. This phenomena was considered in different approaches [3]. Also there exists approach connected with quark basis $q\bar{q} = \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}, s\bar{s}$ [4]. In this case decay constant mixing is considered to be the same as for meson states. There are several ways to define $\eta - \eta'$ mixing parameters:

- The radiative decays of η -mesons [4].
- The radiative J/Ψ - decays [5]

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- Ratios of weak decays constants [3].

Very recently Kloe Collaboration has reported new measurement of the ratio $R = Br(\phi \rightarrow \eta'\gamma)/Br(\phi \rightarrow \eta\gamma)$ [6]. So the subject of admixture of heavy quark or gluonic components in light pseudoscalar mesons has regained interest. Early investigations of this subject already date back to the 70-th, when Kramer [7] and independently Fritzsche and Jackson [8] discussed mixing of light and heavy pseudoscalar mesons in the context of radiative J/Ψ - decays. The rather extensive phenomenological analysis of mixing parameters of $\eta - \eta' - \eta_c$ system have been performed by Chao [5]. There is number of approaches where authors try to explain existing experimental data by admixture of gluonic states (the recent one is [9]).

In this article we study the mixing in the $\eta - \eta' - \eta_c$ system. The analysis of mixing parameters can be performed by study two-photon decays of η, η', η_c and mesons.

There is a problem in is the evaluation of the hadronic matrix elements. In our previous work [10] we perform the calculations in the Quark Confinement Model (QCM) [11]. But the heavy quarks interactions cannot be described in the the framework of this model. So in the present work we use Relativistic Constituent Quark Model (RCQM)[12].

2 The description of η mixing

In order to quantify the mixing in the η, η' system, one have to define appropriate mixing parameters, which can be related to physical observables.

The octet-singlet mixing is based on chiral perturbation theory which traditionally leads to description of η, η' mixing in terms of singlet-octet parameters [3]. In this case the physical states η and η' are the mixture of massive singlet $\eta_0 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$ and massless octet $\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - s\bar{s})$:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix} \quad (1)$$

Mixing in the quark basis (QBM). The parametrization of the decay constants look simpler in another basis, where the two independent axial-vector currents are taken as [4]

$$\begin{aligned} J_{\mu 5}^q &= \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d) \\ J_{\mu 5}^s &= \bar{s}\gamma_\mu\gamma_5 s \end{aligned} \quad (2)$$

In this scheme η, η' mixing is defined as

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \varphi - \sin \varphi \\ \sin \varphi \cos \varphi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} \quad (3)$$

where $\eta_q = \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}, \eta_s = s\bar{s}$.

The numerical value for mixing angle φ varies as $\varphi = 30^\circ \div 45^\circ$ in different approaches [3].

This scheme can be generalized to the $q\bar{q} - s\bar{s} - c\bar{c}$ basis. One can assume behavior for decay constants of $\eta - \eta' - \eta_c$ to be the similar that one in $q\bar{q} - s\bar{s}$ basis. Then we can write:

$$\begin{pmatrix} f_\eta^q & f_\eta^s & f_\eta^c \\ f_{\eta'}^q & f_{\eta'}^s & f_{\eta'}^c \\ f_{\eta_c}^q & f_{\eta_c}^s & f_{\eta_c}^c \end{pmatrix} = U(\varphi, \theta_y, \theta_c) \text{diag}(f_q, f_s, f_c) \quad (4)$$

where $\eta_c = c\bar{c}$. The transformation matrix now involves three angles:

$$U(\varphi, \theta_y, \theta_c) = \begin{pmatrix} \cos \varphi & -\sin \varphi & -\theta_c \sin \theta_y \\ \sin \varphi & \cos \varphi & \theta_c \cos \theta_y \\ -\theta_c \sin(\varphi - \theta_y) & -\theta_c \cos(\varphi - \theta_y) & 1 \end{pmatrix} \quad (5)$$

There terms of order θ_c^2 were neglected, since the mixing between $\eta - \eta'$ and η_c is an effect of the order of $\frac{1}{M_{\eta_c}^2}$. So we have

$$UU^\dagger = 1 + O(\theta_c^2)$$

The two new mixing angles are related with decay constants of $c\bar{c}$

$$f_\eta^c = -f_{\eta_c} \theta_c \sin \theta_y; \quad f_{\eta'}^c = f_{\eta_c} \theta_c \cos \theta_y$$

3 Two-photon decays and η, η' mixing parameters.

The constant of two-photon decay $P \rightarrow \gamma\gamma$ is a very important source of information about η, η' mixing. We can use the experimental data about η, η' decays to fix numerical values of mixing angles φ, θ_c and θ_y .

Analytical expressions for radiative η, η' decay constants in the case of $q\bar{q} - s\bar{s}$ was obtained in QCM [11] both in OSM and QBM in our previous work [10].

In the QBM approach the decay constants were received [10] as

$$g_{\eta\gamma\gamma}(\varphi) = \frac{\sqrt{3h_{\eta}(\varphi)}}{\pi} \left(\frac{1}{\sqrt{2}} \cdot \frac{5}{9} \cdot F_{PVV}(m_{\eta}^2, 0, 0, \Lambda_n) \cdot \cos \varphi - \frac{1}{9} \cdot F_{PVV}(m_{\eta}^2, 0, 0, \Lambda_s) \cdot \sin \varphi \right) \quad (6)$$

$$g_{\eta'\gamma\gamma}(\varphi) = \frac{\sqrt{3h_{\eta'}(\varphi)}}{\pi} \left(\frac{1}{\sqrt{2}} \cdot \frac{5}{9} \cdot F_{PVV}(m_{\eta'}^2, 0, 0, \Lambda_n) \cdot \sin \varphi + \frac{1}{9} \cdot F_{PVV}(m_{\eta'}^2, 0, 0, \Lambda_s) \cdot \cos \varphi \right) \quad (7)$$

The structure integral $F_{PVV}(m^2, 0, 0, \Lambda)$ in (6),(7) arise from triangle diagram with $i\gamma_5, \gamma^{\mu}, \gamma^{\nu}$ vertexes.

In the case of $q\bar{q} - s\bar{s} - c\bar{c}$ basis $g_{\eta\gamma\gamma}, g_{\eta'\gamma\gamma}$ and $g_{\eta_c\gamma\gamma}$ are written as:

$$g_{\eta\gamma\gamma} = \frac{\sqrt{3h_{\eta}(\varphi, \theta_c, \theta_y)}}{\pi} \left(\frac{1}{\sqrt{2}} \frac{5}{9} g_{P_{q\bar{q}} \rightarrow \gamma\gamma}(m_{\eta}^2, \varphi, \theta_c, \theta_y) \cos \varphi - \frac{1}{9} g_{P_{s\bar{s}} \rightarrow \gamma\gamma}(m_{\eta}^2, \varphi, \theta_c, \theta_y) \sin \varphi - \frac{5}{9} \theta_c \sin \theta_y g_{P_{c\bar{c}} \rightarrow \gamma\gamma}(m_{\eta}^2, \varphi, \theta_c, \theta_y) \right) \quad (8)$$

$$g_{\eta'\gamma\gamma} = \frac{\sqrt{3h_{\eta'}(\varphi, \theta_c, \theta_y)}}{\pi} \left(\frac{1}{\sqrt{2}} \frac{5}{9} g_{P_{q\bar{q}} \rightarrow \gamma\gamma}(m_{\eta'}^2, \varphi, \theta_c, \theta_y) \sin \varphi + \frac{1}{9} g_{P_{s\bar{s}} \rightarrow \gamma\gamma}(m_{\eta'}^2, \varphi, \theta_c, \theta_y) \cos \varphi + \frac{5}{9} \theta_c \cos \theta_y g_{P_{c\bar{c}} \rightarrow \gamma\gamma}(m_{\eta'}^2, \varphi, \theta_c, \theta_y) \right) \quad (9)$$

$$g_{\eta_c\gamma\gamma} = \frac{\sqrt{3h_{\eta_c}(\varphi, \theta_c, \theta_y)}}{\pi} \left(-\frac{1}{\sqrt{2}} \frac{5}{9} g_{P_{q\bar{q}} \rightarrow \gamma\gamma}(m_{\eta}^2, \varphi, \theta_c, \theta_y) \theta_c \sin(\varphi - \theta_y) + \frac{1}{9} g_{P_{s\bar{s}} \rightarrow \gamma\gamma}(m_{\eta}^2, \varphi, \theta_c, \theta_y) \theta_c \cos(\varphi - \theta_y) + \frac{5}{9} g_{P_{c\bar{c}} \rightarrow \gamma\gamma}(m_{\eta}^2, \varphi, \theta_c, \theta_y) \right) \quad (10)$$

The $g_{P_{q\bar{q}}}, g_{P_{s\bar{s}}}, g_{P_{c\bar{c}}}$ are the constants of two photon decays of P-states with the corresponding quark contents. The QCM failed in description of heavy quarks, so in the present work we use the Relativistic Constituent Quark Model (RCQM)[12] for description of the hadronic interactions of η mesons.

4 Relativistic Constituent Quark Model

The hadronic interactions will be described in the Relativistic Constituent Quark Model (RCQM)[12]. This model is based on the effective interaction Lagrangian which describes the coupling of hadrons with their constituent quarks:

$$L_{int}(x) = g_H H(x) \int dx_1 \int dx_2 \Phi_H(x, x_1, x_2) \bar{q}(x_1) \Gamma_H \lambda_H q(x_2). \quad (11)$$

Here λ_H and Γ_H are Gell-Mann and Dirac matrices, respectively, which entail the flavor and spin quantum numbers of the hadron H . The function Φ_H is related to the scalar part of Bete-Salpeter amplitude and characterizes the finite size of hadron. In order to provide Lorence invariance of Lagrangian (11) Φ_H have to be invariant under transition $\Phi_H(x + a, x_1 + a, x_2 + a) = \Phi_H(x, x_1, x_2)$.

In the case of an arbitrary pair of quarks with different masses Φ_H is given by

$$\Phi_H(x, x_1, x_2) = \delta(x - \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}) f((x_1 - x_2)^2). \quad (12)$$

The choice of vertex function $f((x_1 - x_2)^2)$ will be specified after transition to momentum space.

Let us consider meson mass function, defined by the diagram in Fig.1.

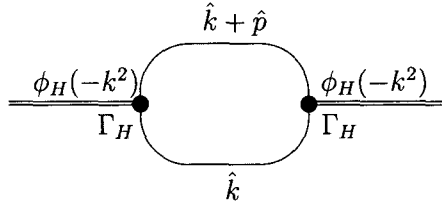


Fig.1

$$\Pi_H(x-y) = \int dx_1 \int dx_2 \Phi_H(x, x_1, x_2) \int dy_1 \int dy_2 \Phi_H(y, y_1, y_2) \cdot \text{Tr}\{S(y_1-x_1)\Gamma_H S(x_2-y_1)\Gamma_H\}. \quad (13)$$

The Fourier-transform of meson mass-function (13) is

$$\begin{aligned} \tilde{\Pi}_H(p) &= \int e^{-ipx} \Pi_H(x) dx = \\ &= \int \frac{dq}{(2\pi)^4} \int \frac{dk_1}{(2\pi)^4} \int \frac{dk_2}{(2\pi)^4} \tilde{\Phi}_H(-p, k_1, k_2) \tilde{\Phi}_H(q, -k_1, k_2) \text{Tr}\{S(\hat{k}_1)\Gamma_H S(\hat{k}_2)\Gamma_H\}. \end{aligned}$$

and finally can be written as

$$\tilde{\Pi}_H(p) = \int \frac{dk}{(2\pi)^4} \phi_H^2(k, p) \text{Tr}\{S(\hat{k} + \hat{p})\Gamma_H S(\hat{k})\Gamma_H\} \quad (14)$$

We assume that the vertex function ϕ_H depends only on the loop momentum k . The function ϕ_H is assumed to be the analytical function which decreases sufficiently fast in the Euclidean momentum space. In this work we employ a Gaussian form for the vertex function $\phi_H(k) = \exp(-k^2/\Lambda_H^2)$. The size parameters Λ_H were determined by fit to experimental data or lattice simulations [13]. We use local quark propagators

$$S_i(\hat{k}) = \frac{1}{m_i - \hat{k}} \quad (15)$$

where m_i is the constituent quark mass. As it discussed in [12], we assume that $m_H < m_{q_1} + m_{q_2}$ in order to avoid the appearance the of imaginary parts in the physical amplitudes. The fit values for the constituent quark masses are taken from [13] and are given as

m_u	m_s	m_c
0.235 (GeV)	0.333 (GeV)	1.67 (Gev)

The coupling constants g_H are determined by the co called compositeness condition [14] and had been used in QCM [11]. It means that the renormalization constant of the meson field is equal to zero

$$Z_M = 1 - \frac{3g_H^2}{4\pi^2} \tilde{\Pi}_H(m_H^2) = 0, \quad (16)$$

where $\tilde{\Pi}'_H(p)$ is the derivative of the mass function (14). It is convenient to use interaction constant in a form:

$$h_H = \frac{3g_H^2}{4\pi^2} = \frac{1}{\tilde{\Pi}'_H(m_H^2)} \quad (17)$$

instead of g_H in the further calculations.

5 Mixing parameters of η, η', η_c - mesons in $q\bar{q} - s\bar{s} - c\bar{c}$ basis.

As it was mentioned above, one has the opportunity to fix the η, η', η_c mixing angles by using the experimental data about the two photon decays of this mesons.

The experimental values of widths of this decays are

$$\begin{aligned} W(\eta \rightarrow \gamma\gamma) &= (0.46 \pm 0.04) \text{KeV} [15] \\ W(\eta' \rightarrow \gamma\gamma) &= (4.27 \pm 0.19) \text{KeV} [15] \\ W(\eta_c \rightarrow \gamma\gamma) &= (26.97 \pm 2.97) \text{KeV} [16] \end{aligned} \quad (18)$$

The matrix element of $\eta, \eta', \eta_c \rightarrow \gamma\gamma$ is defined by the diagram in Fig.2 and can be written as

$$A(P \rightarrow \gamma\gamma) = e^2 g_{P\gamma\gamma} \varepsilon^{\mu\nu\alpha\beta} \varepsilon^\mu(q_1) \varepsilon^\nu(q_2). \quad (19)$$

$g_{P\gamma\gamma}$ are just the decay constants from (8)-(10).

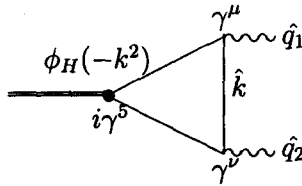


Fig.2

In RCQM $g_{Pq\bar{q}}, g_{Ps\bar{s}}, g_{Pc\bar{c}}$ from (8)-(10) are defined by loop integral

$$g_{Pq\bar{q} \rightarrow \gamma\gamma} = \int \frac{d^4k}{4\pi^2 i} \phi_P\left(-\frac{k^2}{\Lambda_P^2}\right) \text{Tr}[\gamma^5 S_q((\hat{k} - \hat{q}_2) \gamma^\mu S_q((\hat{k}) \gamma^\nu S_q((\hat{k} + \hat{q}_1))] \quad (20)$$

Using integration technique in detailed described in [16], one then arrives at the following analytical representation of $g_{P_{q_i \bar{q}_i}}$ ($q_i = q, s, c$)

$$g_{P_{q_i \bar{q}_i} \rightarrow \gamma\gamma} = m_{q_i} \int_0^\infty dt \left(\frac{t}{t+1} \right)^2 \int_0^1 d^3\alpha \delta(1 - \sum_{i=1}^3 \alpha_i) (-\phi'_P(z_0)), \quad (21)$$

where

$$z_0 = t(m_{q_i}^2 - \alpha_1 \alpha_2 p^2) - \frac{t}{1+t} \alpha_1 \alpha_2 p^2$$

The coupling constants $h_\eta, h_{\eta'}, h_{\eta_c}$ are defined by (17), with $\Pi_{\eta, \eta'}$

$$\Pi_\eta(m_\eta^2) = \Pi(m_\eta^2, m_q) \cos^2 \varphi + \Pi(m_\eta^2, m_s) \sin^2 \varphi + \Pi(m_\eta^2, m_c) \theta_c^2 \sin^2 \theta_y, \quad (22)$$

$$\Pi_{\eta'}(m_{\eta'}^2) = \Pi(m_{\eta'}^2, m_q) \sin^2 \varphi + \Pi(m_{\eta'}^2, m_s) \cos^2 \varphi + \Pi(m_{\eta'}^2, m_c) \theta_c^2 \cos^2 \theta_y \quad (23)$$

$$\Pi_{\eta_c}(m_{\eta_c}^2) = \Pi(m_{\eta_c}^2, m_q) (\theta_c \sin(\varphi - \theta_y))^2 + \Pi(m_{\eta_c}^2, m_s) (\theta_c \cos(\varphi - \theta_y))^2 + \Pi(m_{\eta_c}^2, m_c), \quad (24)$$

Mass functions $\Pi(p^2, m_q)$ in the case of pseudoscalars can be written according to (14)

$$\Pi(p^2, m_q) = \int \frac{dk}{(2\pi)^4} \phi_P^2 \left(-\frac{k^2}{\Lambda_P^2} \right) Tr \{ S_q(\hat{k} + \hat{p}) i\gamma^5 S_q(\hat{k}) i\gamma^5 \} \quad (25)$$

The derivative of mass function $\Pi(p^2, m_q)$ can be received as

$$\frac{d}{dp^2} \Pi(p^2, m_q) |_{p^2=m_H^2} = \frac{1}{2} \int_0^\infty dt \left(\frac{t}{t+1} \right)^2 \int_0^1 d\alpha F(\alpha, t) \quad (26)$$

where

$$F(\alpha, t) = \phi_H^2(z) \frac{1}{1+t} (4 - 3 \frac{\alpha t}{1+t}) - (\phi_H^2(z))' \{ 2m_q^2 + \frac{\alpha t}{1+t} [m_H^2 - m_q^2] - m_H^2 (\frac{\alpha t}{1+t})^2 (2 - \frac{\alpha t}{1+t}) \}$$

with

$$z = t(m_q^2 - \alpha(1-\alpha)m_H^2) - \frac{\alpha^2 t}{1+t} m_H^2 \quad (27)$$

The decay width of two photon decay of pseudoscalar meson is written in standard form

$$W(P \rightarrow \gamma\gamma) = \frac{\pi}{4} m_P^3 \alpha^2 g_{P\gamma\gamma}^2 \quad (28)$$

To define mixing angles we have calculated $g_{P \rightarrow \gamma\gamma}$ for η, η', η_c as described above and have used the experimental values for $g_{\eta, \eta', \eta_c \rightarrow \gamma\gamma}^{exp}$ extracted from (18) by (28). So, the following numerical values were received

$$\varphi = 33.1^\circ; \theta_c = -1.1^\circ; \theta_y = 51.3^\circ \quad (29)$$

Now using the above values for the mixing angles φ, θ_y and θ_c we can find for quark content of the physical mesons

$$\begin{aligned} \eta &= 0.72\eta_q - 0.65\eta_s - 0.005\eta_c \\ \eta' &= 0.65\eta_q + 0.72\eta_s - 0.016\eta_c \\ \eta_c &= 0.012\eta_q + 0.009\eta_s + 0.005\eta_c \end{aligned} \quad (30)$$

6 Radiative decays of η, η' mesons.

Let us calculate the constants of $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ ($P \equiv \eta, \eta'; V \equiv \rho, \omega, \phi$) decays using the mixing parameters received in previous section.

The decay amplitudes are defined as

$$A(V \rightarrow P\gamma) = eg_{VP\gamma} \varepsilon^{\mu\nu\alpha\beta} \varepsilon^\mu(p_V) \varepsilon^\nu(q_\gamma) q_\gamma^\alpha p_V^\beta \quad (31)$$

$$A(P \rightarrow V\gamma) = eg_{PV\gamma} \varepsilon^{\mu\nu\alpha\beta} \varepsilon^\mu(q_\gamma) \varepsilon^\nu(p_V) q_\gamma^\alpha p_V^\beta \quad (32)$$

and the corresponding decay widths are

$$W(P \rightarrow V\gamma) = m_V^3 \alpha g_{PV\gamma}^2 \quad (33)$$

$$W(V \rightarrow P\gamma) = \frac{1}{3} m_P^3 \alpha g_{VP\gamma}^2 \quad (34)$$

The following expressions for decay constants were received :

$$g_{\eta\rho\gamma} = \sqrt{h_\rho h_\eta(\varphi, \theta_c, \theta_y)} \cos \varphi g_{PV\gamma}(m_\eta^2, m_\rho^2, 0, m_q) \quad (35)$$

$$g_{\eta'\rho\gamma} = \sqrt{h_\rho h_{\eta'}}(\varphi, \theta_c, \theta_y) \sin \varphi g_{PV\gamma}(m_{\eta'}^2, m_\rho^2, 0, m_q) \quad (36)$$

$$g_{\eta\omega\gamma} = \sqrt{h_\omega h_\eta(\varphi, \theta_c, \theta_y)} \cos \varphi \frac{1}{3} g_{PV\gamma}(m_\eta^2, m_\omega^2, 0, m_q) \quad (37)$$

$$g_{\eta'\omega\gamma} = \sqrt{h_\omega h_{\eta'}}(\varphi, \theta_c, \theta_y) \sin \varphi \frac{1}{3} g_{PV\gamma}(m_{\eta'}^2, m_\omega^2, 0, m_q) \quad (38)$$

$$g_{\eta\phi\gamma} = \sqrt{h_\phi h_\eta(\varphi, \theta_c, \theta_y)} \sin \varphi \frac{2}{3} g_{PV\gamma}(m_\eta^2, m_\phi^2, 0, m_s) \quad (39)$$

$$g_{\eta'\phi\gamma} = \sqrt{h_\phi h_{\eta'}(\varphi, \theta_c, \theta_y)} \cos \varphi \frac{2}{3} g_{PV\gamma}(m_{\eta'}^2, m_\phi^2, 0, m_s) \quad (40)$$

The analytical expression for function $g_{PV\gamma}(m_P^2, m_V^2, 0, m_q)$ in (35)- (40) can be received by evacuation of one loop integral as described in previous section. The numerical values for $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ decays are given in tables 1 and 2.

Table 1

$g_{V\eta\gamma}(GeV^{-1})$	Experiment [15]	$q\bar{q} - s\bar{s} - c\bar{c}$ basis with $\varphi = 33.1^\circ; \theta_c = -1.1^\circ; \theta_y = 51.3^\circ$
$g_{\rho\eta\gamma}$	$1.47 + 0.25 - 0.28$	1.51
$g_{\omega\eta\gamma}$	0.5 ± 0.04	0.51
$g_{\phi\eta\gamma}$	0.69 ± 0.02	0.72

Table 2

$g_{V\eta'\gamma}(GeV^{-1})$	Experiment [15]	$q\bar{q} - s\bar{s} - c\bar{c}$ basis with $\varphi = 33.1^\circ; \theta_c = -1.1^\circ; \theta_y = 51.3^\circ$
$g_{\rho\eta'\gamma}$	1.31 ± 0.06	1.24
$g_{\omega\eta'\gamma}$	0.45 ± 0.03	0.37
$g_{\phi\eta'\gamma}$	1.00 ± 0.28	0.79

One can see that numerical results are in agreement with experimental data.

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