

5. J. L. Hewett, Phys. Rev. Lett. **82**, 4765 (1999).
6. G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B **544**, 3 (1999).
7. T. Han, J. Lykken and R. Zhang, Phys. Rev. D **59**, 105006 (1999).
8. G. Aad et al. [ATLAS Collaboration], Phys. Rev. D **87**, 015010 (2013).
9. S. Chatrchyan et al. [CMS Collaboration], CMS-PAS-EXO-12-031 (2013).
10. P. Osland, A. A. Pankov and N. Paver, Phys. Rev. D **68**, 015007 (2003).
11. E. W. Dvergsnes, P. Osland, A. A. Pankov and N. Paver, Phys. Rev. D **69**, 115001 (2004).
12. P. Osland, A. A. Pankov, N. Paver, A.V. Tsytrinov, Phys. Rev. D **78**, 035008 (2008).
13. A. Gulov and V. Skalozub, Int. J. Mod. Phys. A **25**, 5787 (2010).

ADVANTAGES OF APT IN QCD STUDY OF HADRONIC TAU DECAYS

V.L. Khandramai, O.P. Solovstsova, V.G. Teplyakov

Gomel State Technical University,
246746 Gomel, Belarus

Introduction. The experimental data on the τ lepton decay into hadrons obtained with a record accuracy for hadronic processes [1, 2, 3] give a unique possibility for testing QCD at low energy scale. The τ lepton is the only lepton known at present whose mass, $M_\tau=1.78$ GeV, is large enough in order to produce decays with a hadronic mode. At the same time, in the context of QCD, the mass is sufficiently small to allow one to investigate perturbative and non-perturbative QCD effects. The theoretical analysis of the hadronic decays of a heavy lepton was performed by Tsai [4] before the experimental discovery of the τ lepton in 1975 and since then this process is intensively studied.

It is known, that perturbation theory (PT), which is a basic tool of calculations in quantum field theory, as a rule cannot be exhaustive in the low energy region of QCD. However, a structure of an initial perturbative approximation of some quantity is not a rigid construction fixed once and for all, but admits a considerable modification due to specific properties of the quantum field theory. Such modification is based on further information of a general character about the sum of the series. In particular,

the properties of renormalization-group (RG) invariance [5], which is lost in a finite order of the initial expansion, allow rearrangements of the perturbative series in terms of the invariant charge. In this case, the properties of the series change essentially. In distinction to the initial expression containing large logarithms, the expansion obtained within the RG method can be used for analyzing the ultraviolet region. However, the perturbative series so derived are ill-defined in the infrared region and the correct analytic properties of the series in the complex Q^2 -plane are violated due to unphysical singularities of the perturbative running coupling, a ghost pole in the one-loop approximation (see discussion in [6, 7]). The difficulty associated with these unphysical singularities is overcome in the analytic approach proposed by Shirkov and Solovtsov [8]. This approach modifies the perturbative expansion on the basis of general properties of the theory so that the new approximations reflect fundamental principles of the theory—renormalization invariance, spectrality, and causality. In the new expansion the correct analytic properties are restored, and the property of RG invariance is preserved [8]. Further developments and applications of the Shirkov–Solovtsov analytic approach have been considered in many papers (see [9] as review).

The original theoretical expression for the hadronic tau decay width $\Gamma(\tau^- \rightarrow \text{hadrons } \nu_\tau)$ involves integration over small values of timelike momentum [4]. The perturbative description with the standard PT running coupling becomes ill-defined in this region and some additional ansatz has to be applied to get a finite result for the hadronic width. To this end, one usually transforms the initial expression, by using Cauchy's theorem, to a contour representation for R_τ [10], which allows one to give meaning to the initial expression and, in principle, perform calculations in the framework of perturbative QCD. Assuming the validity of this transformation it is possible to present results in the form of a truncated power series with $\alpha_s(M_\tau)$ as the expansion parameter [11, 12]. There are also other approaches to evaluating the contour integral. The Le Diberder – Pich prescription [13] allows one to improve the convergence properties of the approximate series and reduce the renormalization scheme (RS) dependence of theoretical predictions. The possibility of using different approaches in the perturbative description of τ lepton decay leads to an uncertainty in the value of $\alpha_s(M_\tau)$ extracted from the experimental data. Moreover, any perturbative description is based on this contour representation, *i.e.*, on the possibility of converting the initial expression

involving integration over timelike momenta into a contour integral in the complex momentum plane. To carry out this transition by using Cauchy's theorem requires certain analytic properties of the hadronic correlator or of the corresponding Adler function [14]. However, the occurrence of incorrect analytic properties in the conventional PT approximation makes it impossible to exploit Cauchy's theorem in this manner and, therefore, prevents rewriting the initial expression for R_τ in the form of a contour integral in the complex momentum-plane. The method based on the Shirkov–Solovtsov analytic approach and called analytic perturbation theory (APT) [15] ensures the correct analytic properties of such important objects as the hadronic correlator or of the corresponding Adler function, leads to equality between the initial theoretical expression for the width $\Gamma(\tau^- \rightarrow \text{hadrons } \nu_\tau)$ and the corresponding contour representation.

The aim of this paper is to reveal features of the application of the PT and APT expansions in studying of hadronic tau decays. We also consider the Adler function which is connected to the correlator of quark-antiquark current and can be used in a description of normalized hadronic decays width of the tau lepton, R_τ .

Analytic perturbation theory. A main object in the description of hadronic tau decays and of many other physical processes is a correlator $\Pi(q^2)$ or the Adler function $D(Q^2)$, which is connected to the correlator by the formula

$$D(Q^2) = -Q^2 \frac{d \Pi(-Q^2)}{d Q^2}. \quad (1)$$

We use the standard convention $Q^2 = -q^2 > 0$ in the Euclidean region. The integral representation for the D -function is given in terms of the function $R(s) \equiv \text{Im } \Pi(s)/\pi$:

$$D(Q^2) = \int_0^\infty \frac{R(s)}{(s+Q^2)^2} ds. \quad (2)$$

The representation (2) defines the function $D(Q^2)$ as the analytic function in the complex Q^2 -plane with the cut along the negative real axis.

It is convenient to separate QCD contributions, $d(Q^2)$ and $r(s)$, in the functions $D \propto 1 + d$ and $R \propto 1 + r$, respectively, which are related by the formulae

$$d(Q^2) = \int_0^\infty \frac{ds}{(s+Q^2)^2} r(s), \quad (3)$$

$$r(s) = -\frac{1}{2\pi i} \int_0^{\infty} \frac{dz}{z} d(-z). \quad (4)$$

The integration contour in (4) lies in the region of analyticity of the integrand and encircles the cut of $d(-z)$ on the positive real z axis.

Within the framework of the APT the basic object is a spectral function $\rho(\sigma)$ which enters into some integral representation. In particular, for two-point functions, it is the Källén–Lehmann representation; whereas for structure functions for inelastic lepton–hadron scattering, the integral representation is the Jost–Lehmann–Dyson one [16]. The spectral function $\rho(\sigma)$ for the objects under consideration here can be obtained by using the perturbative series as a initial approach. Truncated at the fourth-loop level, the perturbative d -function, is written in terms of the perturbative running coupling is

$$d_{PT}(Q^2) = a_{PT}(Q^2) + d_1 a_{PT}^2(Q^2) + d_2 a_{PT}^3(Q^2) + d_3 a_{PT}^4(Q^2), \quad a_{PT}(Q^2) \equiv \frac{\alpha_{PT}(Q^2)}{\pi}, \quad (5)$$

where in the $\overline{\text{MS}}$ -scheme for three active quarks ($n_f=3$) relevant in τ decay, the expansion coefficients are $d_1^{\overline{\text{MS}}} = 1.6398$, $d_2^{\overline{\text{MS}}} = 6.3710$ and $d_3^{\overline{\text{MS}}} = 49.075$ [17]. This expansion generates the following approximation to the spectral function $\rho(\sigma)$:

$$\rho(\sigma) = \rho_1(\sigma) + d_1 \rho_2(\sigma) + d_2 \rho_3(\sigma) + d_3 \rho_4(\sigma) + \dots, \quad (6)$$

where the coefficients d_1 , d_2 and d_3 are the same as in the PT series (5) and the expansion functions are determined by the discontinuity of the corresponding power of the perturbative running coupling, $\rho_k(\sigma) = \text{Im}[a_{PT}^k(-\sigma - i\epsilon)]$.

By using the spectral function (6), we obtain the expression for d -function in the form of the non-power expansion

$$d_{APT}(Q^2) = A_{APT}^{(1)}(Q^2) + d_1 A_{APT}^{(2)}(Q^2) + d_2 A_{APT}^{(3)}(Q^2) + d_3 A_{APT}^{(4)}(Q^2), \quad (7)$$

where $A_{APT}^{(k)}(Q^2)$ are analytic functions:

$$A_{APT}^{(k)}(Q^2) = \frac{1}{\pi} \int_0^{\infty} d\sigma \frac{\rho_k(\sigma)}{\sigma + Q^2}. \quad (6)$$

Note here that the analytic running coupling $\alpha_{APT}(Q^2) \equiv \pi A_{APT}^{(1)}(Q^2)$ (see [18] for details).

A significant source of theoretical uncertainty arises from the Renormalization Scheme (RS) dependence of the results obtained due to the inevitable inclusion of only a finite number of terms in the PT series. In QCD, that uncertainty is the greater, than smaller a value of typical energy of the process. There are no general principles that give preference to a

particular RS, and in this sense, all schemes are equivalent. The APT method improves this situation and gives very stable results over a wide range of RS. To demonstrate this fact, in Fig. 1 we plot functions $d_{PT}(Q^2)$ and $d_{APT}(Q^2)$ in different RS. It is seen that predictions in the perturbative approach for $d(Q^2)$ obtained within different RS diverge considerably (see dashed curves A and B). Note should be made of the fact that the schemes A and B are similar to each other and to the optimal PMS [19] and ECH [20] schemes in the sense of the cancellation index [21]: $C_A = C_B = 2$. For the ECH method, the cancellation index is minimal, equaling unity. The cancellation index for the $\overline{\text{MS}}$ scheme turns out to be somewhat bigger, $C_{\overline{\text{MS}}} = 3.1$.

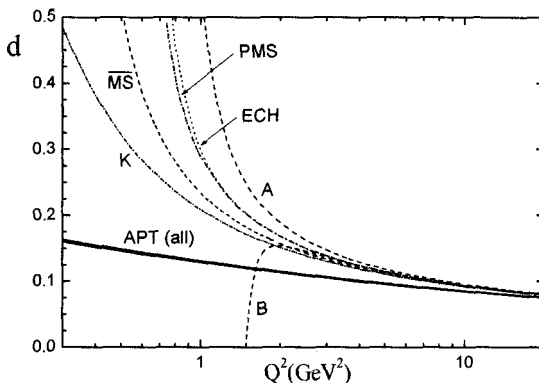


Figure 1 – Renormalization scheme dependence of the d -function (3) as a function of Q^2 in the PT (6) and APT (7) cases. The APT results are shown as solid lines which are very close to each other and practically merge into one curve.

In Fig. 1, we also draw the curves representing PT results in PMS, ECH, $\overline{\text{MS}}$ and K schemes. For the same schemes, in Fig. 1 we also present results obtained in the APT approach. In this case the scheme arbitrariness is extremely small, and all the curves corresponding to the schemes A, B, PMS, ECH, $\overline{\text{MS}}$, and K merge into one thick solid curve. Thus, in the APT, the scheme arbitrariness is very dramatically reduced as compared to that in analogous PT calculations.

The tau ratio R_τ . The total hadronic decays width of the tau lepton is given by difference of its total width and the partial widths for the electronic and muonic decays. In an analogy to well-known Drell-ratio for the e^+e^- annihilation into hadrons, one can define the R_τ -ratio. The theoretical expression for R_τ can be presented as

$$R_\tau = 3 (|V_{ud}|^2 + |V_{us}|^2) S_{EW} (1 + \delta_\tau), \quad (11)$$

where V_{ud} and V_{us} are elements of the CKM quark mixing matrix, S_{EW} is the electroweak factor, and the QCD contribution, δ_τ , is expressed via the function $r(s)$ as follows

$$\delta_\tau(Q^2) = 2 \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left(1 + 2 \frac{s}{M_\tau^2}\right) r(s). \quad (12)$$

This expression is a starting point in our analysis. Within the PT, the integral (12) cannot be evaluated directly due to unphysical singularities of the PT running coupling lying in the range of integration. The most useful trick to rescue the situation is to appeal to analytic properties of the correlator $\Pi(q^2)$. The relations between the functions $r(s)$ and $d(Q^2)$, according Eqs. (3) and (4), allow us to represent δ_τ as a contour integral in the complex z plane by choosing the contour to be a circle of radius $|z| = M_\tau^2$ [11]

$$\delta_\tau(Q^2) = \frac{1}{2\pi i} \int_{|z|=M_\tau^2} \frac{dz}{z} \left(1 - \frac{z}{M_\tau^2}\right)^3 \left(1 + 2 \frac{z}{M_\tau^2}\right) d(-z). \quad (13)$$

It would seem that the transformation to the contour representation (13) allows one to avoid this difficulty, since in this case unphysical singularities of the running coupling lie outside of the contour, and the procedure of integration can formally be easily accomplished. However, in our opinion, this trick ("sweeping the difficulty under the rug") does not mean solve the problem. Actually, incorrect analytic properties of the running coupling result in Eqs. (12) and (13) for δ_τ being no longer equivalent [14, 22], and, if one remains within the PT, nothing can be said about the errors introduced by this transition. The APT may eliminate these problems.

The PT description for the δ_τ is based on the contour representation (13) and can be developed in the following two ways. In the Braaten's (Br) method [12] the quantity (13) is represented in the form of truncated power

series with the expansion parameter a_τ . In this case the three-loop representation for δ_τ is

$$\delta_\tau Br = a_\tau + r_1 a_\tau^2 + r_2 a_\tau^3 + r_3 a_\tau^4, \quad (14)$$

$$a_\tau = \alpha_{pt}(M_\tau^2)/\pi,$$

where coefficients r_1, r_2 and r_3 in the \overline{MS} scheme with three active flavors are $r_1=5.2023, r_2=26.366$ and $r_3=127.079$ [17].

The method proposed by Le Diberder and Pich (LP) [13] uses the PT expansion of the d -function (5). It results to the following non-power representation

$$\delta_\tau LP = A(1)(a_\tau) + d_1 A(2)(a_\tau) + d_2 A(3)(a_\tau) + d_3 A(4)(a_\tau) \quad (15)$$

with

$$A^{(n)}(a) = \frac{1}{2\pi i} \int_{|z|=M_\tau^2} \frac{dz}{z} \left(1 - \frac{z}{M_\tau^2}\right)^3 \left(1 + 2\frac{s}{M_\tau^2}\right) a^{(n)}(z). \quad (16)$$

Both these PT approaches are widely used in the analysis of τ -decay data. However, their status is different. The formula (14) can be obtained self-consistently. In expression (12) one has to use for $r(s)$ the initial perturbative approximation with the expansion parameter a_μ . Then, after integration over s , the logarithmic terms containing $\ln(M_\tau^2/\mu^2)$ are removed by setting $\mu^2 = M_\tau^2$. The same result is obtained if the contour representation (13) is used and the d -function is taken in the form the initial perturbative approximation which preserves the required analytic properties. As for the representation (15), it will be consistent with expressions (12) and (13), if $a(z)$ has analytic properties of the Källén–Lehmann type. The use of the standard PT running coupling with unphysical singularities in (16) breaks this consistency.

The APT description can be equivalently phrased either on the basis of the original expression (12), which involves the Minkowskian quantity $r(s)$, or on the contour representation (13), which involves the Euclidean quantity $d(Q^2)$. Within the framework of the APT approach, both forms can be rewritten in terms of the spectral function $\rho(\sigma)$ as [15]

$$\delta_\tau(Q^2) = \frac{1}{\pi} \int_0^\infty \frac{d\sigma}{\sigma} \rho(\sigma) - \frac{1}{\pi} \int_0^{M_\tau^2} \frac{d\sigma}{\sigma} \left(1 - \frac{\sigma}{M_\tau^2}\right)^3 \left(1 + \frac{\sigma}{M_\tau^2}\right) \rho(\sigma). \quad (17)$$

In Fig. 2, we illustrate the dependence of the R_τ -ratio on the running coupling in the PT(Br) and APT approaches, comparing the convergence

properties in the one-loop (dotted lines), two-loop (dashed lines), and three-loop (solid lines) approximations. Numbers above the curves specify the order of the approximation. The shaded area shows the corridor of experimental errors for $R_\tau^{expt}=3.475\pm 0.011$ [23]. The convergence properties of the APT expansion seem to be much improved compared to those of the PT expansions.

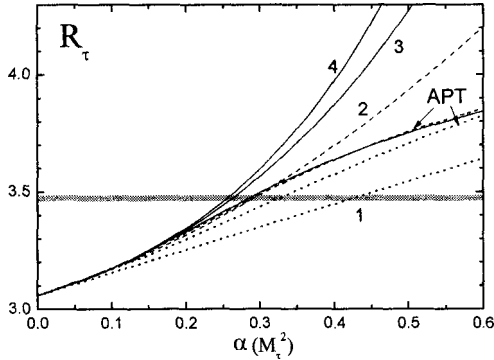


Figure 2: The PT(Br) and APT predictions for the R_τ ratio vs. the running coupling in the \overline{MS} scheme. The numbers labelling the curves denote the level of the loop expansion used.

Our investigation together with other results (see [24]), allows us to formulate the following features of the APT method: (i) this approach maintains the correct analytic properties and leads to a self-consistent procedure of analytic continuation from the spacelike to the timelike region; (ii) it has much improved convergence properties and turns out to be stable with respect to higher-loop corrections; (iii) renormalization scheme dependence of the results obtained within this method is reduced dramatically. Note, for the hadronic τ decay the last statement is easy for understanding if one takes into account the result which is shown in Fig. 1 for d -functions in different RS: instead of RS unstable and rapidly changing PT results, the APT predictions are practically RS independent.

Conclusions. The analytic approach proposed by Shirkov and Solovtsov modifies the perturbative expansions such that the new

approximations reflect basic principles of the theory, such as renormalization invariance, spectrality, and causality. Analytic perturbation theory, which was used here in the hadronic τ decay description, gives a self-consistent description of both the spacelike and timelike regions.

We performed a comparative analysis of the advantages and disadvantages of different forms of perturbative expansion both from the general standpoint and in the context of application to the inclusive τ decay. We presented the arguments in favor of the APT, which not only agrees with the general principles of the theory but also has a number of practical advantages. In the analytic approach, the two methods for describing the inclusive τ lepton decay in terms of timelike or spacelike variables are equivalent.

Within the APT, the dependence of the results on the choice of the renormalization prescription is essentially reduced, and we can speak of the practical independence of the two-loop expressions from the renormalization scheme. The calculations based on the APT thus considerably reduce the theoretical uncertainty of the results. Therefore, using it as the perturbative component increases the reliability of information about the QCD parameters obtained from the experimental data known with high accuracy for the τ lepton decay.

Acknowledgments. It is a pleasure for the authors to thank D. V. Shirkov and O. V. Teryaev for interest in this work and helpful discussions. This research was supported by the JINR-BelRFFR grants F14D-007 and F15RM-072.

References

1. Barate R. *et al.* (ALEPH Collab.), Eur. Phys. J. C. – 1998. – Vol. 4. – P. 409.
2. Ackersta K. *et al.* (OPAL Collab.), Eur. Phys. J. C. – 1999. – Vol. 7. – P. 571.
3. Schael S. *et al.* (ALEPH Collab.), Phys. Rept. – 2005. – Vol. 421. – P. 191.
4. Tsai Y.S., Phys. Rev. D – 1971. – Vol. 4. – P. 2821.
5. Bogoliubov N. N., Shirkov D. V., Introduction to the Theory of Quantized Fields – Wiley, New York – 1959 and 1980.
6. Jones H. F. and Solovtsov I. L., Phys. Lett. B. – 1995. – Vol. 349. – P. 519.

7. Milton K. A. and Solovtsova O. P., Phys. Rev. D. – 1998. – Vol. 57. – P. 5402.
8. Shirkov D. V. and Solovtsov I. L., Phys. Rev. Lett. – 1997. – Vol. 79. – P. 1209.
9. Shirkov D. V. and Solovtsov I. L., Theor. Math. Phys. – 2007. – Vol. 150. – P. 132.
10. Lam C. S. and Yan T. M., Phys. Rev. D – 1977. – Vol. 16. – P. 703.
11. Braaten E., Phys. Rev. Lett. – 1988. – Vol. 60 – P. 1606.
12. Braaten E., Narison S., and Pich A., Nucl. Phys. B. – 1992. – Vol. 373. – P. 581.
13. Le Diberder F. and Pich A., Phys. Lett. B. – 1992. – Vol. 286. – P. 147.
14. Adler S. L., Phys. Rev. B. – 1974. – Vol. 10. – P 3714.
15. Milton K. A., Solovtsov I. L. and Solovtsova O. P., Phys. Lett. B – 1997. – Vol. 415. –P. 104.
16. Jost R., Lehmann H., Nuovo Cimento.–1957.–Vol .5.– P. 1598.
17. Baikov P. A., Chetyrkin K. G., Kuhn J. H., Phys. Rev. Lett. – 2008. – Vol. 101. – P. 012002.
18. Milton K. A., Solovtsov I. L., and Solovtsova O. P., Phys. Rev. D – 2001 – Vol. 64. – P. 016005.
19. Stevenson P. M., Phys. Rev. D. – 1981 – Vol. 23. – P. 2916.
20. Grunberg G., Phys. Rev. D. – Vol. 29. – P. 2315.
21. Raczka P. A., Z. Phys. C – 1995 – Vol. 65. –P. 481.
22. Milton K. A. and Solovtsova O. P., Int J. Mod. Phys. A. – 2002 – Vol. 26. – P. 3789.
23. Davier M., Höcker A., Malaescu B., Yuan C. Z. and Zhang Z., Eur. Phys. J. C – 2014 – Vol. 74, № 3. – P. 2803.
24. Milton K. A., Solovtsov I. L., Solovtsova O. P., and Yasnov V. I., Eur. Phys. J. C. – 2000 – Vol. 14. – P. 495.

MODELING OF VACUUM CIRCUIT BREAKER DETERIORATION PROCESS

Y. Tolstoguzov, V. Krotenok

Gomel State Technical University, Republic of Belarus

Introduction

Ensuring reliable operation of power plants, substations and industrial power systems is largely determined by trouble-free operation of high voltage circuit breakers.